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STABILITY AND CONTROL HANDBOOK FOR HELICOPTERS

By

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August 1967

U. S. ARMY AVIATION MATERIEL LABORATORIES
FORT EUSTIS, VIRGINIA

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BLUE BELL, PENNSYLVANIA

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This report has been reviewed by the U. S. Army Aviation Materiel Laboratories and is considered to be technically sound. The work was performed under Contract DA 44-177-AMC-197(T). The main objective was to update the Stability and Control Handoook for Helicopters, TREC Report 60-43, and to provide information required as a guide for preliminary design calculations of helicopter stability and control characteristics.

Analytical methods are presented for determining the dynamic stability and control characteristics of generalized helicopter configurations. This handbook also contains information suitable for extensive digital and analog computer studies for the inexperienced stability analyst and a good reference manual for the specialist.

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STABILITY AND CONTROL HANDBOOK FOR HELICOPTERS

Dynasciences Report No. DCR-186

by
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ABSTRACT

Analytical methods are presented for determining the dynamic stability and control characteristics of generalized helicopter configurations. The methods utilize calculation procedures which are considerably simplified through the extensive use of information presented in graphs and charts. These charts are applicable to flight conditions from hover to high forward speeds.

The charts for low forward speeds (advance ratios, $\mu \le 0.2$) were obtained from the rotor performance data based on classical rotor theory. However, the high-speed charts ($\mu \ge 0.3$) exclude the major assumptions of classical theory and include blade compressibility, stall, reverse flow, large inflow ratios, etc.

This handbook contains information suitable for extensive digital and analog computer studies and also provides rapid procedures for predicting helicopter stability and control characteristics for preliminary design applications.

FOREWORD

This handbook was prepared by the Dynasciences Corporation, Blue Bell, Pennsylvania for the U. S. Army Aviation Materiel Laboratories, Fort Eustis, Virginia under Contract No. DA 44-177-AMC-197(T), during the period from June 1964 through June 1967.

The work contained in this report incorporates recently available rotor performance and stability information and represents a revision and extension of the U. S. Army Stability and Control Handbook for Helicopters published as TRECOM Report TREC 60-43 in August 1960.

The Army technical representative was Mr. J. Yeates, who was assisted by Mr. R. Piper and Mr. R. P. Smith. The contributions of the Army personnel to this work are gratefully acknowledged. The following Dynasciences Corporation personnel contributed to this work:

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SYMBOLS

A, a_m	constants as defined in the text
AR	aspect ratio
$A_{X_{FUS}}$	fuselage frontal area, ft^2
$A_{Y_{FUS}}$	fuselage side area, ft^2
$A_{Z_{FUS}}$	fuselage planform area, ft^2
A_1	lateral cyclic tilt of the control axis relative to the shaft, positive to the right, rad
a	rotor blade lift curve slope, or lift curve slope of an aerodynamic surface
a_{mn}	element of a determinant, m^{th} row and n^{th} column
a_0	blade coning angle, rad
a_1	longitudinal flapping angle, positive when flapping up at front, rad
B, b_m	constants as defined in the text
B_T	tip loss factor (0.97)
B_1	longitudinal cyclic tilt of control axis relative to shaft, positive when forward, rad
b	number of blades
b_1	lateral flapping angle, positive when flapping down to the right, rad
C	constant as defined in the text
C_D	drag coefficient of an aerodynamic surface

C_D'	drag coefficient of a rotor $= D/T.F.$
$C_{D_{FUS}}$	fuselage drag coefficient $= D_{FUS}/\frac{1}{2}\rho V_c^2 A_{X_{FUS}}$
C_{D_O}	profile drag coefficient of an aerodynamic surface
C.G.	aircraft C.G. position
C_H	rotor H-force coefficient $= H/T.F.$
C_L	lift coefficient of an aerodynamic surface $= L/\frac{1}{2}\rho V_0^2 S$
C_L'	lift coefficient of a rotor $= L/T.F.$
$C_{L_{FUS}}$	fuselage lift coefficient $= L_{FUS}/\frac{1}{2}\rho V_0^2 A_{Z_{FUS}}$
$C_{\mathcal{L}_{FUS}}$	fuselage rolling moment coefficient $=\mathcal{L}_{FUS}/\frac{1}{2}\rho V_0^2 A_{X_{FUS}} \ell_{FUS}$
$C_{M_{FUS}}$	fuselage pitching moment coefficients $=M_{FUS}/\frac{1}{2}\rho V_0^2 A_{X_{FUS}} \ell_{FUS}$
$C_{N_{FUS}}$	fuselage yawing moment coefficient $=N_{FUS}/\frac{1}{2}\rho V_0^2 A_{X_{FUS}} \ell_{FUS}$
C_Q	rotor torque coefficient $= Q/(T.F.)R$
C_Y'	rotor side force coefficient $= Y/T.F.$
$C_{Y_{FUS}}$	fuselage side force coefficient $= Y_{FUS}/\frac{1}{2}\rho V_0^2 A_{Y_{FUS}}$
c	blade chord, ft
D	aerodynamic drag force of an aircraft component, positive in the direction of wind, lb
D_n, d_n	constants as defined in the text

E, e_n	constants as defined in the text
e	blade hinge offset, ft
F, f_n	constants as defined in the text
$\vec{F}$	force vector = $X\vec{i} + Y\vec{j} + Z\vec{k}$ lb
$F(\Lambda)$	stability characteristic equation
$f_n(\Lambda)$	operator function for aircraft response
G_n, g_n	constants as defined in the text
g	acceleration due to gravity, ft/sec^2
H	rotor H-force, component of the resultant rotor force perpendicular to the control axis, lb
H_n, h_n	constants as defined in the text
I	rotor moment of inertia about axis of rotation, slug-ft^2
I_b	blade moment of inertia about flapping hinge, slug-ft^2
I_{XX}	aircraft moment of inertia about the body X-axis, slug-ft^2
I_{YY}	aircraft moment of inertia about the body Y-axis, slug-ft^2
I_{ZZ}	aircraft moment of inertia about the body Z-axis, slug-ft^2
I_{XY}, I_{YZ}, I_{XZ}	aircraft products of inertia pertaining to body X, Y, Z system of axis, slug-ft^2
i	geometric incidence of an aircraft component relative to the body X-axis, rad

i, j, k	unit vectors along body X, Y and Z axis respectively
$J_1, J_2, \ldots$	pilot authority ratios pertaining to stability augmentation system
K_n, k_n	constants as defined in the text
$K_{FR}, K_{FFUS}, \ldots$	downwash interference factors pertaining to various aerodynamic components as defined by the subscripts
K_V	fuselage download factor
L	aerodynamic lift force of an aircraft component, perpendicular to local wind vector, lb
$\mathcal{L}$	rolling moment of an aircraft component, positive down to the right, ft-lb
$\mathcal{L}_w, \mathcal{L}_\phi, \ldots$	rolling moment total derivatives
$\vec{\ell}$	position vector of an aircraft component, relative to aircraft C.G. position, ft
ℓ_x	longitudinal moment arm, positive when the point of application of the force vector is forward from the C.G. position, ft
ℓ_Y	lateral moment arm, positive when the point of application of the force vector is to the right from the C.G. position, ft
ℓ_z	normal (vertical) moment arm, positive when the point of application of the force vector is below C.G. position, ft
M	pitching moment of an aircraft component, positive nose-up, ft-lb
$\vec{M}$	moment vector $=\mathcal{L}\vec{i} + M\vec{j} + N\vec{k}$, ft-lb
$M_u, M_\theta, \ldots$	pitching moment total derivatives

M_s	first moment of blade mass about the flapping hinge, slug-ft
M_T	Mach number of advancing blade tip
N	yawing moment of an aircraft component, positive to the right, ft-lb
$N_v, N_\psi, \ldots$	yawing moment total stability derivatives
n	number of propellers
P	period of oscillation, sec
p	angular rolling velocity ($\dot{\phi}$), positive down to the right, rad/sec
Q	rotor/propeller torque, ft-lb
q	angular pitching velocity ($\dot{\theta}$), positive nose up, rad/sec
q_0	dynamic pressure $= \frac{1}{2}\rho V_0^2$, lb/ft^2
R	rotor radius, ft
R^*	Routh discriminant
r	angular yawing velocity ($\dot{\psi}$), positive nose to the right, rad/sec
S	area of an aerodynamic surface, ft^2
S^*	constant as defined in the text
T	rotor/propeller thrust, force acting along the shaft or control axis, lb
T.F.	thrust factor $= \rho\pi R^2(\Omega R)^2$, lb
t	time, sec
u	longitudinal velocity component, along body X-axis, $= u_0+\bar{u}$ positive forward, ft/sec

$\vec{V}$	instantaneous velocity vector $= u\vec{i} + v\vec{j} + w\vec{k}$, ft/sec
V_0	steady state, or trim value of the resultant velocity vector $= \sqrt{u_0 + v_0 + w_0}$, ft/sec
V_s	velocity of sound in standard atmospheric condition, ft/sec
v	lateral velocity component along body Y-axis $= v_0 + \bar{v}$, positive to the right, ft/sec
v_i	rotor induced velocity, ft/sec
W	aircraft gross weight, lb
w	normal velocity component along body Z-axis, $= w_0 + \bar{w}$, positive down, ft/sec
X	longitudinal force along the body X-axis, positive forward, lb
$X_u, X_\theta, \ldots..$	total stability derivatives of the longitudinal X-force
Y	lateral force along the body Y-axis, positive to the right, lb
$Y_v, Y_\phi, \ldots..$	total stability derivatives of the lateral Y-force
Z	normal force along the body Z-axis, positive down, lb
$Z_u, Z_\theta, \ldots..$	total stability derivatives of the normal Z-force
A	amplitude of an oscillation
α	remote wind angle of attack relative to body X-axis, $\tan^{-1}(w/u)$ positive nose up, rad

α_c — rotor angle of attack; angle between axis of no feathering and a plane perpendicular to flight path, positive when axis is inclined rearward, rad

β — blade flapping angle = $a_0 - a_1 \cos\psi - b_1 \sin\psi$, rad

β_s — aircraft side slip angle = $\tan^{-1}(v/u)$, positive when wind vector is to the right of body X-axis, rad

Γ — rotor dihedral angle = $i_F - i_R$, rad

γ — Lock inertia number = $\rho \sigma c R^4 / I_b$

γ_c — aircraft climb angle, rad

Δ — discriminant, or increment, or perturbation from trim

δ_r — rudder pedal motion, rad

$\delta_0, \delta_1, \delta_2$ — blade drag constants defining drag polar

ϵ — downwash interference angle, rad

ζ, η — constants as defined in the text

Θ — blade collective pitch = $J\theta_c + \theta_s$, rad

θ — pitch altitude, positive nose up, rad

θ_c — blade collective pitch due to pilot control input, rad

θ_s — blade collective pitch due to stability augmentation system input, rad

$\theta_{.75}$ — blade section pitch angle at 0.75 radius, rad

θ_0 — collective pitch at blade root, rad

θ_1 — blade twist angle per unit spanwise distance, rad

K_n	constants for solidity correction of local stability derivatives
Λ	operator = $d(\ )/dt$
λ	rotor inflow ratio = $(V_0 \sin \alpha_c - v_i)/\Omega R$
μ	rotor tip speed ratio = $V_0 \cos \alpha_c/\Omega R$
ν	constant as defined in the text
Π_n	constants as defined in the text
π	constant = 3.14
ρ	air density, slug/ft^3
Σ	summation
σ	rotor solidity = $bc/\pi R$
ς	constant as defined in the text
τ	time constant
Φ	phase angle, rad
ϕ	aircraft roll attitude, positive to the right, rad
X	rotor wake angle = $a_1 + \tan^{-1}(-\mu/\lambda)$, rad
χ	generalized body space angle, rad
$\vec{\chi}$	vectorial, angular displacement relative to body X, Y, Z axes = $\phi\vec{i} + \theta\vec{j} + \psi\vec{k}$, rad
Ψ	blade azimuth position, rad
ψ	aircraft yaw attitude, positive nose to the right, rad
Ω	rotor angular velocity, rad/sec

$\vec{\omega}$	instantaneous angular velocity vector $= p\vec{i} + q\vec{j} + r\vec{k}$, rad/sec

SUBSCRIPTS

A	aerodynamic
C	control
F	pertaining to front rotor
FR, FFUS	effects of front rotor on rear rotor, front rotor on fuselage, etc.
G	pertaining to gravity
HUB	pertaining to rotor hub
I	pertaining to inertia
i	an integer 1, 2, 3 ..., or i^{th} aircraft component
L	pertaining to lift
$\mathcal{L}$	pertaining to rolling moment
M	pertaining to pitching moment
m,n	integers as defined in the text
N	pertaining to yawing moment
O	pertaining to initial condition or steady state
P	propeller
R	rear rotor
S	stability augmentation system
T	horizontal tailplane

TR	tail rotor
VT	vertical tailplane
W	wing
X	longitudinal direction
Y	lateral direction
Z	normal (vertical) direction

Dots denote time rate of change of variables

Bars denote perturbation values

SECTION 1. INTRODUCTION

The purpose of this Stability and Control Handbook for Helicopters is to provide a systematic summary of methods for estimating the stability and control characteristics of generalized helicopter configurations.

The information contained herein represents a revision and expansion of the material presented in TRECOM Report TREC 60-43, published in August 1960. That work utilized rotor performance data which derived from classical rotor theory and covered the forward speed range from hovering to advance ratios of $\mu = 0.3$. The present volume incorporates these results up to advance ratios of $\mu = 0.2$ and utilizes recently published rotor data for advance ratios ranging from $\mu = 0.3$ to $\mu = 1.0$. The latter data includes the effects of rotor blade compressibility, reverse flow, and blade stall.

The available rotor performance data, as discussed above, was utilized to develop comprehensive stability charts which considerably simplify the calculation procedures for helicopter stability and response characteristics.

Helicopter equations of motion presented in this handbook are derived without resorting to the simplifying assumptions of decoupling the longitudinal from the lateral directional degrees of freedom. These equations incorporate the contributions of the wings, the auxiliary propulsion system, and the horizontal and vertical tailplanes; they also include the effects of various stability augmentation systems. The resulting aircraft equations of motion can be directly utilized for digital or analog computer studies.

Comments concerning this work are invited and should be directed to the U. S. Army Aviation Materiel Laboratories, Fort Eustis, Virginia 23604.

SECTION 2. GUIDE TO THE HANDBOOK

The main objective of this handbook is to provide under one cover a comprehensive summary of analytical methods for predicting stability and control characteristics of generalized helicopter configurations. The information contained herein is intended to be used for preliminary design purposes, but it is also suitable for detailed digital or analog computer studies.

The handbook is organized in such a way that it is self-sufficient. For a given flight condition and a configuration, the complete set of stability derivatives can be calculated and the required helicopter stability and response characteristics can be determined. The use of reliable test data, especially for the fuselage characteristics, is strongly recommended.

The various sections of the handbook have been numbered with a decimal system which provides maximum flexibility for revising, deleting or supplementing any of the material, with a minimum disturbance to the remainder of the volume. The following pattern was developed for the numbering system:

Section:	An orderly numbering system is used, with numbers having not more than two parts separated by a decimal point, e.g., 3.2 or 10.1.
Subsection:	Subsections have numbers with more than two parts, e.g., 3.1.2 or 10.2.1.
Page:	The page number consists of the section number followed by a dash. Example: Page 3.1-20.
Figures:	Figure numbers follow a numerical sequence starting from 1 for each section.

Tables: Table numbers follow a numerical sequence starting from I for each section.

Equations: Equation numbers (where required) follow a numerical sequence starting from 1 for each section.

References: References are located at the end of each section. References are numbered in sequence starting from 1.

The overall organization of the handbook proceeds in a conputational sequence from the general to the particular. Hence, the equations of motion are presented before the total derivatives, which in turn precede the local and isolated derivatives.

For digital computer work, the general equations of motion, Section 4, can be used directly. For analog computer work or hand calculations, the stability characteristics of a helicopter are obtained by proceeding as follows:

Determine trim conditions	Section 5
Determine the isolated derivatives	Section 7.5
Correct the isolated derivatives for rotor solidity	Section 7.4
Determine local derivatives	Section 7.3
Determine total derivatives	Section 7.1
Determine characteristic equation	Section 8.1
Determine roots of the characteristic equation	Section 8.5

Determine control
derivatives . . . . . . Section 7.2

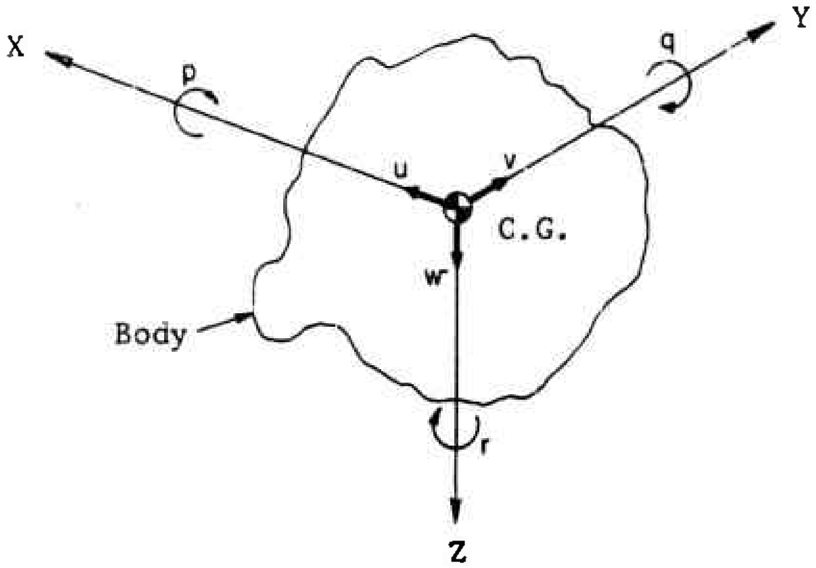
Determine response
to control input . . . . . . Section 9

Rapid methods for estimating the longitudinal stability of conventional single and tandem rotor helicopters are presented in Section 9.

SECTION 3. DEFINITIONS

3.1 DEFINITION OF AXIS SYSTEM

Sketch 1 below shows a right-angled coordinate axis system commonly used in stability work.



Sketch 1. Definition of Axis System

In analyzing aircraft stability, a variety of reference axes can be utilized. Descriptions of various axes systems are presented in References 1 and 2.

In general, the choice of the appropriate reference axis depends on the nature of the stability problem and the aircraft configuration to be analyzed.

The most common systems of reference axes presently in use are:

(a) Gravity Axes
(b) Stability (Wind) Axes
(c) Body Axes

The following subsections contain brief descriptions of these axes systems.

3.1.1 Gravity Axes

Gravity axes refer to a right-handed system of Cartesian coordinates with the origin either fixed at a point on the surface of the earth or at the aircraft C.G. (moving with the aircraft).

In each case the Z-axis is pointing to the center of the earth (positive downward), the X-axis is directed along the horizon (positive forward), and the Y-axis is oriented to form a right-handed orthogonal axes system (positive towards right).

The gravity or earth axes are primarily useful as a reference system for the gravity vector, aircraft altitude, horizontal distance and orientation. The use of these axes introduces certain simplifications in the stability analyses, in that the linear velocity components (u, v, w) along X, Y, Z axes are independent of aircraft rotation about the C.G. and are only functions of aircraft translation and the climb angle (γ_c). It follows that in the derivation of the equations of motion the aerodynamic force contribution is accounted for through the sine or cosine of climb angle (γ_c). Further simplifications occur for level flight ($\gamma_c = 0$). However, the use of gravity axes introduces rather cumbersome corrections to aircraft inertia terms and products of inertia in accounting for aircraft rotation.

3.1.2 Stability Axes

The stability axes represent a right-handed system of Cartesian coordinates, with the origin located at the aircraft C.G. and with the axes oriented such that the X-axis is coincident with the velocity vector and is positive pointing into the relative wind. The Z-axis is perpendicular to the relative wind and is positive downward, and the Y-axis is oriented to form a right-handed orthogonal axis system (positive to the right). The use of stability axes eliminates the terms containing w_o and v_o and thus introduces substantial simplifications into the aerodynamic terms. In this case, the only existing linear velocity component is u, which is independent of aircraft rotation (as in the case of gravity axes) and which represents perturbation of the forward velocity vector. However, the moment of inertia and product of

inertia terms vary for each flight condition. In general, these terms are assumed to be constant in the equations of motions. This limits the use of the stability axis system to small disturbance motions.

In addition to simplification of the aerodynamic terms, the use of stability axes systems has a specific application in correlating the theoretically calculated stability results with wind tunnel results, which are automatically resolved parallel and perpendicular to wind.

3.1.3 Body Axes

The body axis system refers to a right-handed, orthogonal system of axes fixed at aircraft C.G., rotating and translating with the aircraft. The X-axis is aligned along a reference line (datum line) fixed to the vehicle (positive pointing forward). The Z-axis is perpendicular to X-axis, positive towards the bottom of the vehicle. The Y-axis is mutually perpendicular to X and Z, positive when pointing to the right.

The use of the axes fixed to the vehicle insures that the inertia terms in the equations of motion are constant (independent of flight conditions); furthermore, by coinciding one of the body axes with a principal axis of inertia, certain products of inertia terms can be eliminated. In this axis system, the aerodynamic forces and moments depend on relative velocity orientation with respect to the body as defined by the angles α and β_s.

Body axes are particularly useful in the study of aircraft dynamics, since velocities and accelerations with respect to these axes are the same as those that would be experienced by a pilot or would be measured by the instruments mounted in the aircraft.

3.1.4 Choice of Axes

Since it is more convenient to express the aerodynamic and gravitational forces and moments with respect to body axes than to express inertia forces and moments with respect to wind or gravity axes, a body axis coordinate system has been selected for the work in this handbook.

For this axis system, the following definitions are made:

(a) Linear Velocities

$$\vec{V} = u\vec{i} + v\vec{j} + w\vec{k}$$

In the above definition the velocity components u, v, and w consist of the sum of initial (trim) values u_o, v_o, and w_o and of their perturbation values, respectively.

(b) Angular Displacements

$$\vec{\chi} = \phi\vec{i} + \theta\vec{j} + \psi\vec{k}$$

(c) Angular Velocities About C.G.

$$\vec{\omega} = p\vec{i} + q\vec{j} + r\vec{k}$$

(d) Forces

$$\vec{F} = X\vec{i} + Y\vec{j} + Z\vec{k}$$

(e) Moments

$$\vec{M} = \mathcal{L}\vec{i} + M\vec{j} + N\vec{k}$$

(f) Moment Arms

$$\vec{\ell} = \ell_X\vec{i} + \ell_Y\vec{j} + \ell_Z\vec{k}$$

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1. Thelander, J. A., Aircraft Motion Analysis, Technical Documentary Report FDL-TDR-64-70, Air Force Flight Dynamics Laboratory, Air Force System Command, Wright-Patterson Air Force Base, Ohio, March 1965.

2. Dynamics of the Airframe, Bu Aer Report AE-61-4 II, Bureau of Aeronautics, Navy Department (presently, Bureau of Naval Weapons, Naval Air System Command Headquarter), Washington, D.C., September 1952.

3.2 STABILITY VARIABLES

3.2.1 Independent Variables

Following are the selected independent stability variables:

(a) Linear Velocity Components (ft/sec) u, v, and w - defined in Subsection 3.1.4(a)

(b) The Angular Displacements (radians) ϕ , θ and ψ - defined in Subsection 3.1.4(b)

3.2.2 Dependent Variables

(a) Free Stream Angle of Attack (radians)

$$\alpha = \tan^{-1}\left(\frac{w}{u}\right)$$

The perturbation angle of attack is given by:

$$\bar{\alpha} \simeq \frac{\bar{w}}{u_0}$$

(b) Sideslip Angle (radians)

$$\beta_s = \tan^{-1}\left(\frac{v}{u}\right)$$

The perturbation sideslip angle is given by:

$$\bar{\beta}_s \simeq \frac{\bar{v}}{u_0}$$

(c) Interference Angles (radians)

Changes of local velocity due to aerodynamic interactions are accounted for by the interference angles

$\epsilon_F, \epsilon_R, \epsilon_{FUS}, \epsilon_W, \epsilon_T, \epsilon_{TR}, \epsilon_{VT}$ etc.

3.3 ILLUSTRATION OF PARAMETERS AND SIGN CONVENTION

Typical single and tandem rotor configurations together with the definition of parameters and the illustration of the sign convention utilized herein are presented in Figures 1 and 2 respectively.

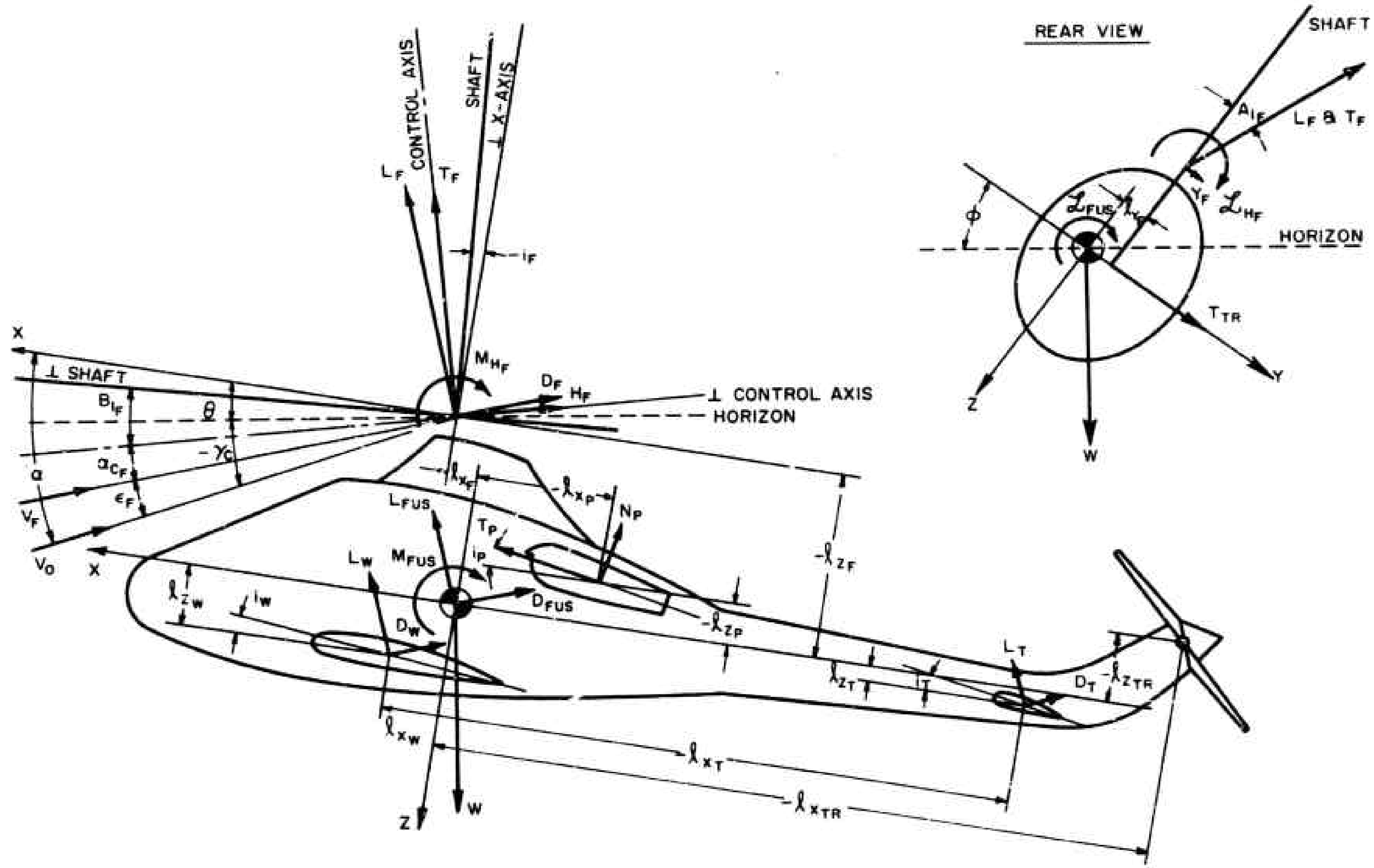

Figure 1. Definition of Parameters and Sign Convention for a Compound Single Rotor Helicopter.

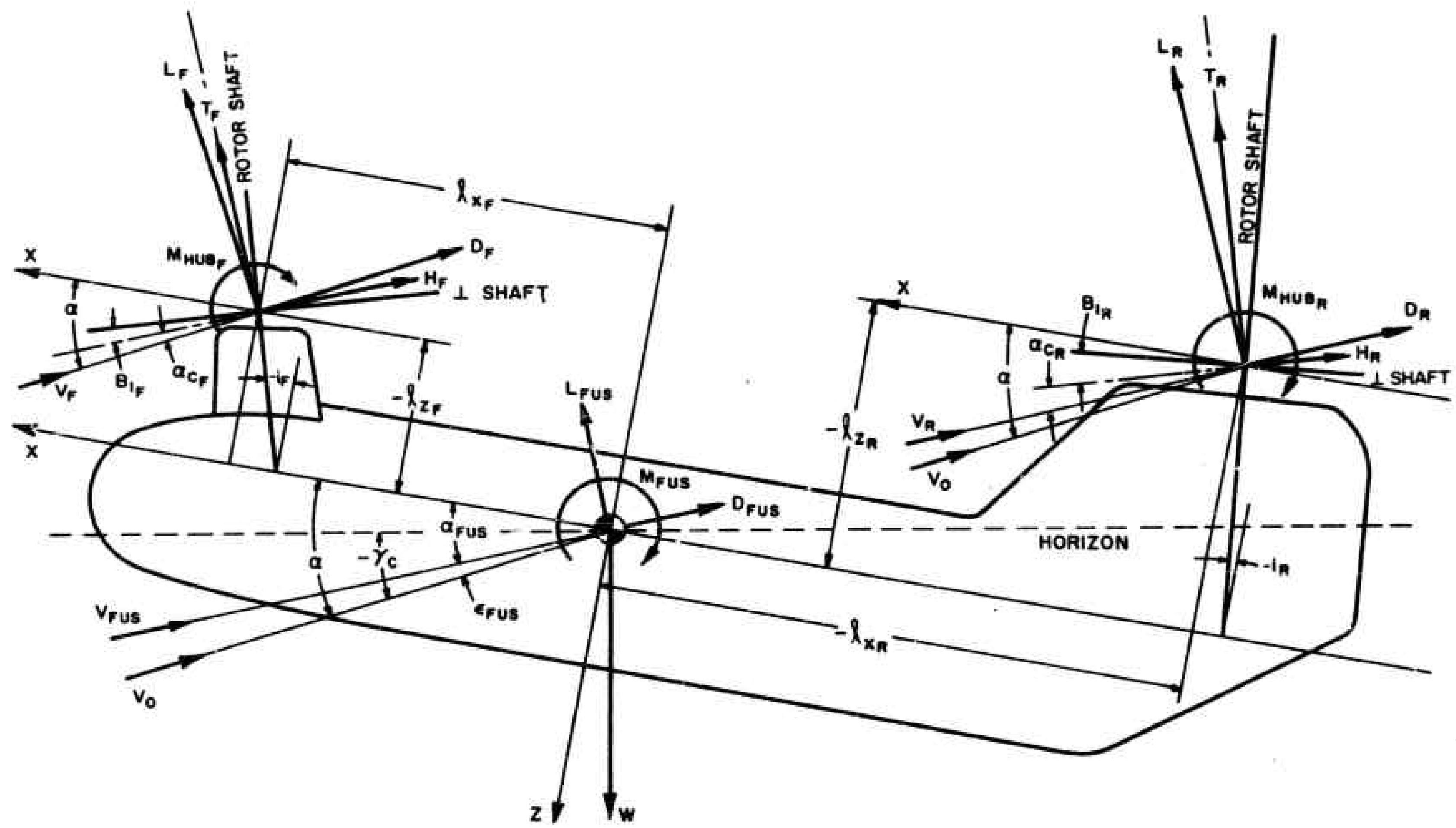

Figure 2. Definition of Parameters and Sign Convention for a Tandem Rotor Helicopter.

SECTION 4. EQUATIONS OF MOTION

The generalized equations presented herein pertain to 6 degrees of freedom of coupled longitudinal and lateral aircraft motions, about the body system of axes described in Subsection 3.1.3. Included also in this section are the equations of motion of various stabilization devices.

The analysis is performed for generalized aircraft configuration, which may consist of the following components:

(a) Single rotor
(b) Two rotors in tandem rotor configuration
(c) Fuselage
(d) Horizontal tailplane
(e) Vertical tail
(f) Tail rotor
(g) Propellers or jet engines
(h) Wings
(i) Various stabilization devices

The equations of motion presented herein can be adapted to single rotor helicopters, tandem rotor helicopters, and various types of compound helicopters by selecting those aerodynamic and design components which pertain to the aircraft under consideration and by eliminating the components which do not apply. To insure the generality of the equations, all products of inertia are retained. A detailed derivation of the equations of motion is presented in Section 11.

Hohenemser's quasi-static assumption, References 1 and 2, is utilized in the analysis. This assumption implies that the flapping motion of the rotor may be specified in terms of fuselage angular motion. According to Reference 1, this assumption is valid if $\omega_b/\Omega < 0.1$, where ω_b is the fuselage angular velocity and Ω is the rotor rotational speed. Present-day helicopters generally satisfy this criterion.

From the theoretical derivations presented in Section 11, the equations of motion for a generalized aircraft configuration are:

(a) The X-Force Equation

$$X = (X)_F + (X)_R + (X)_{FUS} + (X)_W + (X)_T + (X)_{VT} + (X)_{TR} + \sum_{i=1}^{n} (X)_{P_i} + W \sin\phi \sin\psi$$

$$- W \cos\phi \sin\theta \cos\psi - \frac{W}{g}(\dot{u} + qw - rv) = 0$$

where

$$(X)_F = \left[(L_F \cos A_{I_F} - Y_F \sin A_{I_F}) \sin(\alpha - \epsilon_F) - D_F \cos(\alpha - \epsilon_F) \right] \cos\beta_S$$

$$- (L_F \sin A_{I_F} + Y_F \cos A_{I_F}) \sin\beta_S$$

$$(X)_R = \left[(L_R \cos A_{I_R} + Y_R \sin A_{I_R}) \sin(\alpha - \epsilon_R) - D_R \cos(\alpha - \epsilon_R) \right] \cos\beta_S$$

$$- (L_R \sin A_{I_R} - Y_R \cos A_{I_R}) \sin\beta_S$$

$$(X)_{FUS} = \left[L_{FUS} \sin(\alpha - \epsilon_{FUS}) - D_{FUS} \cos(\alpha - \epsilon_{FUS}) \right] \cos\beta_S - Y_{FUS} \sin\beta_S$$

$$(X)_W = \left[L_W \sin(\alpha - \epsilon_W) - D_W \cos(\alpha - \epsilon_W) \right] \cos\beta_S$$

$$(X)_T = \left[L_T \sin(\alpha - \epsilon_T) - D_T \cos(\alpha - \epsilon_T) \right] \cos\beta_S$$

$$(X)_{VT} = - D_{VT} \cos(\alpha - \epsilon_{VT}) \cos\beta_S + L_{VT} \sin\beta_S$$

$$(X)_{TR} = \left[Y_{TR} \sin(\alpha - \epsilon_{TR}) - D_{TR} \cos(\alpha - \epsilon_{TR}) \right] \cos\beta_S - T_{TR} \sin\beta_S$$

$$(X)_{P_i} = T_{P_i} \cos i_{P_i} - N_{P_i} \sin i_{P_i}$$

(b) The Y-Force Equation

$$Y = (Y)_F + (Y)_R + (Y)_{FUS} + (Y)_W + (Y)_T + (Y)_{VT} + (Y)_{TR} + \sum_{i=1}^{n}(Y)_{P_i} + W \sin\phi \cos\psi$$

$$+ W \cos\phi \sin\theta \sin\psi - \frac{W}{g}(\dot{v} + ru - pw) = 0$$

where

$$(Y)_F = \left[(L_F \cos A_{I_F} - Y_F \sin A_{I_F}) \sin(\alpha - \epsilon_F) - D_F \cos(\alpha - \epsilon_F)\right] \sin\beta_S$$

$$+ (L_F \sin A_{I_F} + Y_F \cos A_{I_F}) \cos\beta_S$$

$$(Y)_R = \left[(L_R \cos A_{I_R} + Y_R \sin A_{I_R}) \sin(\alpha - \epsilon_R) - D_R \cos(\alpha - \epsilon_R)\right] \sin\beta_S$$

$$+ (L_R \sin A_{I_R} - Y_R \cos A_{I_R}) \cos\beta_S$$

$$(Y)_{FUS} = \left[L_{FUS} \sin(\alpha - \epsilon_{FUS}) - D_{FUS} \cos(\alpha - \epsilon_{FUS})\right] \sin\beta_S + Y_{FUS} \cos\beta_S$$

$$(Y)_W = \left[L_W \sin(\alpha - \epsilon_W) - D_W \cos(\alpha - \epsilon_W)\right] \sin\beta_S$$

$$(Y)_T = \left[L_T \sin(\alpha - \epsilon_T) - D_T \cos(\alpha - \epsilon_T)\right] \sin\beta_S$$

$$(Y)_{VT} = -D_{VT} \cos(\alpha - \epsilon_{VT}) \sin\beta_S - L_{VT} \cos\beta_S$$

$$(Y)_{TR} = \left[Y_{TR} \sin(\alpha - \epsilon_{TR}) - D_{TR} \cos(\alpha - \epsilon_{TR})\right] \sin\beta_S + T_{TR} \cos\beta_S$$

$$(Y)_{P_i} = Y_{P_i}$$

(c) The Z-Force Equation

$$Z = (Z)_F + (Z)_R + (Z)_{FUS} + (Z)_W + (Z)_T + (Z)_{VT} + (Z)_{TR} + \sum_{i=1}^{n} (Z)_{P_i} + W \cos\phi \cos\theta$$

$$- \frac{W}{g}(\dot{w} + pv - qu) = 0$$

where

$$(Z)_F = -\left[D_F \sin(\alpha - \epsilon_F) + (L_F \cos A_{I_F} - Y_F \sin A_{I_F}) \cos(\alpha - \epsilon_F)\right]$$

$$(Z)_R = -\left[D_R \sin(\alpha - \epsilon_R) + (L_R \cos A_{I_R} + Y_R \sin A_{I_R}) \cos(\alpha - \epsilon_R)\right]$$

$$(Z)_{FUS} = -\left[D_{FUS} \sin(\alpha - \epsilon_{FUS}) + L_{FUS} \cos(\alpha - \epsilon_{FUS})\right]$$

$$(Z)_W = -\left[D_W \sin(\alpha - \epsilon_W) + L_W \cos(\alpha - \epsilon_W)\right]$$

$$(Z)_T = -\left[D_T \sin(\alpha - \epsilon_T) + L_T \cos(\alpha - \epsilon_T)\right]$$

$$(Z)_{VT} = -D_{VT} \sin(\alpha - \epsilon_{VT})$$

$$(Z)_{TR} = -\left[D_{TR} \sin(\alpha - \epsilon_{TR}) + Y_{TR} \cos(\alpha - \epsilon_{TR})\right]$$

$$(Z)_{P_i} = -\left[T_{P_i} \sin i_{P_i} + N_{P_i} \cos i_{P_i}\right]$$

(d) The Rolling Moment Equation ($\mathcal{L}$)

$$\mathcal{L} = \sum_{i=1}^{n} (\mathcal{L})_i = \sum_{i=1}^{n} \left[(Z)_i \ell_{Y_i} - (Y)_i \ell_{Z_i} + (\mathcal{L}_0)_i \right] + \mathcal{L}_I$$

$$\mathcal{L} = (Z)_F \ell_{Y_F} - (Y)_F \ell_{Z_F} + (Z)_R \ell_{Y_R} - (Y)_R \ell_{Z_R}$$

$$+ (Z)_W \ell_{Y_W} - (Y)_W \ell_{Z_W} + (Z)_T \ell_{Y_T} - (Y)_T \ell_{Z_T}$$

$$+ (Z)_{VT} \ell_{Y_{VT}} - (Y)_{VT} \ell_{Z_{VT}} + (Z)_{TR} \ell_{Y_{TR}} - (Y)_{TR} \ell_{Z_{TR}}$$

$$+ \sum_{i=1}^{n} \left[(Z)_{P_i} \ell_{Y_{P_i}} - (Y)_{P_i} \ell_{Z_{P_i}} + Q_{P_i} \right] + \mathcal{L}_{FUS} + \mathcal{L}_{HUB_F} - \mathcal{L}_{HUB_R}$$

$$- \dot{p} I_{XX} + I_{XZ} (\dot{r} + pq) + rq (I_{YY} - I_{ZZ}) + I_{XY} (\dot{q} - rp)$$

$$+ I_{YZ} (q^2 - r^2) = 0$$

where i refers to the i^{th} aircraft component and $(\ \ )_i$ is evaluated by letting i = 1, 2, 3, etc., or the appropriate component designation.

Also $(\ \ )_I$ refers to inertia terms. Similar notation is utilized in the pitching and yawing moment equations given below.

(e) The Pitching Moment Equation (M)

$$M = \sum_{i=1}^{n} (M)_i = \sum_{i=1}^{n} \left[(X)_i \ell_{Z_i} - (Z)_i \ell_{X_i} + (M_0)_i \right] + M_I$$

$$M = (X)_F \ell_{Z_F} - (Z)_F \ell_{X_F} + (X)_R \ell_{Z_R} - (Z)_R \ell_{X_R}$$

$$+(X)_W \ell_{Z_W} - (Z)_W \ell_{X_W} + (X)_T \ell_{Z_T} - (Z)_T \ell_{X_T}$$

$$+(X)_{VT} \ell_{Z_{VT}} - (Z)_{VT} \ell_{X_{VT}} + (X)_{TR} \ell_{Z_{TR}} - (Z)_{TR} \ell_{X_{TR}}$$

$$+\sum_{i=1}^{n} \left[(X)_{P_i} \ell_{Z_{P_i}} - (Z)_{P_i} \ell_{X_{P_i}} + M_{P_i} \right] + M_{FUS} + M_{HUB_F} + M_{HUB_R} + Q_{TR}$$

$$-\dot{q} I_{YY} + I_{XZ}(r^2 - p^2) - rp(I_{XX} - I_{ZZ})$$

$$+ I_{XY}(\dot{p} + rq) + I_{YZ}(\dot{r} - pq) = 0$$

(f) <u>The Yawing Moment Equation (N)</u>

$$N = \sum_{i=1}^{n} (N)_i = \sum_{i=1}^{n} \left[(Y)_i \ell_{X_i} - (X)_i \ell_{Y_i} + (N_O)_i \right] + N_I$$

$$N = (Y)_F \ell_{X_F} - (X)_F \ell_{Y_F} + (Y)_R \ell_{X_R} - (X)_R \ell_{Y_R}$$

$$+ (Y)_W \ell_{X_W} - (X)_W \ell_{Y_W} + (Y)_T \ell_{X_T} - (X)_T \ell_{Y_T}$$

$$+ (Y)_{VT} \ell_{X_{VT}} - (X)_{VT} \ell_{Y_{VT}} + (Y)_{TR} \ell_{X_{TR}} - (X)_{TR} \ell_{Y_{TR}}$$

$$+\sum_{i=1}^{n} \left[(Y)_{P_i} \ell_{X_{P_i}} - (X)_{P_i} \ell_{Y_{P_i}} \right] + N_{FUS} + Q_F - Q_R$$

$$-\dot{r} I_{ZZ} + I_{XZ}(\dot{p} - qr) - pq(I_{YY} - I_{XX})$$

$$+ I_{XY}(p^2 - q^2) + I_{YZ}(\dot{q} + pr) = 0$$

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SECTION 5. EVALUATION OF TRIM CONDITIONS

The trim, or steady-state, equilibrium conditions for a helicopter can be obtained by simultaneously solving the equations of motion with all acceleration and inertia terms set equal to zero.

In order to evaluate the trim conditions for a generalized helicopter configuration, the following design parameters must be known:

(a) The aircraft gross weight W, lb
(b) The location of rotor hub (or hubs) relative to aircraft C.G. position, ft

$\ell_{X_F}, \ell_{Y_F}, \ell_{Z_F}$ and $\ell_{X_R}, \ell_{Y_R}, \ell_{Z_R}$

(c) The rotor radii R_F and R_R, ft
(d) The rotor solidities σ_F and σ_R
(e) The rotor rotational speeds $(\Omega R)_F$ and $(\Omega R)_R$, rad/sec
(f) The blade Lock inertia numbers γ_F and γ_R and blade twists θ_{1_F} and θ_{1_R}
(g) The blade mass moments of inertia M_{S_F} and M_{S_R}, slugs-ft
(h) The flapping hinge offsets e_F and e_R
(i) The number of blades b, per rotor
(j) The tip loss factor, $B_T = 0.97$
(k) The fuselage projected areas, ft^2, frontal $A_{X_{FUS}}$, side $A_{Y_{FUS}}$ and planform $A_{Z_{FUS}}$
(l) The fuselage overall length ℓ_{FUS}, ft
(m) The geometric, fixed incidences of wing i_W, tail i_T, and rotor shaft inclinations relative to the fuselage i_F and i_R
(n) The horizontal, vertical tailplane areas, ft^2
(o) The wing area, ft^2
(p) The lift curve slopes of rotor blades, horizontal tail, vertical tail, wing, etc.
(q) The moment arms ℓ_X, ℓ_Y and ℓ_Z of horizontal tail, vertical tail, wing, tail rotor, etc, ft
(r) The tail rotor tip speed $(\Omega R)_{TR}$, and the tail rotor twist $(\theta_1)_{TR}$
(s) The propeller geometry

5.1 TRIM CONDITIONS FOR SINGLE ROTOR HELICOPTERS

5.1.1 Hovering

The calculation procedure given below utilizes the expressions for thrust, constant inflow and coning angle presented in Reference 1.

The simplifying assumptions made in Reference 1, such as constant induced velocity, no radial flow, tip loss factor $B_T = 0.97$, etc., are incorporated in this procedure.

The vertical trim condition for hovering can be calculated as follows:

$$T_F = K_V W$$

where

K_V = fuselage download factor.

$$C_{T_F} = \left(\frac{T}{T.F.}\right)_F$$

where

$$(T.F.)_F = \rho \pi \left[R^2 (\Omega R)^2\right]_F$$

$$\lambda_F = -\sqrt{\frac{C_{T_F}}{2}}$$

$$\theta_{.75} = \frac{\left(\frac{2C_T}{a\sigma} - 0.4704\lambda\right)_F}{0.3042}$$

$$a_{0_F} = \left[\frac{\gamma}{2}(0.2213\,\theta_{.75} + 0.3042\lambda)\right]_F$$

5.1.2 Forward Speed

The trim procedure for forward speed utilizes the performance charts. Such charts, for helicopter high forward speeds, corresponding to advance ratios from $\mu = 0.25$ to $\mu = 1.4$ are presented in Reference 2. These charts, as pointed out in Reference 2, incorporate the effects of blade stall, flow compressibility and large inflow angles. The low-speed performance charts, corresponding advance ratios of $\mu \leq 0.2$, were obtained from the results of Reference 3 and are herein presented in Section 5.3.

The trim procedure for forward flight is performed as follows:

(a) Determine the required design parameters for a single rotor helicopter as specified on page 5-1.

(b) Establish the helicopter operating conditions such as $V_O, (\Omega R)_F, (\Omega R)_{TR}, \rho, V_S$, etc.

Then compute

$$\mu_F = \frac{V_O}{(\Omega R)_F} \qquad \mu_{TR} = \frac{V_O}{(\Omega R)_{TR}}$$

$$(M_T)_F = \frac{V_O + (\Omega R)_F}{V_S} \qquad (M_T)_{TR} = \frac{V_O + (\Omega R)_{TR}}{V_S}$$

$$(T.F.)_F = \left[\rho \pi R^2 (\Omega R)^2\right]_F \qquad (T.F.)_{TR} = \left[\rho \pi R^2 (\Omega R)^2\right]_{TR}$$

$$q_O = \frac{1}{2} \rho V_O^2$$

(c) Obtain fuselage lift and drag coefficients $C_{L_{FUS}}$ and $C_{D_{FUS}}$ for $\alpha_{FUS} = 0$, then calculate

$$D_{FUS} = C_{D_{FUS}} q_O A_{X_{FUS}}, \quad L_{FUS} = C_{L_{FUS}} q_O A_{Z_{FUS}}$$

(d) Calculate the first approximation for the main rotor lift and drag coefficients, thus:

$$\left(\frac{C_L'}{\sigma}\right)_F = \frac{W - L_{FUS}}{\left[(T.F.)\sigma\right]_F}$$

$$\left(\frac{C_D'}{\sigma}\right)_F = - \frac{D_{FUS}}{\left[(T.F.)\sigma\right]_F}$$

Also compute rotor lift and drag using

$$L_F = \left[(T.F.)\frac{C_L'}{\sigma}\right]_F$$

$$D_F = \left[(T.F.)\frac{C_D'}{\sigma}\right]_F$$

(e) Using Reference 2, calculate the chart values of rotor lift and drag coefficients corresponding to rotor solidity of $\sigma = 0.1$, thus:

$$\left[\left(\frac{C_L'}{\sigma}\right)_{0.1}\right]_F = \left(\frac{C_L'}{\sigma}\right)_F$$

$$\left[\left(\frac{C_D'}{\sigma}\right)_{0.1}\right]_F = \left[\frac{C_D'}{\sigma} - \frac{\Delta\sigma}{2\mu^2}\left(\frac{C_L'}{\sigma}\right)^2\right]_F$$

where

$$(\Delta\sigma)_F = \sigma_F - 0.1$$

(f) Using the values of $\left[(C_L'/\sigma)_{0.1}\right]_F$ and $\left[(C_D'/\sigma)_{0.1}\right]_F$ from step (e) and θ_{1_F}, M_{T_F} and μ_F from steps (a) and (b), enter the appropriate trim charts, presented in Section 5.3 or Reference 2, and obtain the first approximations for the following rotor trim parameters corresponding to $\sigma = 0.1$:

$\left[(\alpha_C)_{0.1}\right]_F$, a_{1_F}, b_{1_F}, $\theta_{.75_F}$, λ_F, $(C_Q/\sigma)_F$

(g) Calculate main rotor angle of attack α_{C_F} and rotor torque Q_F as follows:

$$\alpha_{C_F} = \left[(\alpha_C)_{0.1} + \frac{\Delta\sigma}{2\mu^2}\left(\frac{C_L'}{\sigma}\right)\right]_F$$

$$Q_F = \left[(T.F.)\sigma\left(\frac{C_Q}{\sigma}\right)\right]_F$$

(h) Using λ_F from step (f) and α_{C_F} from step (g), obtain the first approximations for the following interference angles:

$$\epsilon_{FUS} = K_{FFUS}\left(\tan\alpha_C - \frac{\lambda}{\mu}\right)_F$$

$$\epsilon_T = K_{FT}\left(\tan\alpha_C - \frac{\lambda}{\mu}\right)_F$$

$$\epsilon_{TR} = K_{FTR}\left(\tan\alpha_C - \frac{\lambda}{\mu}\right)_F$$

where K_{FFUS}, K_{FT}, K_{FTR} are the downwash interference factors discussed in Section 7.6.

(i) Using the trim parameters obtained in the steps above, assume two values of $C_{M_{FUS}}$ and calculate α_{FUS} from the following equation:

$$\alpha_{FUS} = \frac{\ell_{x_F} L_F - \ell_{z_F} D_F + \ell_{x_T} q_0 S_T a_T (i_T - \epsilon_T) + \left[\frac{eb\Omega^2 M_S}{2}(a_1 - B_1)\right]_F + M_{FUS}}{-\ell_{x_F} D_F - \ell_{z_F} L_F - \ell_{x_T} q_0 S_T a_T} - \epsilon_{FUS}$$

where

$$B_{1_F} = (-\alpha_C + i)_F$$

The straight line obtained by connecting the two points thus calculated is superimposed on the experimental fuselage pitching moment curve of ($C_{M_{FUS}}$ vs. α_{FUS}). The point of intersection will yield the fuselage trim angle of attack α_{FUS} .

(j) Using α_{FUS} from step (i), enter fuselage charts and obtain

$C_{L_{FUS}}$, $C_{D_{FUS}}$, $C_{Y_{FUS}}$, $C_{\ell_{FUS}}$, $C_{M_{FUS}}$ and $C_{N_{FUS}}$. Then calculate the following fuselage trim values:

$$L_{FUS} = C_{L_{FUS}} q_0 A_{Z_{FUS}} , \qquad D_{FUS} = C_{D_{FUS}} q_0 A_{X_{FUS}}$$

$$Y_{FUS} = C_{Y_{FUS}} q_0 A_{Y_{FUS}} , \qquad \mathcal{L}_{FUS} = C_{\ell_{FUS}} q_0 A_{X_{FUS}} \ell_{FUS}$$

$$M_{FUS} = C_{M_{FUS}} q_0 A_{X_{FUS}} \ell_{FUS} , \qquad N_{FUS} = C_{N_{FUS}} q_0 A_{X_{FUS}} \ell_{FUS}$$

(k) Using the values of N_{FUS} from step (j) and Q_F from step (g), determine the following tail-rotor parameters:

$$T_{TR} = \frac{N_{FUS} + Q_F}{-\ell_{X_{TR}}}$$

$$\left(\frac{C_L'}{\sigma}\right)_{TR} = \left[\frac{T}{(T.F)\sigma}\right]_{TR}$$

(l) Knowing the tail-rotor parameters $\mu_{TR} = \mu_F$, $M_{T\,TR} = \left[V_D + (\Omega R)_{TR}\right]/V_S$ and $\theta_{1\,TR}$ and using the value of $(C_L'/\sigma)_{TR}$ from step (k) and $\alpha_{C\,TR} = 0$, enter the appropriate performance charts and obtain the following tail-rotor trim values:

$$\left[\left(\frac{C_D'}{\sigma}\right)_{D.I}\right]_{TR}, \left(\frac{C_Q}{\sigma}\right)_{TR}, \lambda_{TR}, \theta_{.75\,TR}, a_{0\,TR}, a_{1\,TR}, b_{1\,TR}$$

then compute

$$\left(\frac{C_D'}{\sigma}\right)_{TR} = \left[\left(\frac{C_D'}{\sigma}\right)_{D.I} + \frac{\Delta\sigma}{2\mu^2}\left(\frac{C_L'}{\sigma}\right)^2\right]_{TR}$$

$$D_{TR} = \left[(T.F.)\sigma\,\frac{C_D'}{\sigma}\right]_{TR}$$

and

$$Q_{TR} = \left[(T.F.)\,\sigma\,R\left(\frac{C_Q}{\sigma}\right)\right]_{TR}$$

(m) Using values of ϵ_{FUS} from step (h) and α_{FUS} from step (i), calculate

$$\alpha = \alpha_{FUS} + \epsilon_{FUS}$$

and then obtain

$$\alpha_T = \alpha + i_T - \epsilon_T$$

$$C_{L_T} = a_T \; \alpha_T$$

$$C_{D_T} = (C_{D_0} + \frac{C_L^2}{\pi A\!\!R})_T$$

where

$$(C_{D_0})_T \simeq 0.01$$

$$L_T = C_{L_T} \, q_0 \, S_T$$

$$D_T = C_{D_T} \, q_0 \, S_T$$

(n) Using the trim parameters obtained above and assuming $A_{l_F} = \phi = Y_{TR} = \gamma_C = 0$, solve simultaneously the X and Z equations from Section 4 to obtain a better approximation for the main rotor drag and lift, thus:

$$D_F = L_F \, \alpha - K_1$$

$$L_F = \frac{K_1 \alpha - K_2}{1 + \alpha^2}$$

where

$$K_1 = W\alpha - L_{FUS}(\alpha - \epsilon_{FUS}) - L_T(\alpha + i_T - \epsilon_T)$$

$$+ D_{FUS} + D_T + D_{TR}$$

and

$$K_2 = D_{FUS}(\alpha - \epsilon_{FUS}) + D_T(\alpha + i_T - \epsilon_T)$$

$$+ D_{TR}(\alpha - \epsilon_{TR}) + L_{FUS} + L_T - W$$

Then obtain

$$\left(\frac{C_L'}{\sigma}\right)_F = \left[\left(\frac{C_L'}{\sigma}\right)_{0.1}\right]_F = \left[\frac{L}{(T.F.)\sigma}\right]_F$$

$$\left(\frac{C_D'}{\sigma}\right)_F = \left[\frac{D}{(T.F.)\sigma}\right]_F$$

and compute

$$\left[\left(\frac{C_D'}{\sigma}\right)_{0.1}\right]_F = \left[\frac{C_D'}{\sigma} - \frac{\Delta\sigma}{2\mu^2}\left(\frac{C_L'}{\sigma}\right)^2\right]_F$$

(o) Repeat steps (f) through (n) with new values of $[(C_L'/\sigma)_{0.1}]_F$ and $[(C_D'/\sigma)_{0.1}]_F$ until convergence is achieved, yielding the final trim values.

(p) Calculate main rotor side force, thus:

$$Y_F = \Big[(T.F.)\sigma \frac{a}{2}\Big(-\frac{3}{4}\mu\theta_{.75}a_0 + \frac{1}{3}\theta_{.75}b_1 + \frac{3}{8}\mu^2\theta_{.75}b_1 + \frac{3}{4}\lambda b_1 + \frac{1}{6}a_0a_1 - \frac{3}{2}\mu\lambda a_0 - \mu^2 a_0 a_1 + \frac{1}{4}\mu a_1 b_1 + \frac{1}{8}\mu^2\lambda b_1\Big)\Big]_F$$

(q) Using the latest trim values, compute the main rotor lateral cyclic pitch A_{1_F} (from rolling moment equation) and aircraft roll attitude ϕ (from side force equation), as follows:

$$A_{1_F} = \frac{\ell_{Z_F} Y_F + \ell_{Z_{TR}} T_{TR} + \ell_{Y_{TR}} D_{TR}(\alpha - \epsilon_{TR}) - \mathcal{L}_{FUS} - \frac{(eb\Omega^2 M_S)_F}{2} b_{1_F}}{-\ell_{Z_F} L_F + \frac{(eb\Omega^2 M_S)_F}{2}}$$

and

$$\phi = -\left(\frac{L_F A_{1_F} + Y_F + Y_{FUS} + T_{TR}}{W}\right)$$

(r) Finally, compute the main rotor longitudinal cyclic pitch B_{1_F} using latest trim values, thus:

$$B_{1_F} = \alpha - \alpha_{C_F} + i_F$$

In general, for $\alpha \leq 5^{\circ}$, the above iteration procedure is very rapidly convergent, and therefore one or two iterations are sufficient to obtain the final trim conditions.

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5.2 TRIM CONDITIONS FOR TANDEM ROTOR HELICOPTERS

5.2.1 Hovering

The hovering trim conditions for tandem rotcr helicopters are obtained by simultaneously solving the Z-force and pitching moment equations for the non-accelerated condition:

$$T_F = \frac{K_V W}{1 - \dfrac{\ell_{X_F} \cos i_F - \ell_{Z_F} \sin i_F}{\ell_{X_R} \cos i_R - \ell_{Z_R} \sin i_R}}$$

and

$$T_R = K_V W - T_F$$

The thrust coefficients for each rotor are obtained by using the simplified aerodynamic expressions of Reference 1.

$$C_{T_F} = \frac{T_F}{(T.F.)} \qquad C_{T_R} = \frac{T_R}{(T.F)}$$

$$\lambda_F = -\sqrt{\frac{C_{T_F}}{2}} \qquad \lambda_R = -\sqrt{\frac{C_{T_R}}{2}}$$

$$\theta_{.75_F} = \frac{\frac{2}{a}\left(\frac{C_T}{\sigma}\right)_F - \frac{B_T^2}{2}\lambda_F}{\frac{B_T^3}{3}} \qquad \theta_{.75_R} = \frac{\frac{2}{a}\left(\frac{C_T}{\sigma}\right)_R - \frac{B_T^2}{2}\lambda_R}{\frac{B_T^3}{3}}$$

where

B_T = 0.97 - tip loss factor

a = 5.73 - blade lift curve slcpe

and

$$\left(\frac{a_0}{\gamma}\right)_F = (0.1521\lambda + 0.111\,\theta_{.75})_F$$

$$\left(\frac{a_0}{\gamma}\right)_R = (0.1521\lambda + 0.111\,\theta_{.75})_R$$

5.2.2 Forward Speed

The trim procedure for tandem rotor helicopters is in principle similar to that of single rotor helicopters described in Section 5.1.2. The major difference arises from the fact that some tandem rotor helicopters operate with predetermined values of longitudinal cyclic pitch (B_{1C}) on the front and rear rotors. Knowing the longitudinal cyclic schedule, which for those helicopters is a function of forward speed, simplifies the trim procedure for tandem rotor helicopters.

Generally, the front and rear rotor geometric parameters are identical.

Based on this assumption, the trim procedure for tandem rotor helicopters is as follows:

(a) Determine the required design parameters for a tandem rotor configuration as specified on page 5.1.

(b) Establish the helicopter operating conditions such as V_0, ΩR, σ, ρ, V_S , etc.

Then compute

$$\mu = \frac{V_0}{\Omega R}$$

$$M_T = \frac{V_0 + \Omega R}{V_S}$$

$$T.F. = \rho \pi R^2 (\Omega R)^2$$

$$q_0 = \frac{1}{2} \rho V_0^2$$

(c) Obtain $C_{L_{FUS}}$ and $C_{D_{FUS}}$ from the appropriate fuselage characteristic charts assuming values for α . Start with $\alpha = \alpha_{FUS} = 0$; then calculate

$$D_{FUS} = C_{D_{FUS}} q_0 A_{X_{FUS}}$$

$$L_{FUS} = C_{L_{FUS}} q_0 A_{Z_{FUS}}$$

It is recommended that the fuselage characteristics be extracted from the appropriate experimental data.

(d) Using Z-force equation, calculate, $(C_L'/\sigma)_F$, assuming arbitrary values for $(C_L'/\sigma)_R$, thus:

$$\left(\frac{C_L'}{\sigma}\right)_F = \frac{W - L_{FUS} + D_{FUS}\, \epsilon_{FUS}}{(T.F.)\sigma} - \left(\frac{C_L'}{\sigma}\right)_R$$

Also compute

$$\alpha_{C_F} = \alpha + i_F - B_{1_F} - \epsilon_F$$

$$\alpha_{C_R} = \alpha + i_R - B_{1_R} - \epsilon_R$$

where α is given in step (c).

Assume initially that

$$\epsilon_F = \epsilon_R = \epsilon_{FUS} = 0$$

(e) Using X-force equation, calculate $(C_O'/\sigma)^*_{TOTAL}$ using values of steps (c) and (d), thus:

$$\left(\frac{C_O'}{\sigma}\right)^*_{TOTAL} = \left(\frac{C_O'}{\sigma}\right)_F + \left(\frac{C_D'}{\sigma}\right)_R = -\frac{D_{FUS} + L_{FUS}\epsilon_{FUS}}{(T.F.)\sigma} - \epsilon_R\left(\frac{C_L'}{\sigma}\right)_R$$

Then compute

$$(\alpha_{C_{0.1}})_F = \alpha_{C_F} - \frac{\Delta\sigma}{2\mu^2}\left(\frac{C_L'}{\sigma}\right)_F$$

$$(\alpha_{C_{0.1}})_R = \alpha_{C_R} - \frac{\Delta\sigma}{2\mu^2}\left(\frac{C_L'}{\sigma}\right)_R$$

where

$$\Delta\sigma = \sigma - 0.1$$

Assume initially that

$$\epsilon_R = \epsilon_{FUS} = 0$$

(f) With appropriate values of $(C_L'/\sigma)_F$ and $(\alpha_{C_{0.1}})_F$ and $(C_L'/\sigma)_R$ and $(\alpha_{C_{0.1}})_R$ from steps (d) and (e), enter the trim charts and read off

$$\left[\left(\frac{C_O'}{\sigma}\right)_{0.1}\right]_F \qquad \left[\left(\frac{C_O'}{\sigma}\right)_{0.1}\right]_R$$

Then compute

$$(\frac{C_D'}{\sigma})_F = \left[(\frac{C_D'}{\sigma})_{0.1} + \frac{\Delta\sigma}{2\mu^2}(\frac{C_L'}{\sigma})^2\right]_F$$

$$(\frac{C_D'}{\sigma})_R = \left[(\frac{C_D'}{\sigma})_{0.1} + \frac{\Delta\sigma}{2\mu^2}(\frac{C_L'}{\sigma})^2\right]_R$$

and

$$(\frac{C_D'}{\sigma})_F + (\frac{C_D'}{\sigma})_R = (\frac{C_D'}{\sigma})_{TOTAL}$$

(g) Plot $(C_L'/\sigma)_F$, $(C_D'/\sigma)_F$, $(C_L'/\sigma)_R$, $(C_D'/\sigma)_R$ and $(C_D'/\sigma)^*_{TOTAL}$ obtained from steps (d), (e), and (f) vs. $(C_D'/\sigma)_{TOTAL}$ from step (f). Draw a straight line of $(C_D'/\sigma)^*_{TOTAL} = (C_D'/\sigma)_{TOTAL}$. At the point of intersection of this line with $(C_D'/\sigma)^*_{TOTAL}$ vs. $(C_D'/\sigma)_{TOTAL}$ curve read off: $(C_L'/\sigma)_F$, $(C_D'/\sigma)_F$, $(C_L'/\sigma)_R$ and $(C_D'/\sigma)_R$. Convert $(C_D'/\sigma)_F$ and $(C_D'/\sigma)_R$ thus obtained into $[(C_D'/\sigma)_{0.1}]_F$ and $[(C_D'/\sigma)_{0.1}]_R$, using equations from step (f).

(h) Using the values of $(C_L'/\sigma)_F$, $[(C_D'/\sigma)_{0.1}]_F$ $(C_L'/\sigma)_R$ and $[(C_D'/\sigma)_{0.1}]_R$ enter the appropriate rotor trim charts and obtain

$$a_{1_F},\ \lambda_F,\ a_{1_R} \text{ and } \lambda_R$$

(i) Compute all necessary interference angles, thus:

$$\epsilon_F = K_{RF}(\tan \alpha_C - \frac{\lambda}{\mu})_R$$

$$\epsilon_R = K_{FR}(\tan\ \alpha_c - \frac{\lambda}{\mu})_F$$

$$\epsilon_{FUS} = K_{FFUS}(\tan\alpha_c - \frac{\lambda}{\mu})_F + K_{RFUS}(\tan\ \alpha_c - \frac{\lambda}{\mu})_R$$

where K_{RF}, K_{FR}, etc., are the rotor downwash interference factors discussed in Section 7.6. The values of α_{C_F}, α_{C_R} are obtained in step (d), and λ_F, λ_R are obtained in step (h).

(j) Solve the pitching moment equation and compute $C_{M_{FUS}}$ using the values obtained in steps (h) and (i), thus:

$$C_{M_{FUS}} = \frac{(T.F.)\sigma}{q_a A_{X_{FUS}} \ell_{FUS}} \left[(A \sin\alpha - B\cos\alpha) - \frac{(M_{HUB_F} + M_{HUB_R})}{(T.F.)\sigma}\right]$$

where

$$A = -\left[\ell_X(\frac{C_D'}{\sigma}) + \ell_Z(\frac{C_L'}{\sigma})\right]_F - \left[\ell_X(\frac{C_D'}{\sigma}) + \ell_Z(\frac{C_L'}{\sigma})\right]_R$$

$$- \left[\ell_X(\frac{C_L'}{\sigma}) - \ell_Z(\frac{C_D'}{\sigma})\right]_F \epsilon_F - \left[\ell_X(\frac{C_L'}{\sigma}) - \ell_Z(\frac{C_D'}{\sigma})\right]_R \epsilon_R$$

$$B = \left[\ell_X(\frac{C_L'}{\sigma}) - \ell_Z(\frac{C_D'}{\sigma})\right]_F + \left[\ell_X(\frac{C_L'}{\sigma}) - \ell_Z(\frac{C_D'}{\sigma})\right]_R$$

$$- \left[\ell_X(\frac{C_D'}{\sigma}) + \ell_Z(\frac{C_L'}{\sigma})\right]_F \epsilon_F - \left[\ell_X(\frac{C_D'}{\sigma}) + \ell_Z(\frac{C_L'}{\sigma})\right]_R \epsilon_R$$

$$M_{HUB_F} + M_{HUB_R} = \frac{eb\Omega^2 M_S}{2} \ (a_{1_F} + a_{1_R} - B_{1_F} - B_{1_R})$$

Also obtain

$$\alpha_{FUS} = \alpha - \epsilon_{FUS}$$

where α is given in step (c) and ϵ_{FUS} is obtained in step (i).

(k) Repeat steps (c) through (j) for one or two different values of α and obtain a cross plot of $(C_L'/\sigma)_F$, $(C_D'/\sigma)_F$, ϵ_F, $(C_L'/\sigma)_R$, $(C_D'/\sigma)_R$, ϵ_R ϵ_{FUS} and $C_{M_{FUS}}$ versus α_{FUS}.

(l) Superimpose the available fuselage data (preferably test data) of $C_{M_{FUS}}$ versus α_{FUS} on the cross plot obtained in step (k). The point of intersection of the available fuselage moment data ($C_{M_{FUS}}$ versus α_{FUS}) with the corresponding values computed in step (k), will yield the first approximation for the trim values of α_{FUS} and $C_{M_{FUS}}$.

(m) Enter α_{FUS} from step (l) on the cross plot of step (k) and read off trim values of

$$\left(\frac{C_L'}{\sigma}\right)_F, \left(\frac{C_D'}{\sigma}\right)_F, \epsilon_F, \left(\frac{C_L'}{\sigma}\right)_R, \left(\frac{C_D'}{\sigma}\right)_R, \epsilon_R \text{ and } \epsilon_{FUS}$$

Then compute

$$\alpha = \alpha_{FUS} + \epsilon_{FUS}$$

$$\alpha_{C_F} = \alpha + i_F - B_{1_F} - \epsilon_F$$

$$\alpha_{C_R} = \alpha + i_R - B_{1_R} - \epsilon_R$$

(n) Repeat steps (c) through (m) using values of α_{FUS} , ϵ_{FUS} , ϵ_F and ϵ_R from steps (i) and (m) and obtain the final trim values for front and rear rotor as follows:

$$\left(\frac{C_L'}{\sigma}\right), \left(\frac{C_D'}{\sigma}\right), \left(\frac{C_Q}{\sigma}\right), \alpha_C, \epsilon, \theta_{.75}, \lambda, a_0, a_1, b_1, \text{ etc.}$$

Obtain the following fusélage characteristics using the available fuselage data:

$$C_{L_{FUS}}, C_{D_{FUS}}, C_{Y_{FUS}}, C_{\mathcal{L}_{FUS}}, C_{M_{FUS}}, C_{N_{FUS}}, \alpha_{FUS}, \epsilon_{FUS}$$

Then compute

$$L_{FUS} = C_{L_{FUS}} q_0 A_{Z_{FUS}} \quad , \quad D_{FUS} = C_{D_{FUS}} q_0 A_{X_{FUS}}$$

$$Y_{FUS} = C_{Y_{FUS}} q_0 A_{Y_{FUS}} \quad , \quad \mathcal{L}_{FUS} = C_{\mathcal{L}_{FUS}} q_0 A_{X_{FUS}} \ell_{FUS}$$

$$M_{FUS} = C_{M_{FUS}} q_0 A_{X_{FUS}} \ell_{FUS} \quad , \quad N_{FUS} = C_{N_{FUS}} q_0 A_{X_{FUS}} \ell_{FUS}$$

(o) Calculate rotor side force (C_Y'/σ) for front and rear rotors using

$$\left(\frac{C_Y'}{\sigma}\right) = \frac{a}{2}\left[-\frac{3}{4}\mu\theta_{.75}a_0 + \frac{1}{3}\theta_{.75}b_1 + \frac{3}{8}\mu^2\theta_{.75}b_1 + \frac{3}{4}\lambda b_1 + \frac{1}{6}a_0 a_1 - \frac{3}{2}\mu\lambda a_0 - \mu^2 a_0 a_1 + \frac{1}{4}\mu a_1 b_1 + \frac{1}{8}\mu^2\lambda b_1\right]$$

(p) Solve rolling and yawing moment equations of motion and obtain A_{1_F} and A_{1_R} as follows:

$$A_{1_F} = \Big\{ -\ell_{x_R}\left(\frac{C_L'}{\sigma}\right)_R \left[-\ell_{z_F}\left(\frac{C_Y'}{\sigma}\right)_F + \ell_{z_R}\left(\frac{C_Y'}{\sigma}\right)_R + \frac{eb\Omega^2 M_S}{2\sigma(T.F.)}\left(b_{1_F} + b_{1_R}\right)\right.$$

$$\left. + \frac{\mathcal{L}_{FUS}}{\sigma(T.F.)}\right] + \left[-\ell_{z_R}\left(\frac{C_L'}{\sigma}\right)_R + \frac{eb\Omega^2 M_S}{2\sigma(T.F.)}\right]\left[R\left(\frac{C_Q}{\sigma}\right)_F - R\left(\frac{C_Q}{\sigma}\right)_R + \ell_{x_F}\left(\frac{C_Y'}{\sigma}\right)_F\right.$$

$$\left.\left. - \ell_{x_R}\left(\frac{C_Y'}{\sigma}\right)_R + \frac{N_{FUS}}{\sigma(T.F.)}\right]\right\} \Big/ \left\{ \left(\frac{C_L'}{\sigma}\right)_F \left(\frac{C_L'}{\sigma}\right)_R \left[\ell_{x_F}\ell_{z_R} - \ell_{z_F}\ell_{x_R}\right]\right.$$

$$\left. - \frac{eb\Omega^2 M_S}{2\sigma(T.F.)}\left[\ell_{x_F}\left(\frac{C_L'}{\sigma}\right)_F - \ell_{x_R}\left(\frac{C_L'}{\sigma}\right)_R\right]\right\}$$

$$A_{1_R} = \Big\{ -\left[-\ell_{z_F}\left(\frac{C_L'}{\sigma}\right)_F + \frac{eb\Omega^2 M_S}{2\sigma(T.F.)}\right]\left[R\left(\frac{C_Q}{\sigma}\right)_F - R\left(\frac{C_Q}{\sigma}\right)_R + \ell_{x_F}\left(\frac{C_Y'}{\sigma}\right) - \ell_{x_R}\left(\frac{C_Y'}{\sigma}\right)_R\right.$$

$$\left. + \frac{N_{FUS}}{\sigma(T.F.)}\right] + \ell_{x_F}\left(\frac{C_L'}{\sigma}\right)_F \left[-\ell_{z_F}\left(\frac{C_Y'}{\sigma}\right)_F + \ell_{z_R}\left(\frac{C_Y'}{\sigma}\right)_R + \frac{eb\Omega^2 M_S}{2\sigma(T.F.)}\left(b_{1_F} + b_{1_R}\right)\right.$$

$$\left.\left. + \frac{\mathcal{L}_{FUS}}{\sigma(T.F.)}\right]\right\} \Big/ \left\{ \left(\frac{C_L'}{\sigma}\right)_F \left(\frac{C_L'}{\sigma}\right)_R \left[\ell_{x_F}\ell_{z_R} - \ell_{z_F}\ell_{x_R}\right]\right.$$

$$\left. - \frac{eb\Omega^2 M_S}{2\sigma(T.F.)}\left[\ell_{x_F}\left(\frac{C_L'}{\sigma}\right)_F - \ell_{x_R}\left(\frac{C_L'}{\sigma}\right)_R\right]\right\}$$

(q) From the Y-force equation, solve for the roll attitude, thus:

$$\phi = \frac{\left(\frac{C_Y'}{\sigma}\right)_R - \left(\frac{C_Y'}{\sigma}\right)_F - \left(\frac{C_L'}{\sigma}\right)_F A_{1F} - \left(\frac{C_L'}{\sigma}\right)_R A_{1R}}{\frac{W}{(T.F.)\sigma}}$$

It is found from experience that adequate accuracy is obtained without additional iterations.

REFERENCE

1. Gessow, A., and Meyers, G. C., Jr., Aerodynamics of the Helicopter, The MacMillan Company, New York, 1952.

5.3 TRIM CHARTS FOR ROTOR SOLIDITY, $\sigma = 0.1$

Classical rotor aerodynamic theories, such as those presented in References 1 and 2, utilize several simplifying assumptions which limit the applicability of the resulting equations to low forward speeds. To increase the range of applicability, some of these assumptions have been eliminated in Reference 3, which presents charts of pertinent aerodynamic rotor parameters for the tip speed ratios ranging from $\mu = 0.3$ to $\mu = 1.4$. These charts include the effects of blade compressibility and retreating blade stall and do not rely upon small angle assumptions of the classical theory. However, the charts are prepared for only one value of rotor solidity $\sigma = 0.1$ and do not include the rotor Y-force data.

In applying the above performance charts for rotor solidity different from $\sigma = 0.1$, appropriate solidity correction factors were utilized as presented in Reference 3. The required Y-force data were generated by utilizing the equation of Reference 2 together with the pertinent performance results obtainable from Reference 4. The charts for rotor inflow ratio λ and the blade flapping parameters a_0 and b_1, which were not included in Reference 3, were derived from the results of Reference 4 and are presented in this section.

All low-speed performance charts for $\mu = 0.1$ and 0.2 were derived from the classical rotor performance results of Reference 5, and are presented in Figures 1 and 2. The high-speed charts which are not included in Reference 3 are presented in Figures 3 through 13.

The performance charts of Reference 3 and those presented here are derived for constant values of μ, M_T, and θ_1. The relationships between the basic rotor performance parameters, such as C_L'/σ, C_D'/σ, C_Q/σ, a_1, α_C and $\theta_{.75}$, are presented in the form of carpet plots. The parameters such as λ, a_0 and b_1 for all values of μ are presented as a function of rotor angle of attack, α_C, for constant values of C_L'/σ. Using the above parameters, the rotor side force coefficient can be computed from the following equation:

$$\frac{C_Y'}{\sigma} = \frac{a}{2}\left(-\frac{3}{4}\mu\,\theta_{.75}a_0 + \frac{1}{3}\theta_{.75}b_1 + \frac{3}{8}\mu^2\theta_{.75}b_1 + \frac{3}{4}\lambda b_1\right.$$

$$\left. + \frac{1}{6}a_0 a_1 - \frac{3}{2}\mu\lambda a_0 - \mu^2 a_0 a_1 + \frac{1}{4}\mu a_1 b_1 + \frac{1}{8}\mu^2\lambda b_1\right)$$

The utilization of the above mentioned performance charts with the analytical expressions wherever necessary constitutes an integral part of the stability method presented in this Handbook.

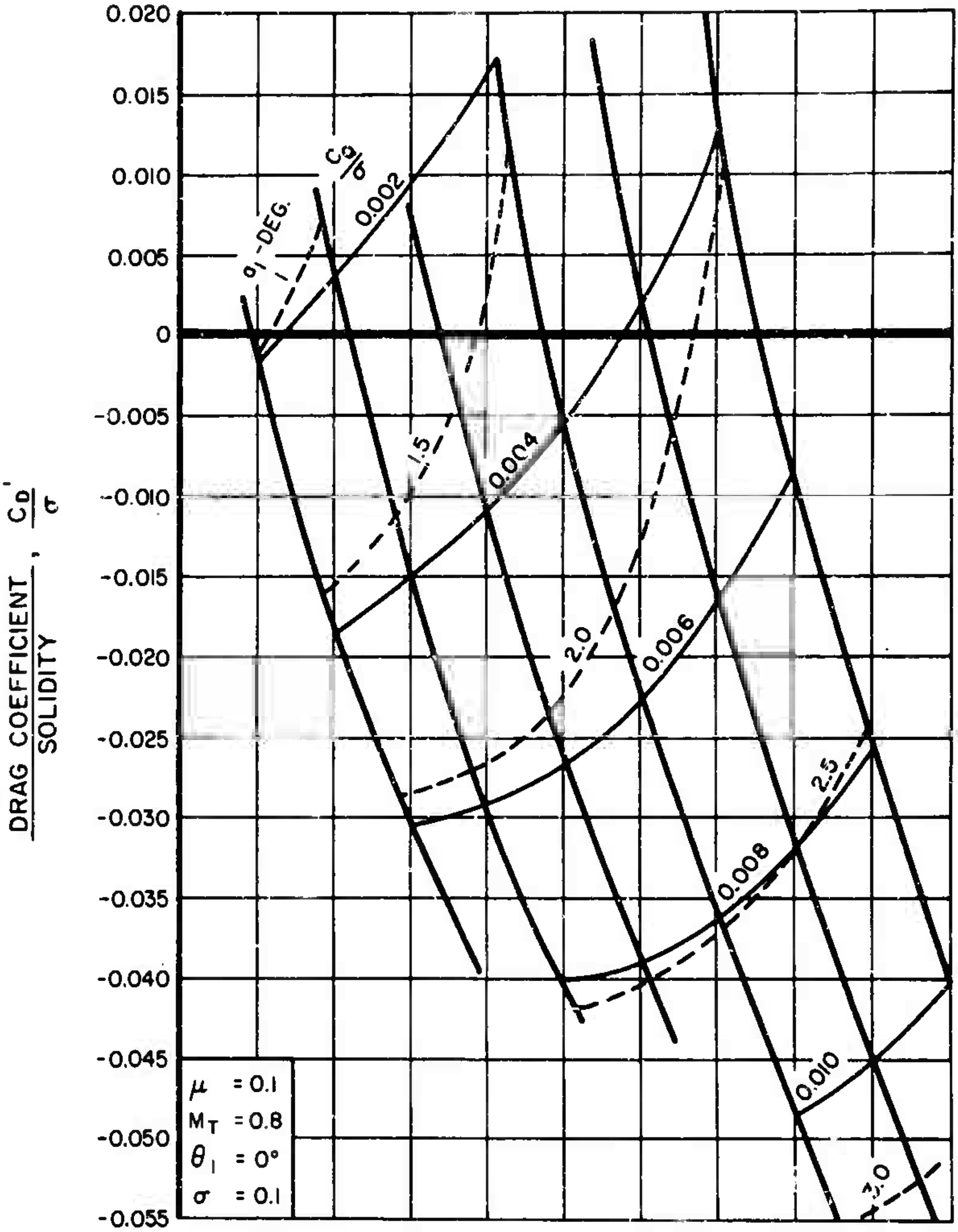

Figure 1. Calculated Characteristics of a Rotor With 0° Twist for μ = 0.1 and M_T = 0.8.

(a) $\frac{C_Q}{\sigma}$ and a_1

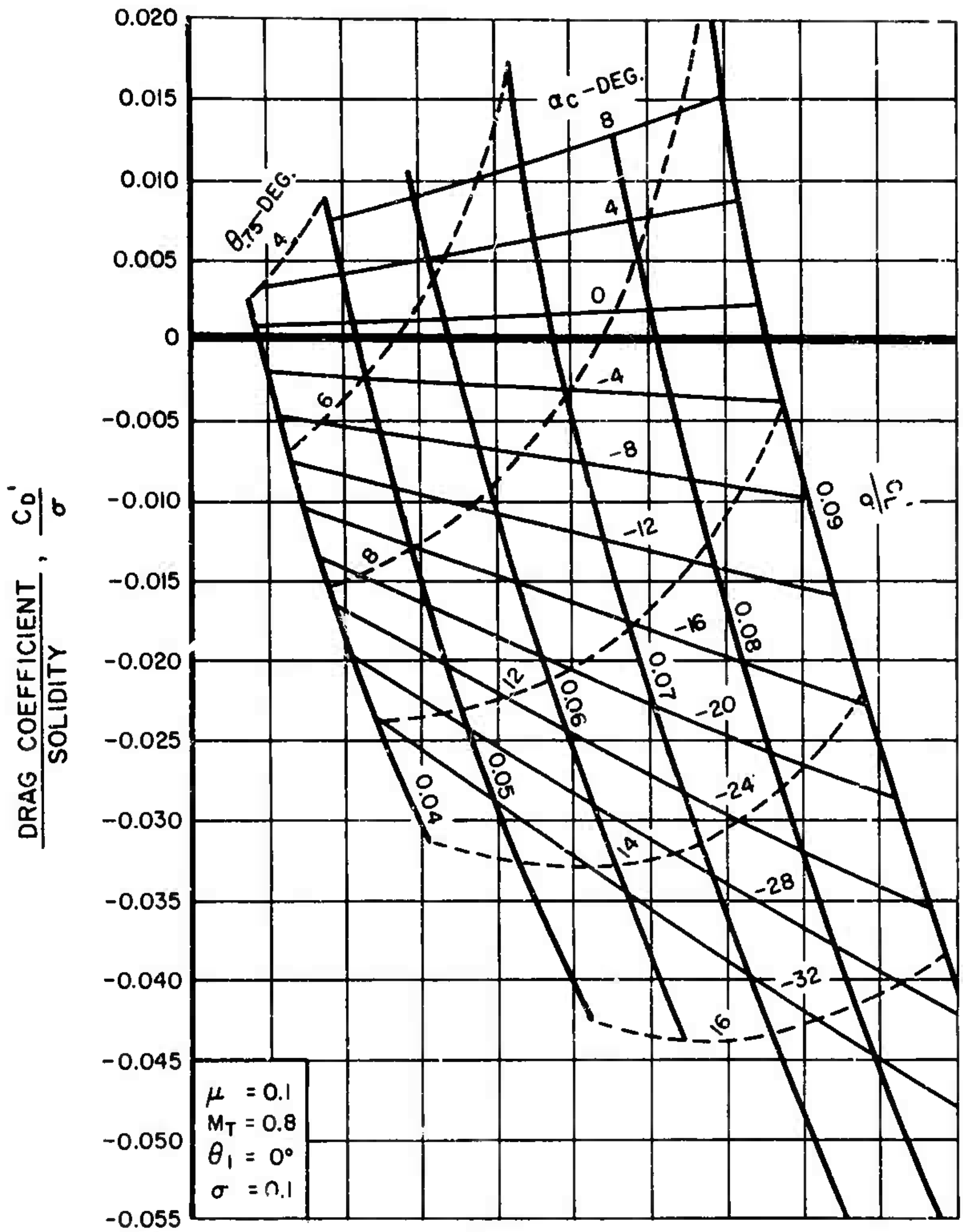

Figure 1. Continued

(b) α_C and $\theta_{.75}$

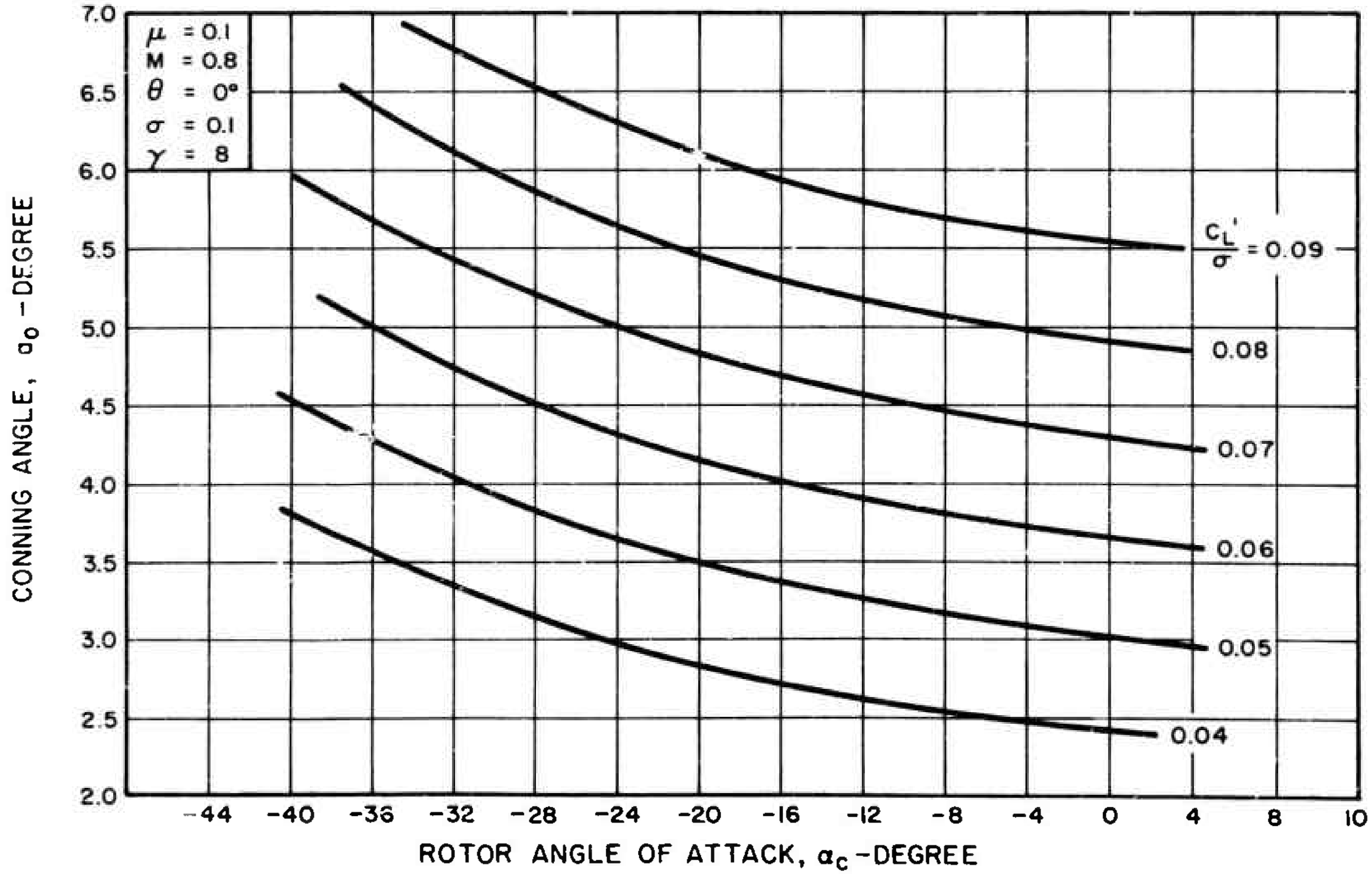

Figure 1. Continued

(c) a_0

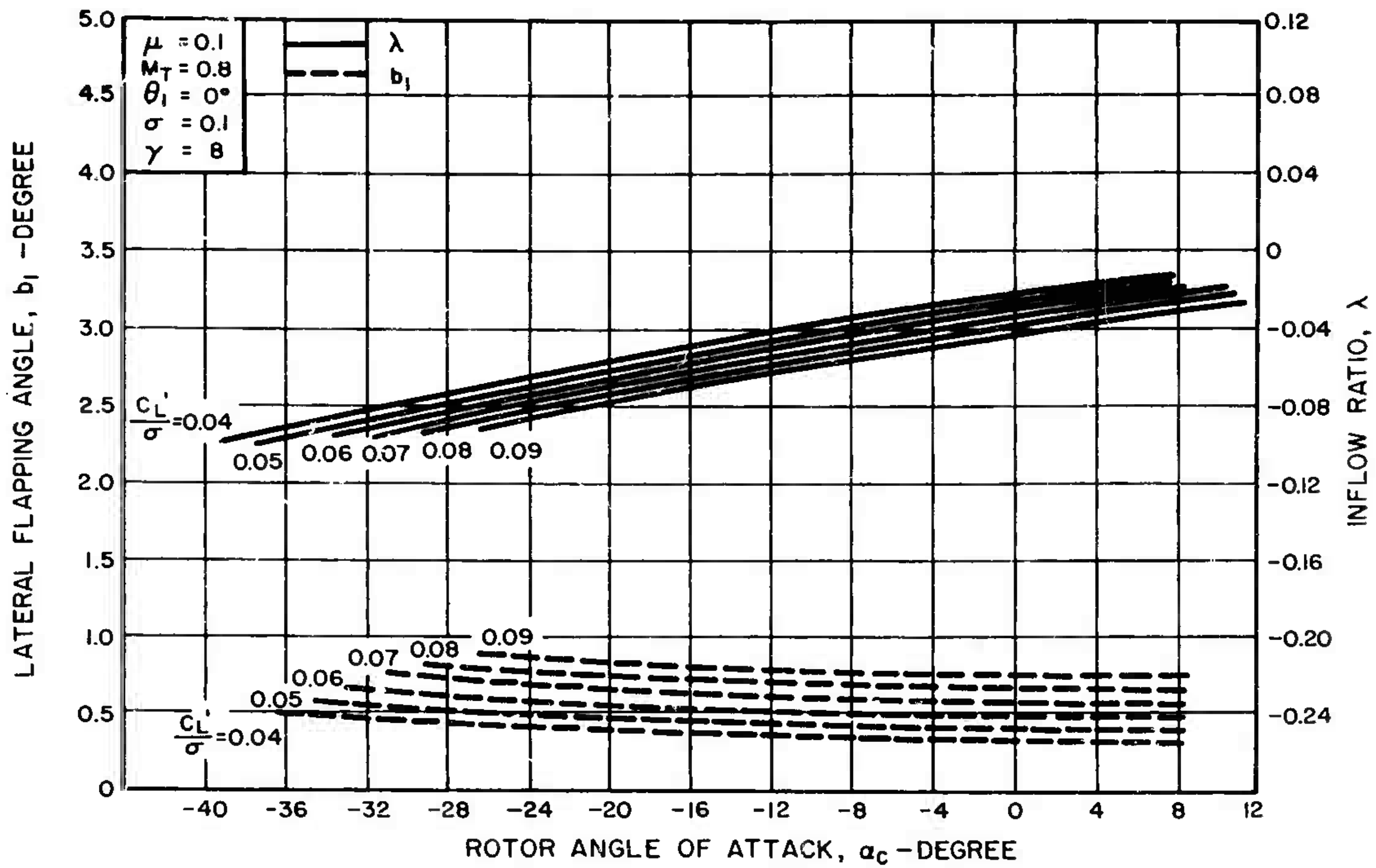

Figure 1. Concluded

(d) b_1 and λ

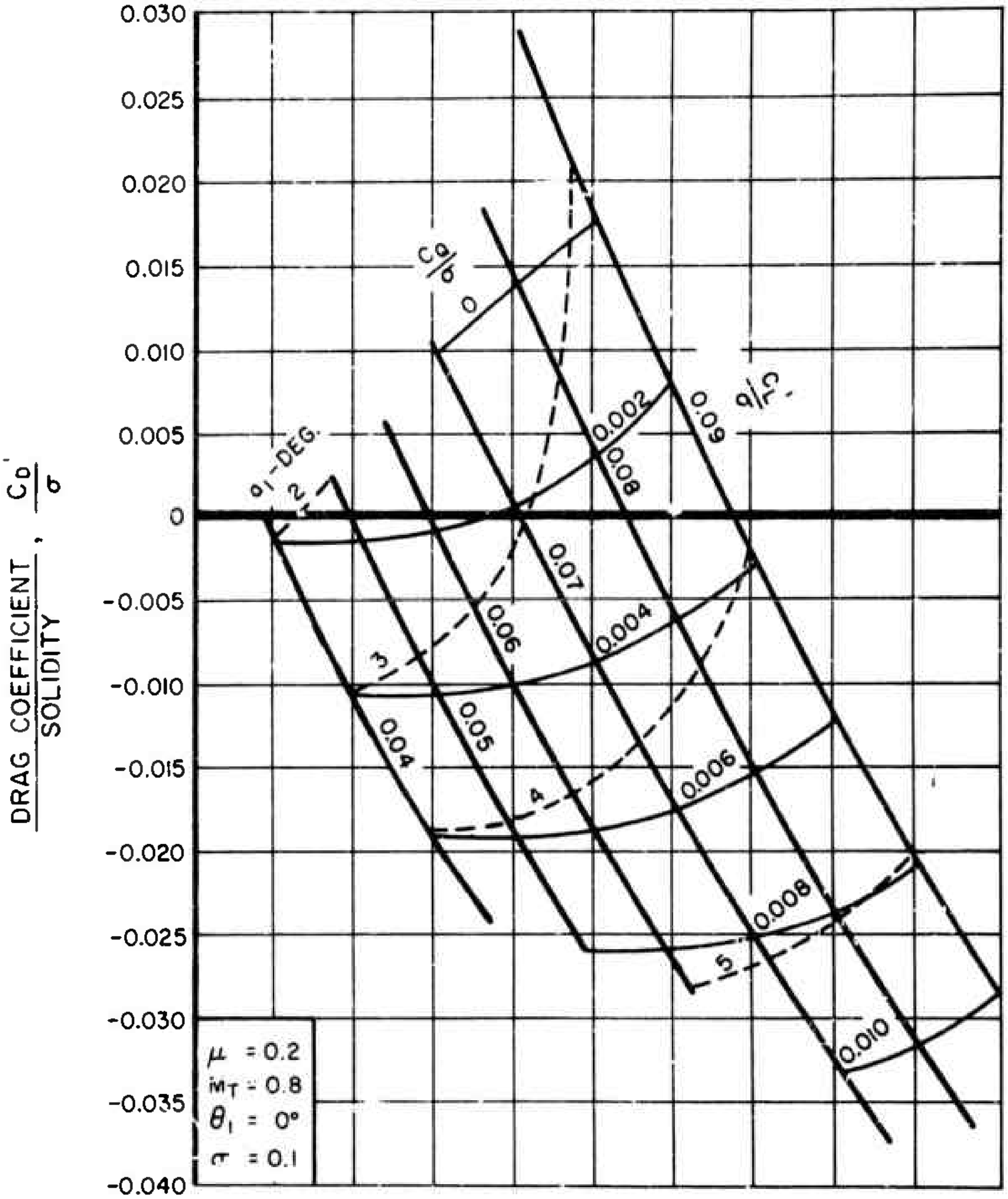

Figure 2. Calculated Characteristics of a Rotor With 0° Twist for μ = 0.2 and M_T = 0.8.

(a) $\frac{C_Q}{\sigma}$ and a_1

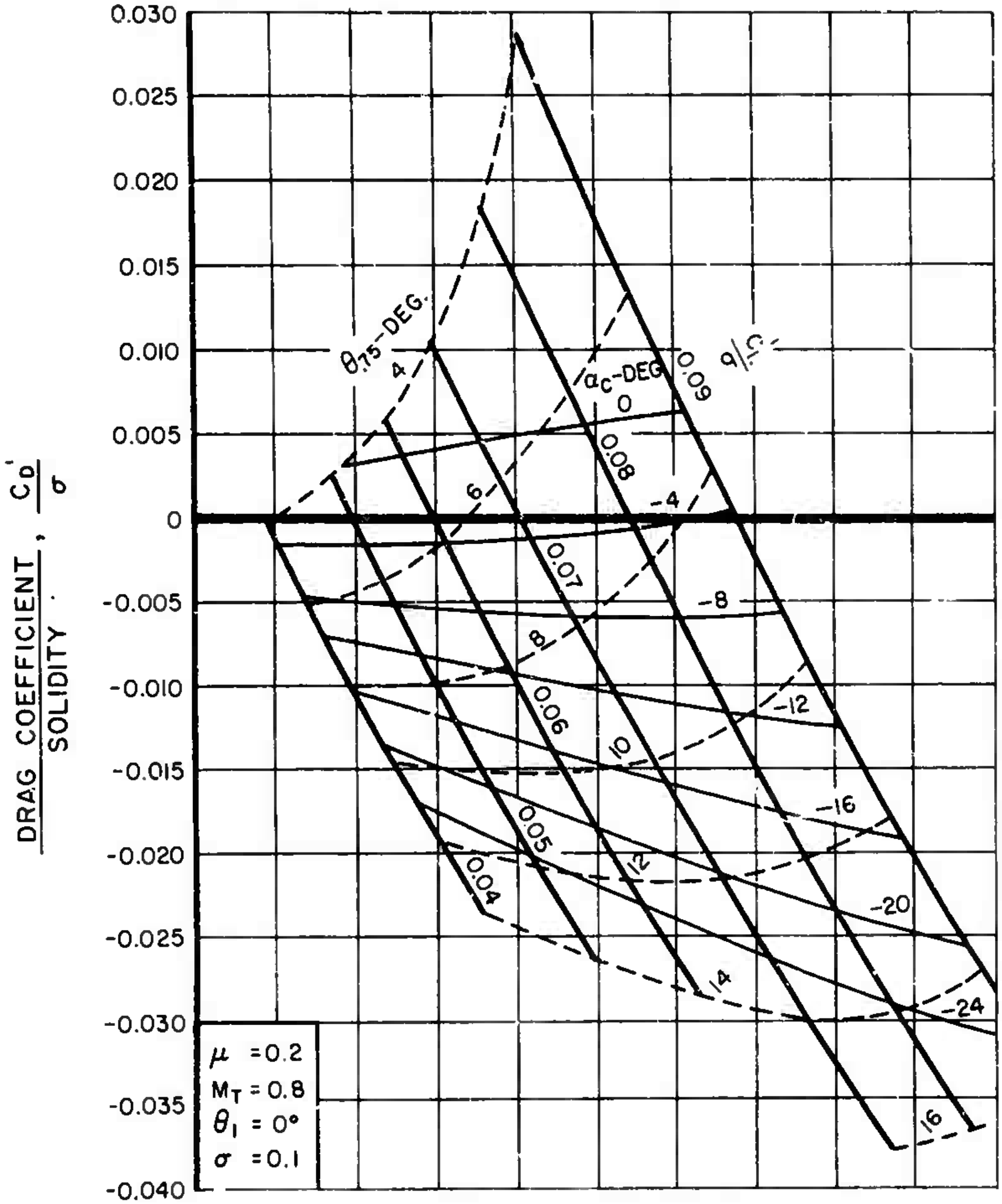

Figure 2. Continued

(b) α_c and $\theta_{.75}$

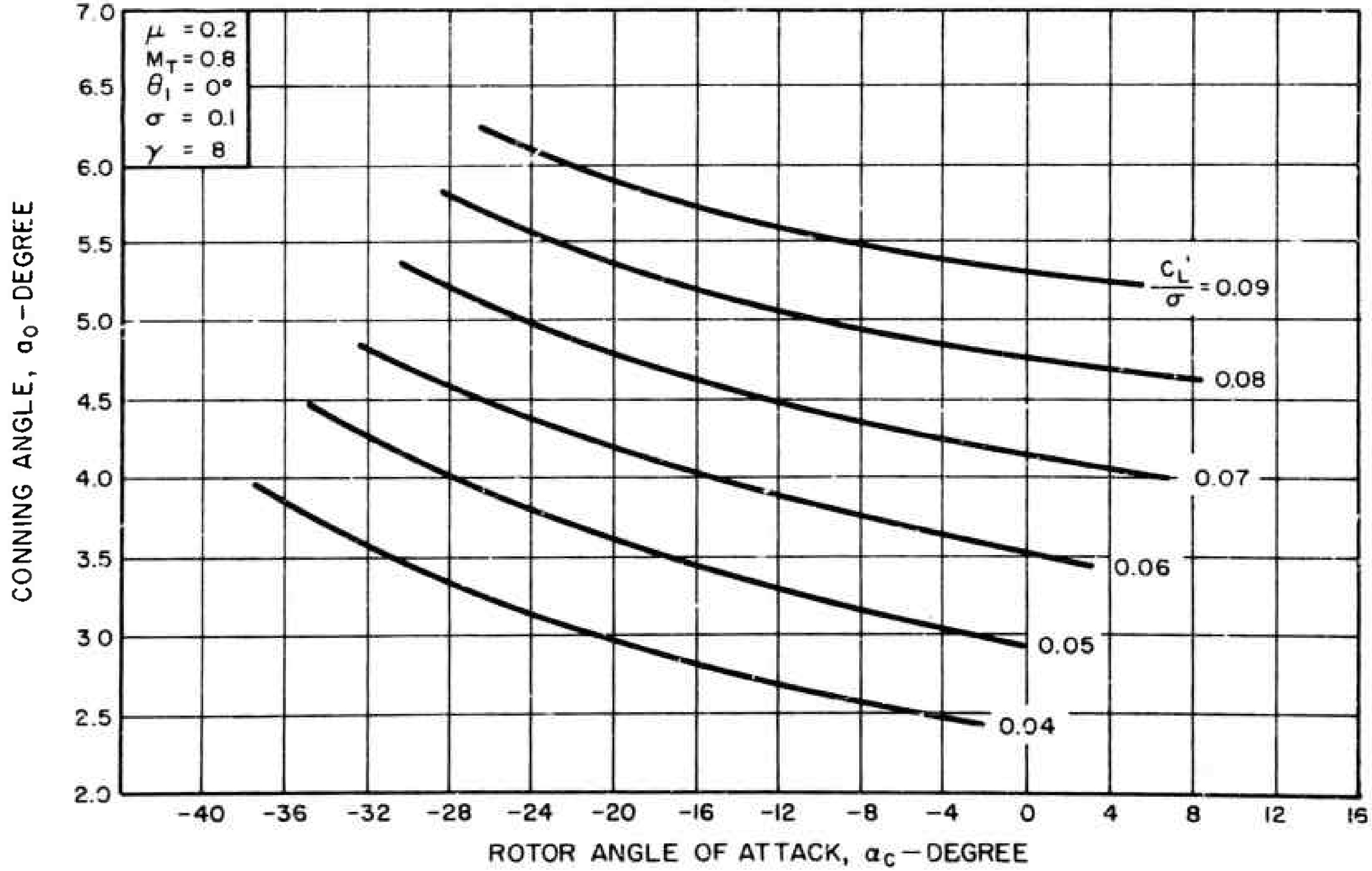

Figure 2. Continued

(c) a_0

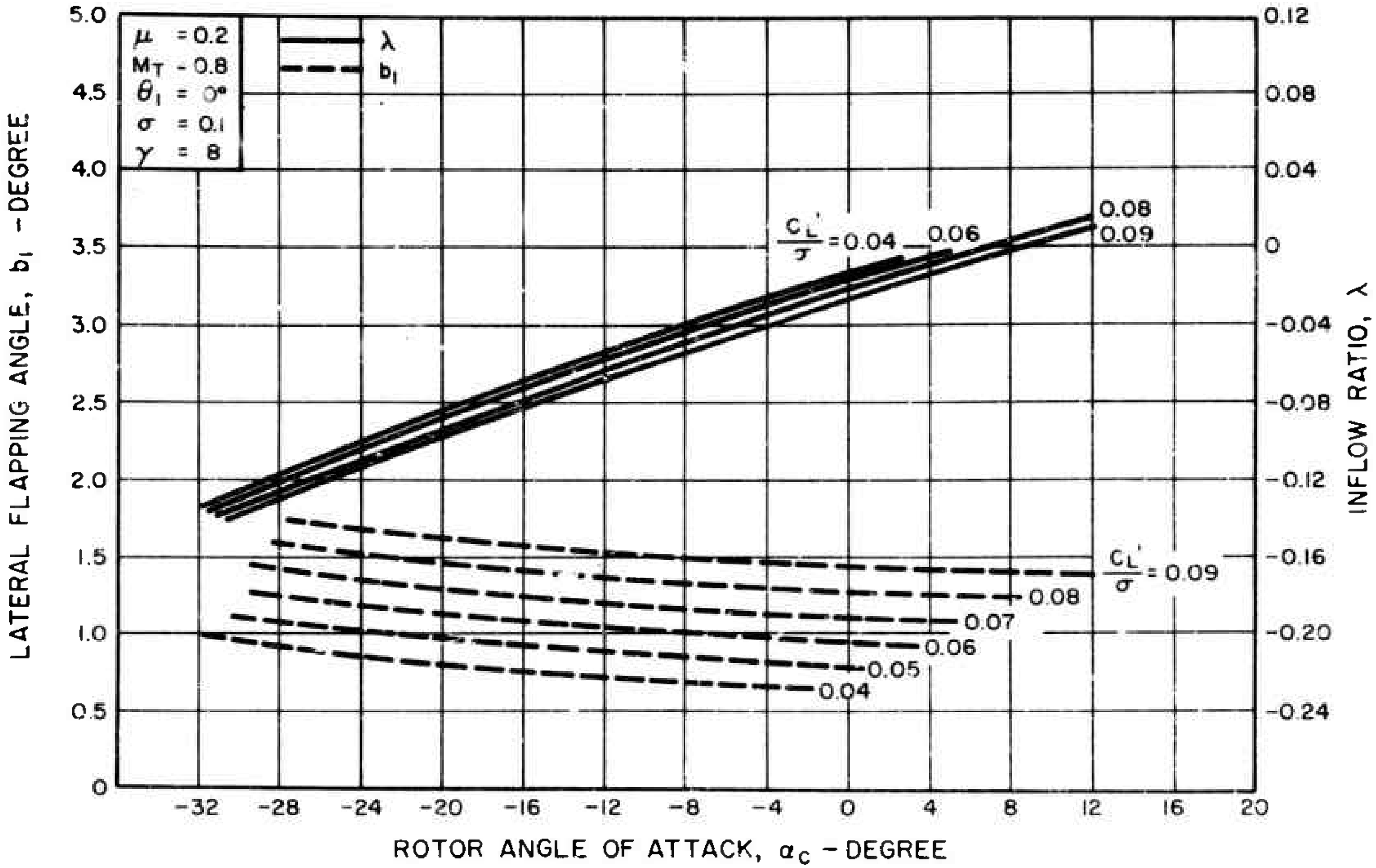

Figure 2. Concluded

(d) b_1 and λ

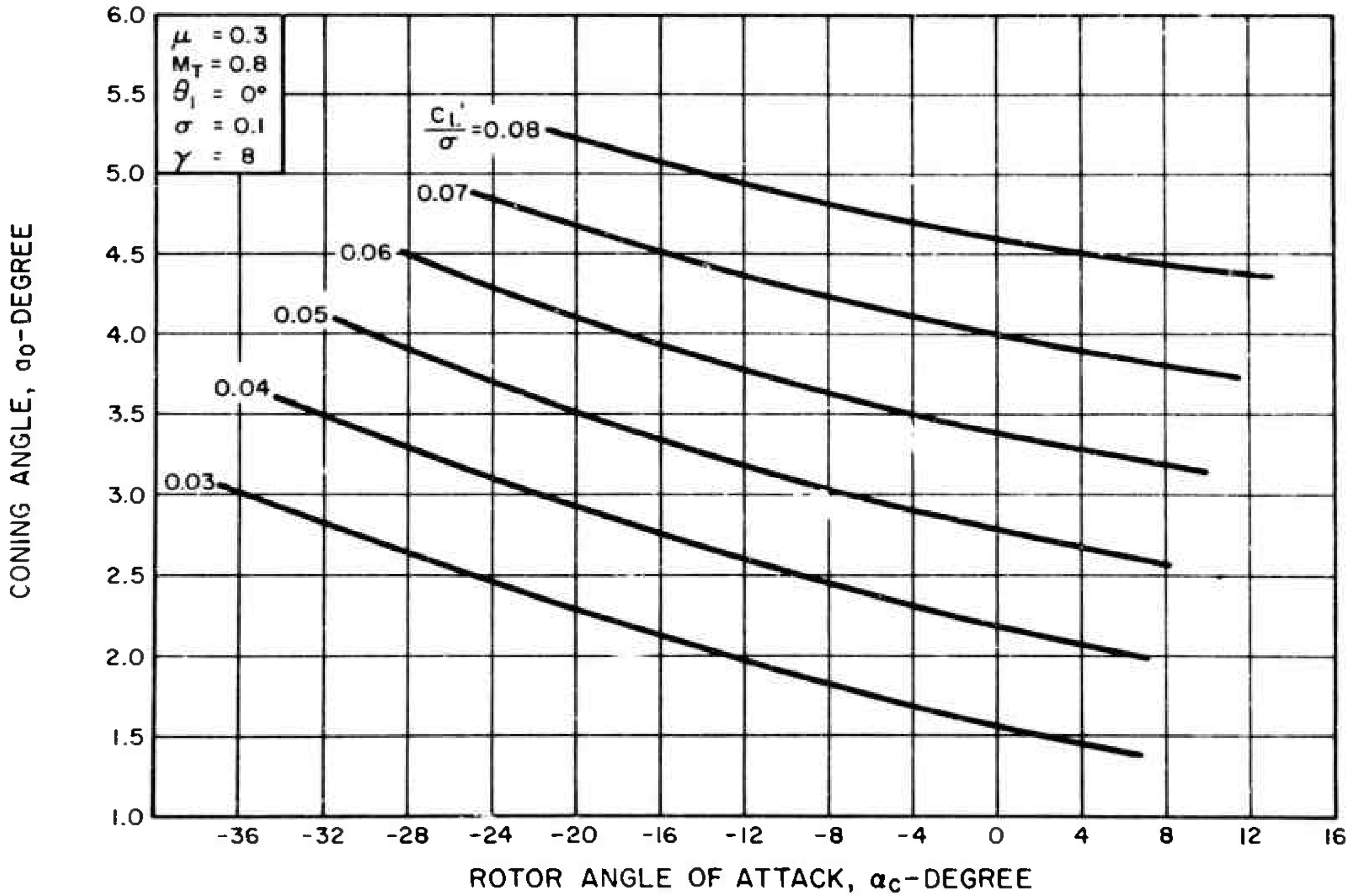

Figure 3. Calculated Characteristics of a Rotor With 0° Twist for $\mu = 0.3$ and $M_T = 0.8$.

(a) a_o

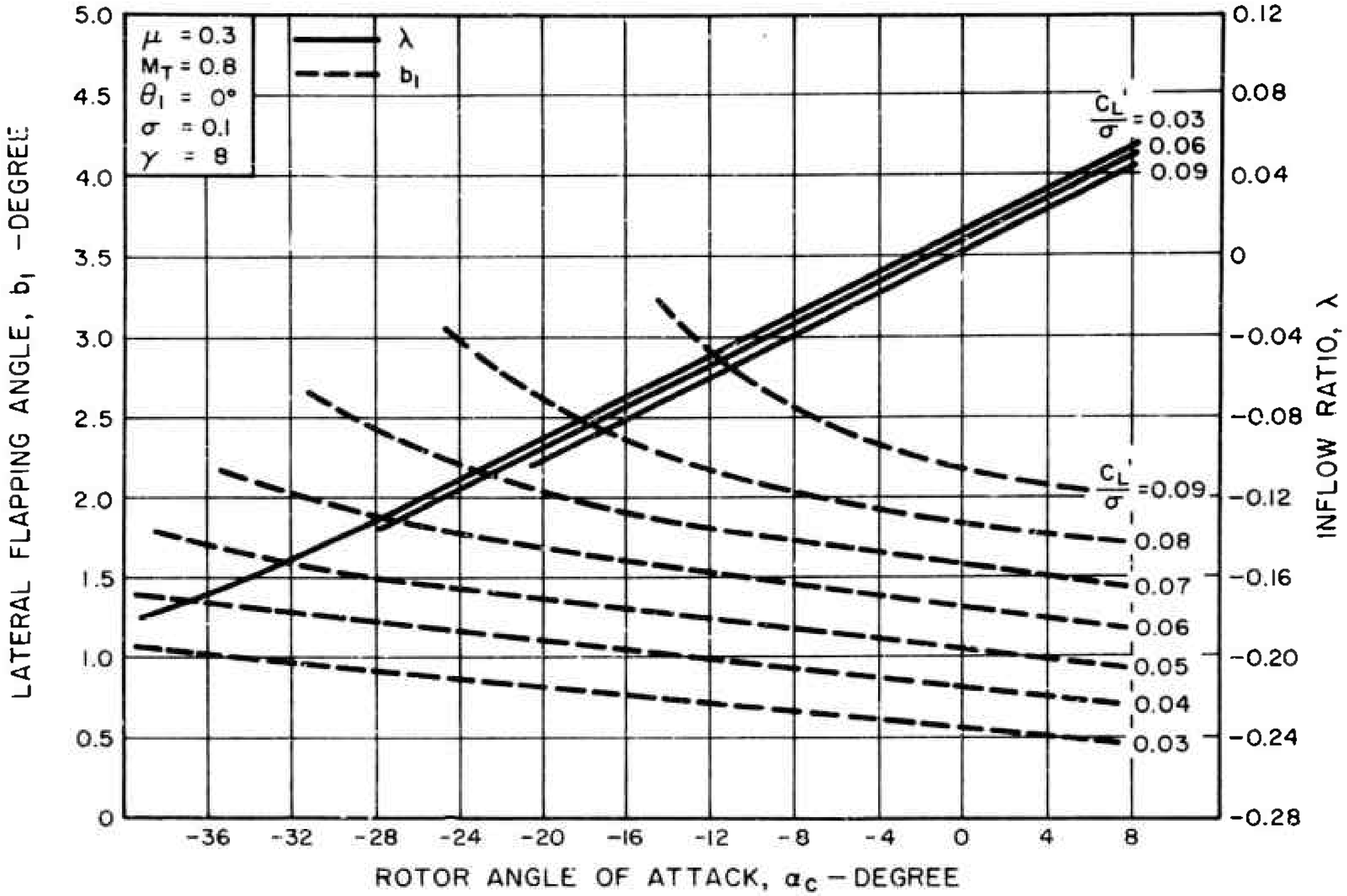

Figure 3. Concluded
(b) b_1 and λ

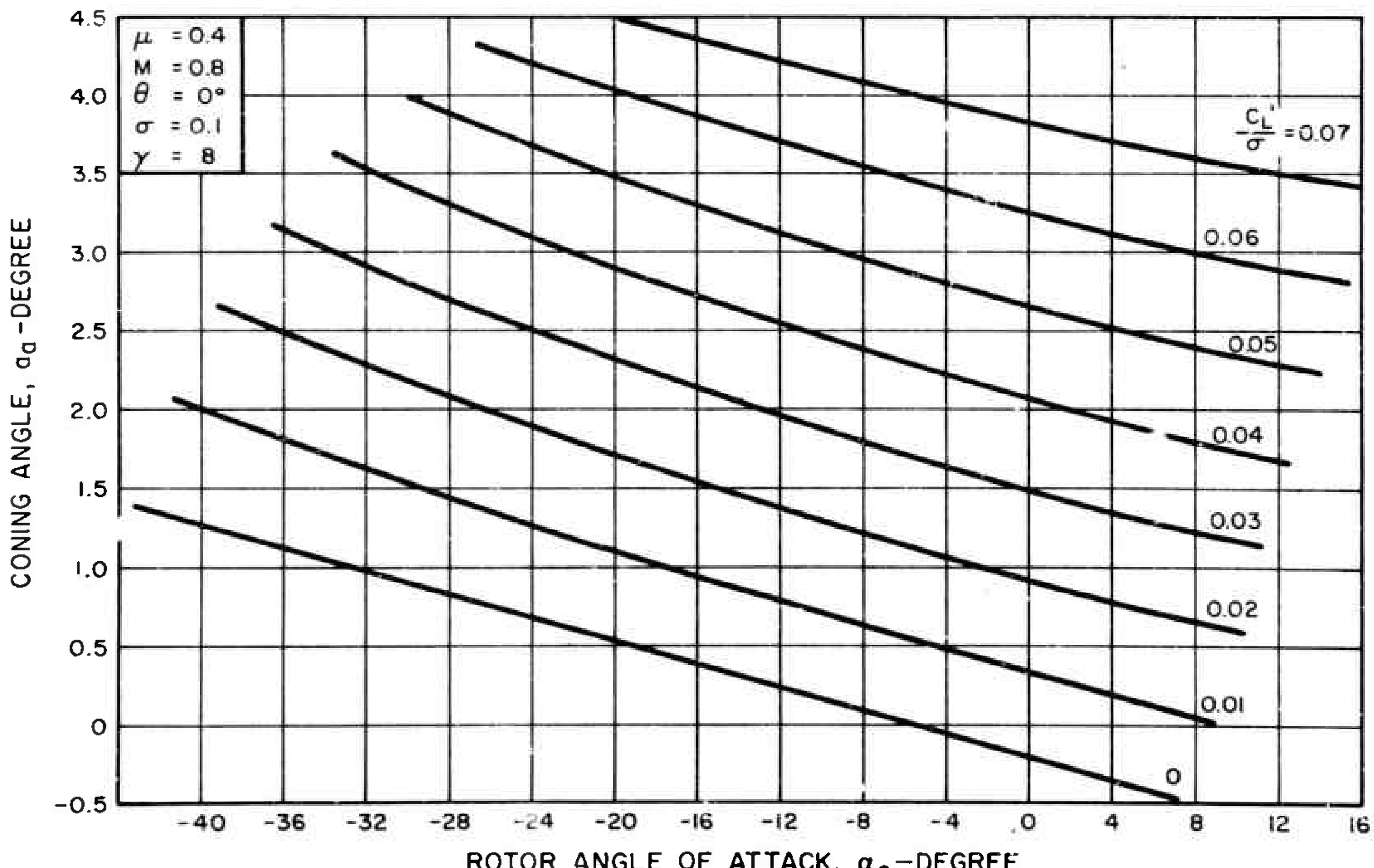

Figure 4. Calculated Characteristics of a Rotor With 0° Twist for $\mu = 0.4$ and $M_T = 0.8$.

(a) a_0

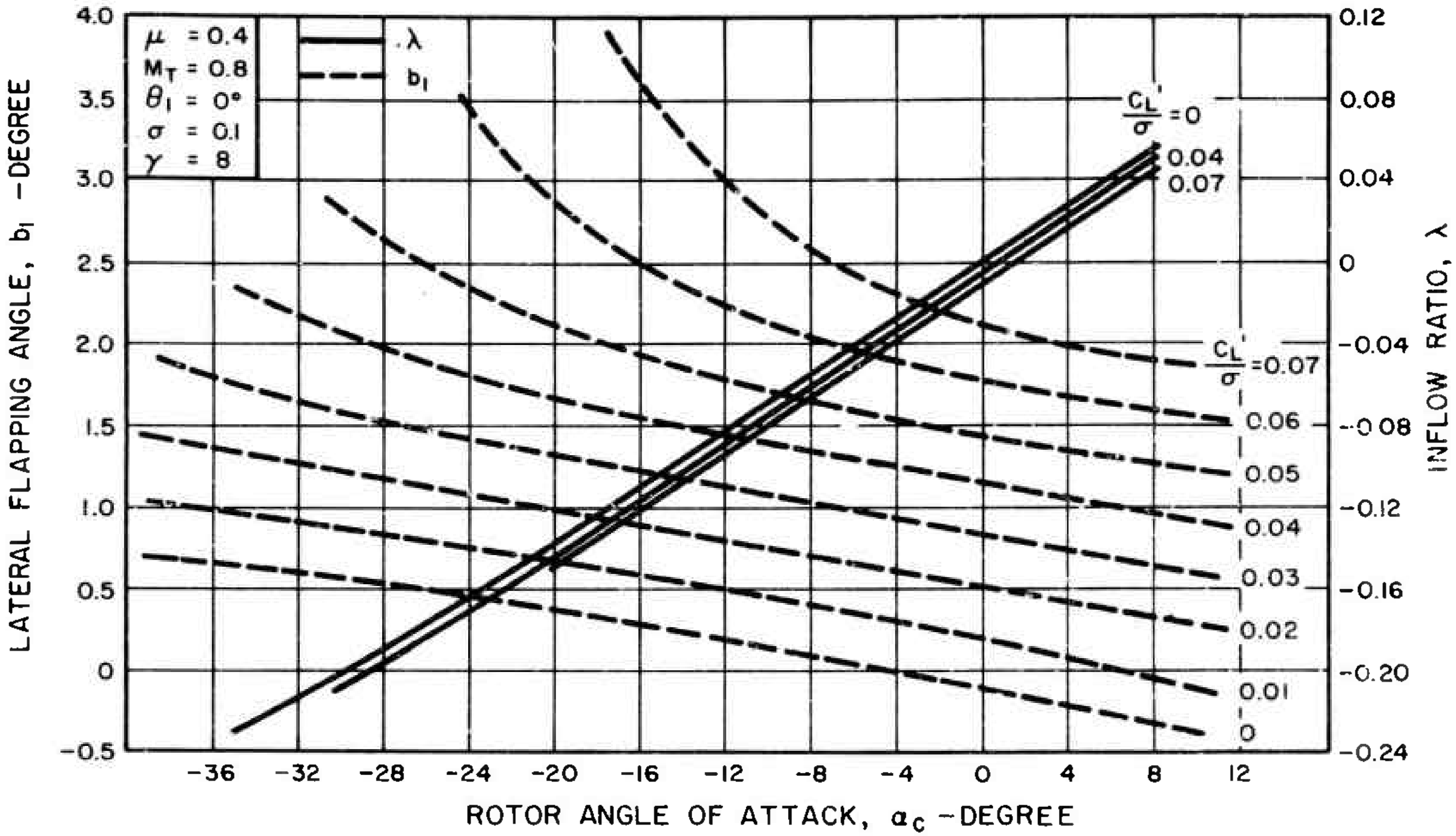

Figure 4. Concluded.

(b) b_1 and λ

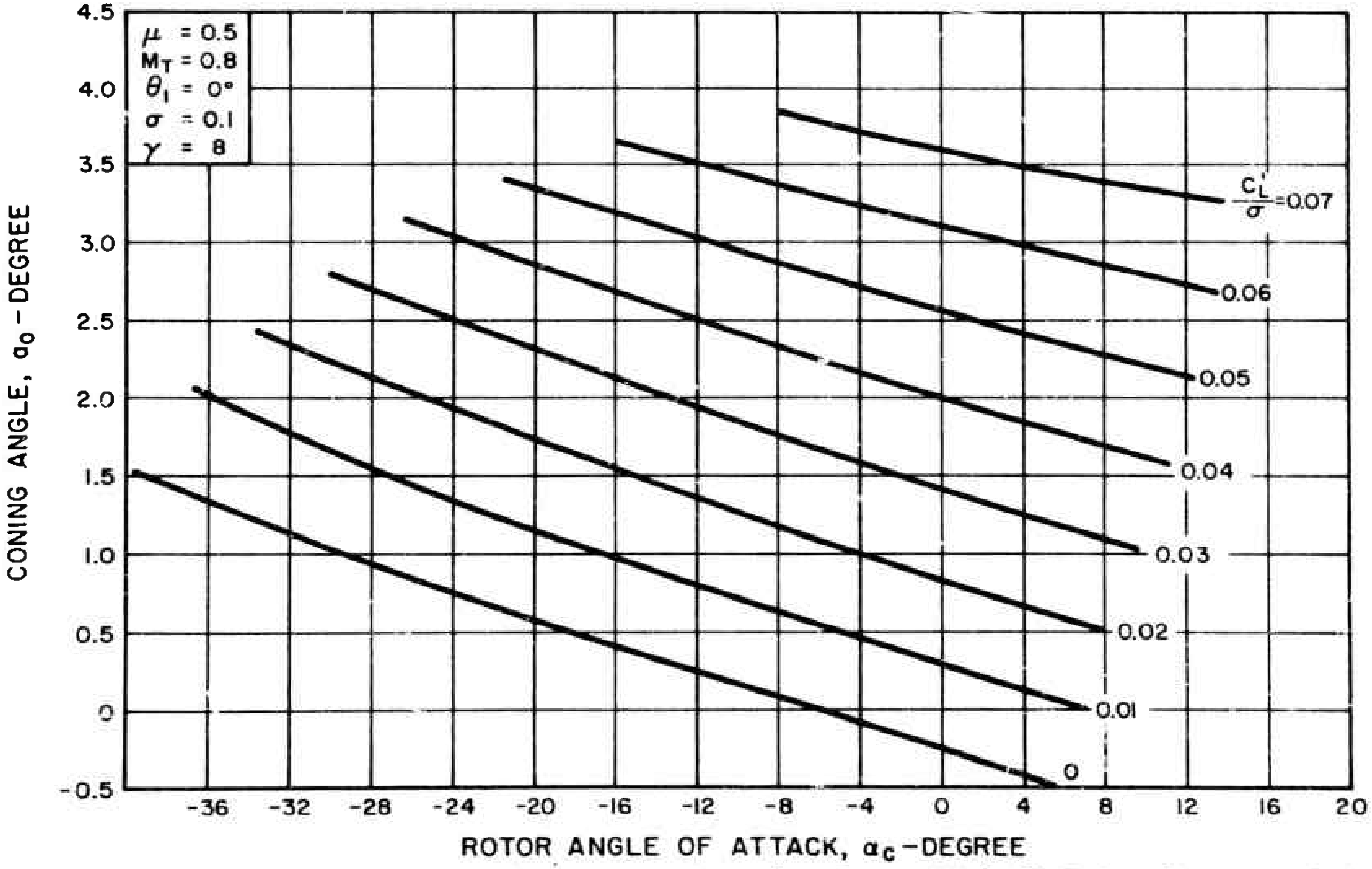

Figure 5. Calculated Characteristics of a Rotor With 0° Twist for μ = 0.5 and M_T = 0.8.

(a) a_0

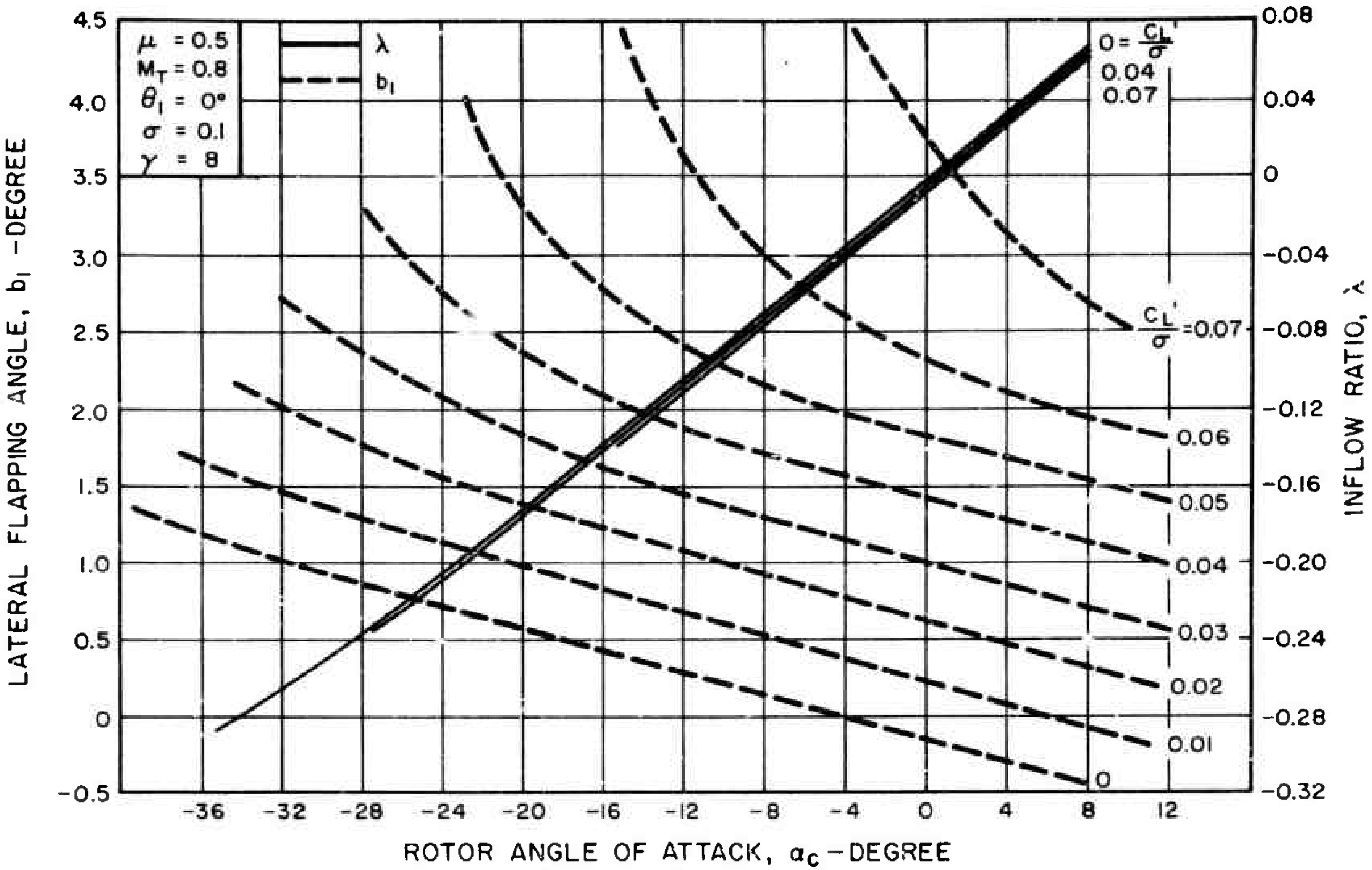

Figure 5. Concluded

(b) b_1 and λ

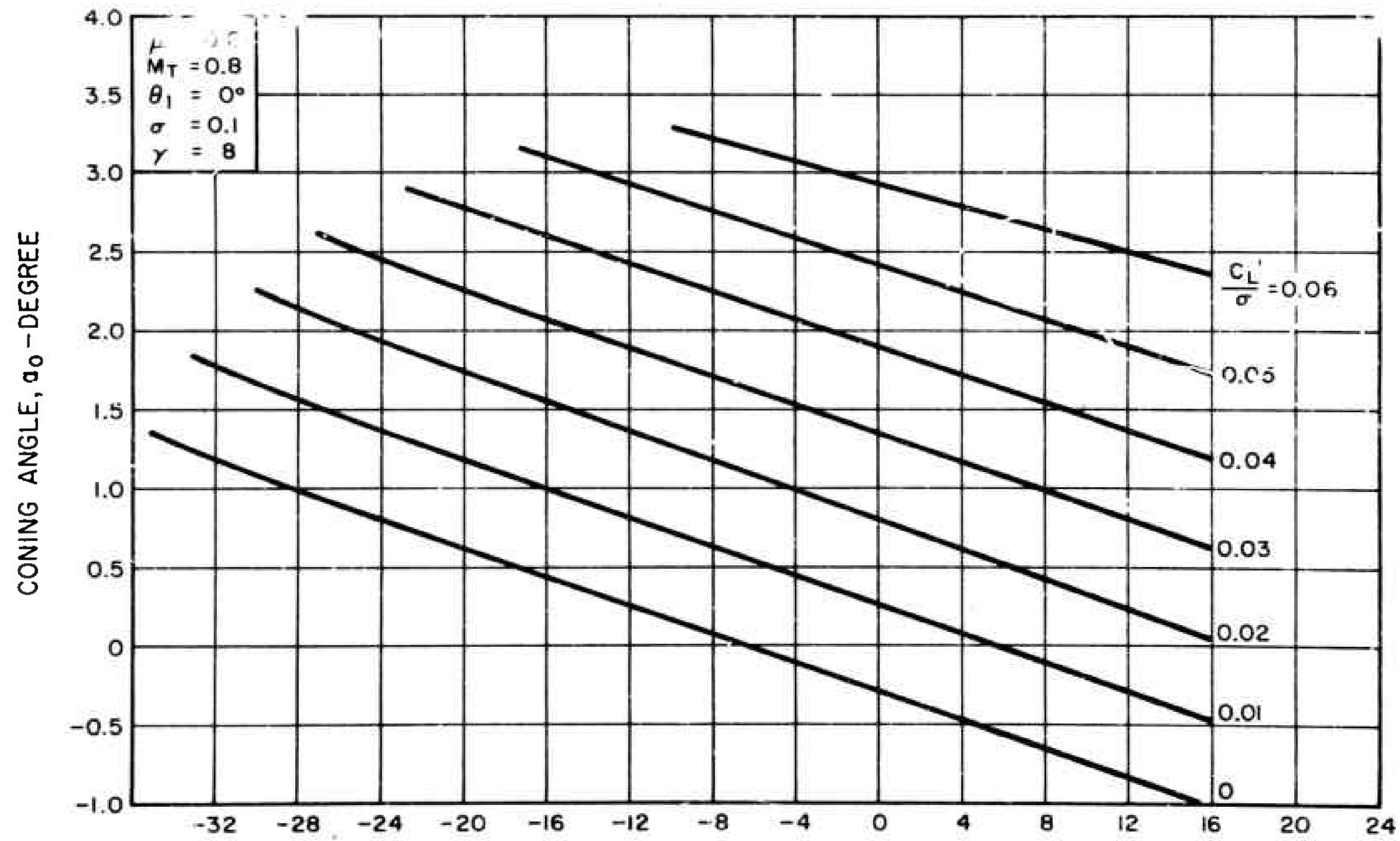

Figure 6. Calculated Characteristics of a Rotor With 0° Twist for μ = 0.6 and M_T = 0.8.

(a) a_0

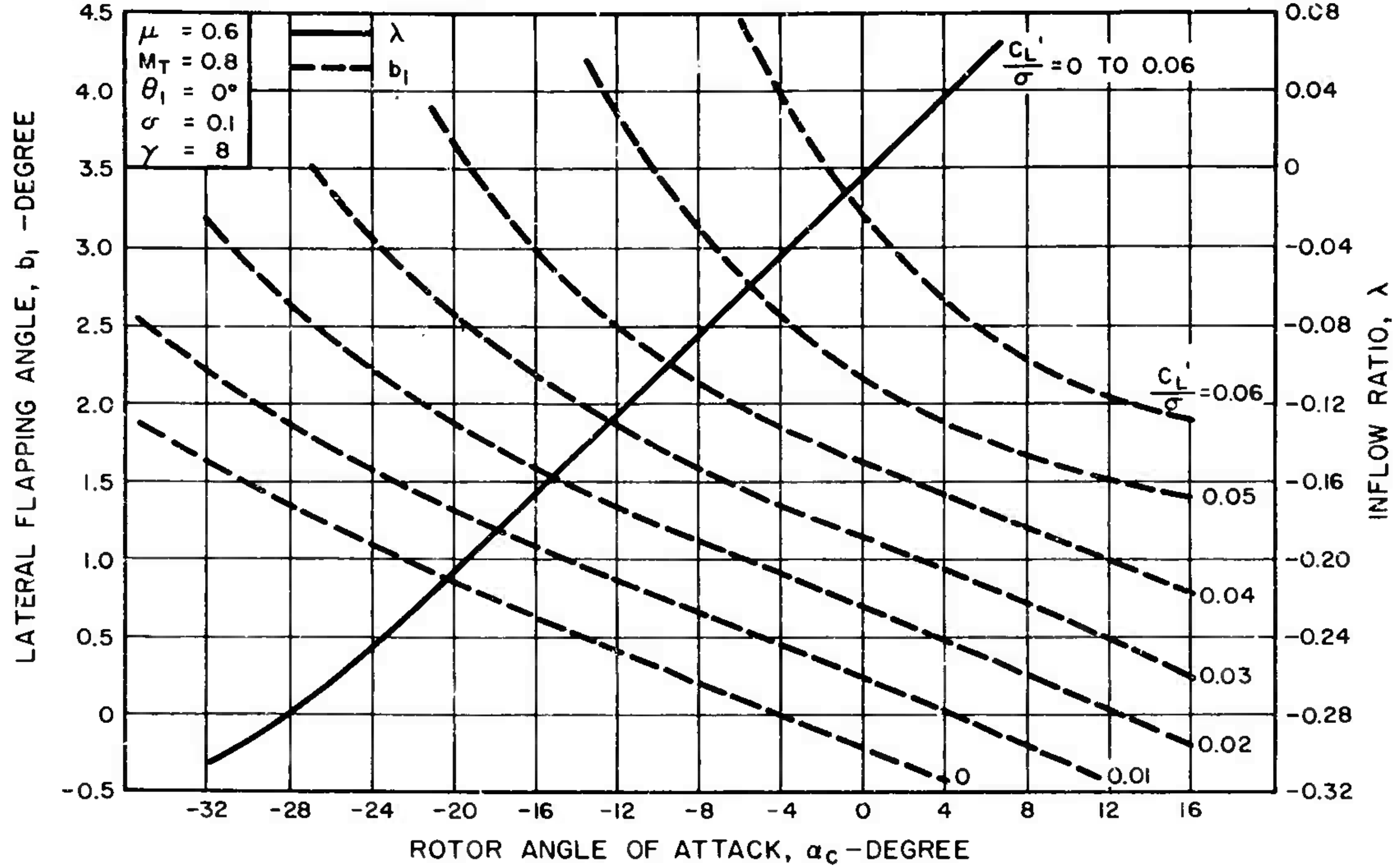

Figure 6. Concluded.

(b) b_1 and λ

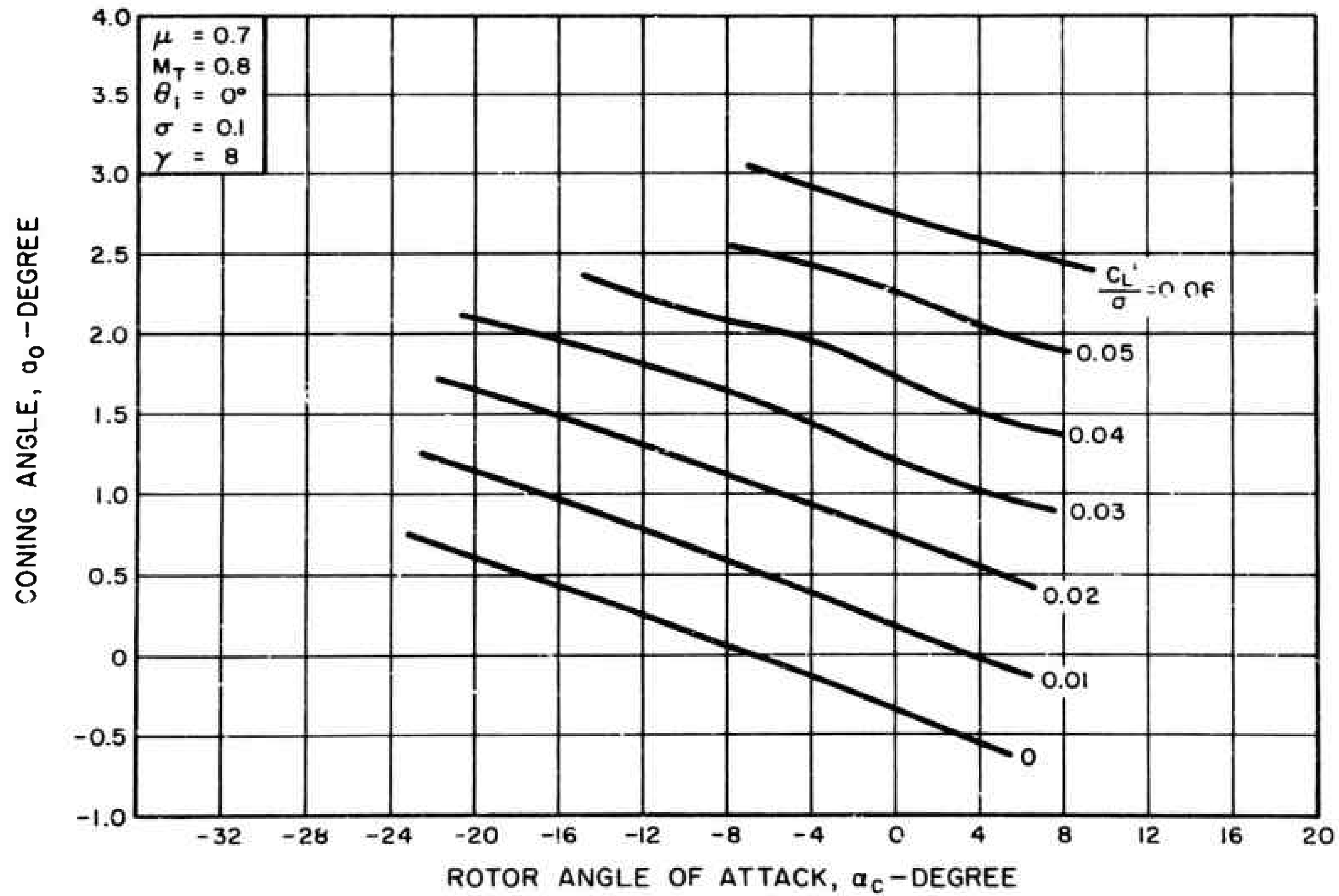

Figure 7. Calculated Characteristics of a Rotor With 0° Twist for μ = 0.7 and M_T = 0.8.

(a) a_0

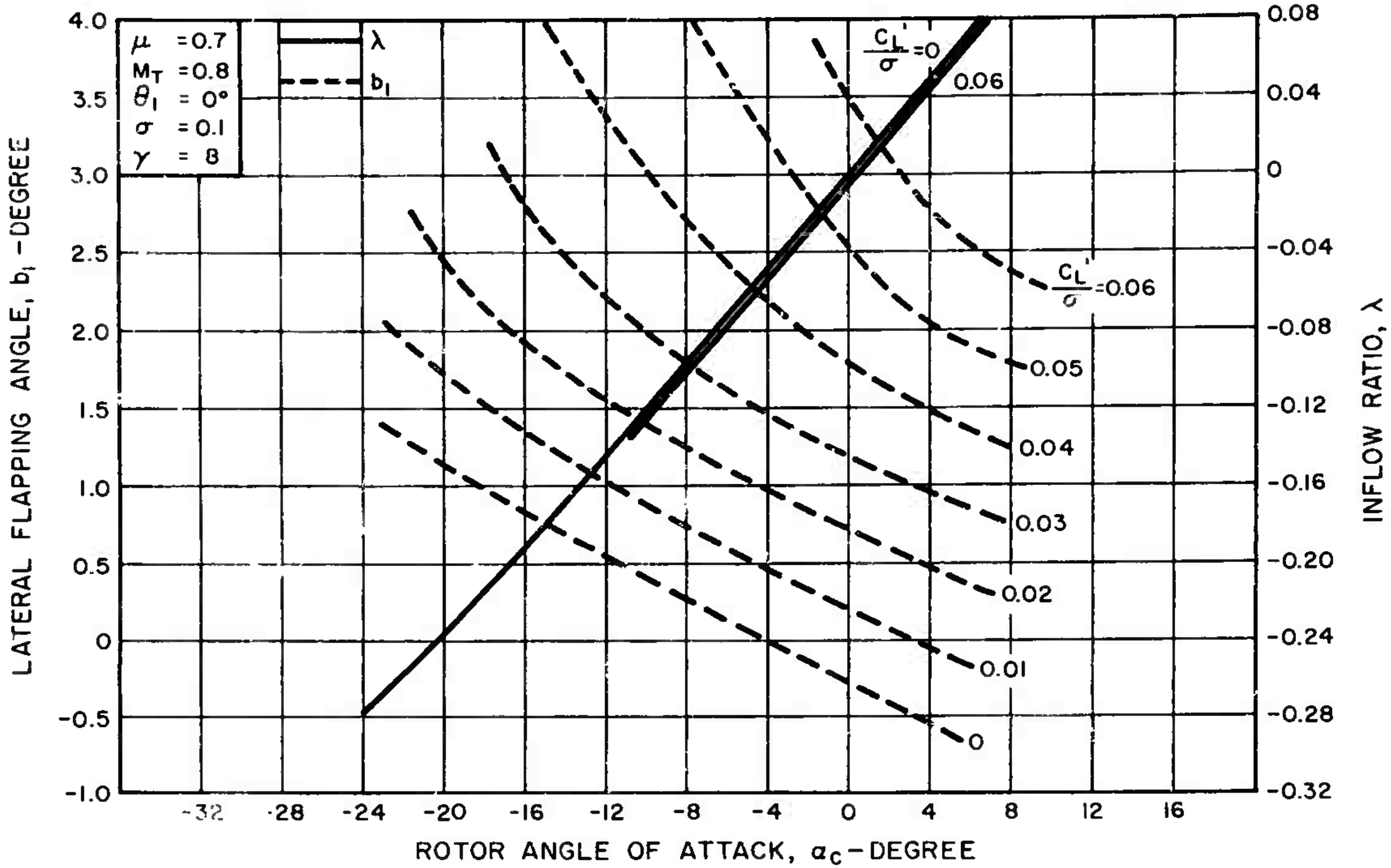

Figure 7. Concluded

b) b_1 and λ

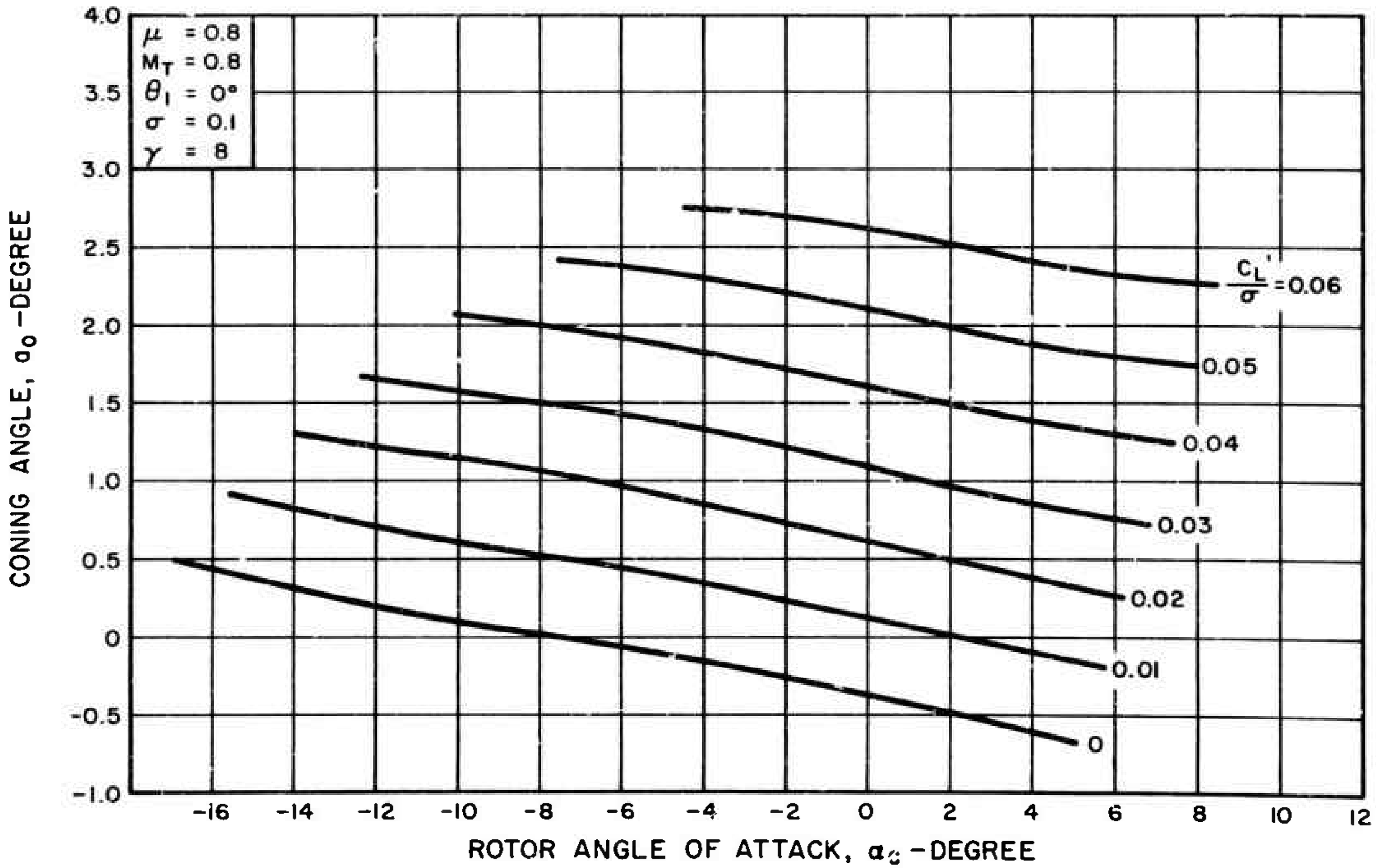

Figure 8. Calculated Characteristics of a Rotor With 0° Twist for $\mu = 0.8$ and $M_T = 0.8$.

(a) a_0

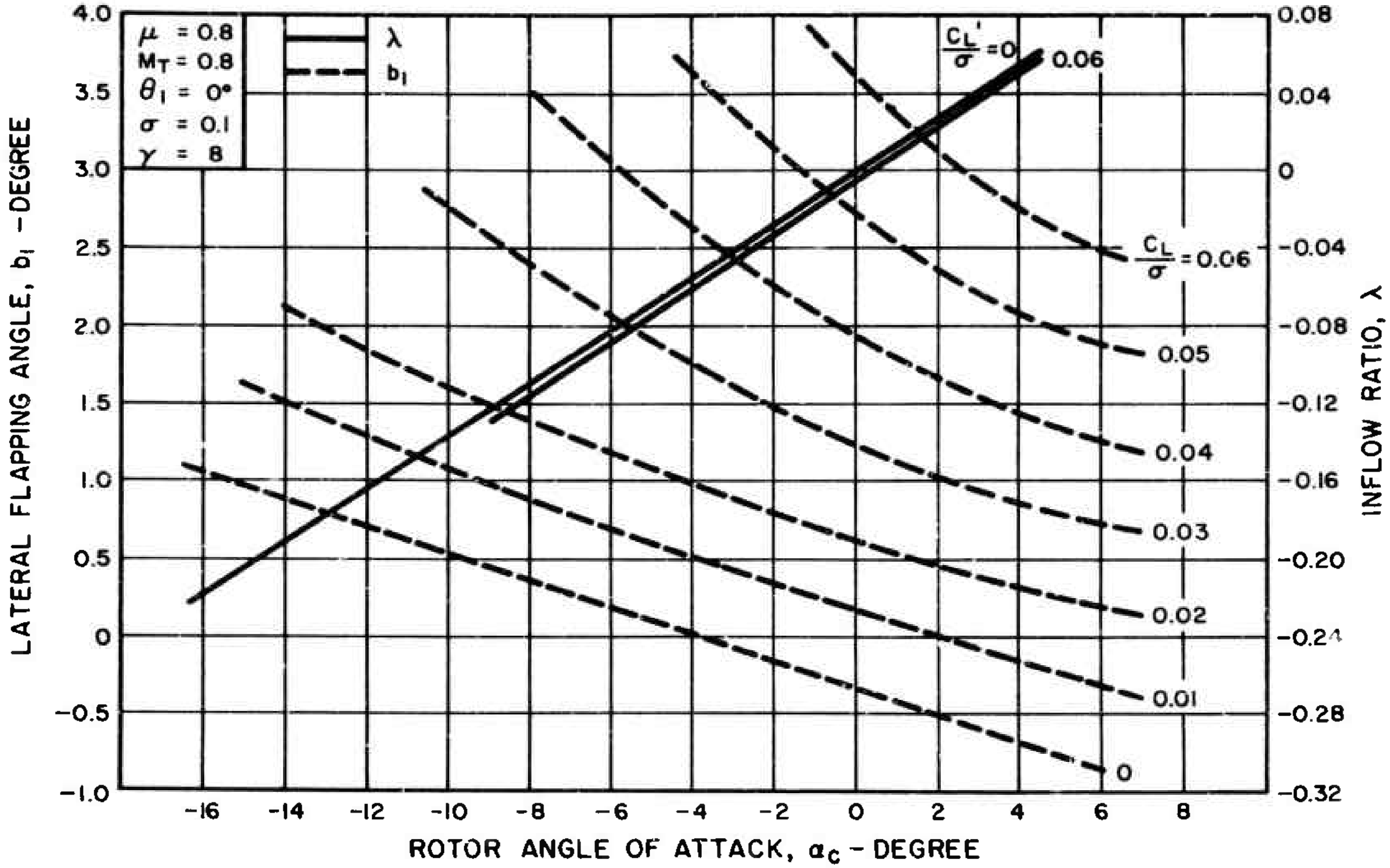

Figure 8. Concluded.

(b) b_1 and λ

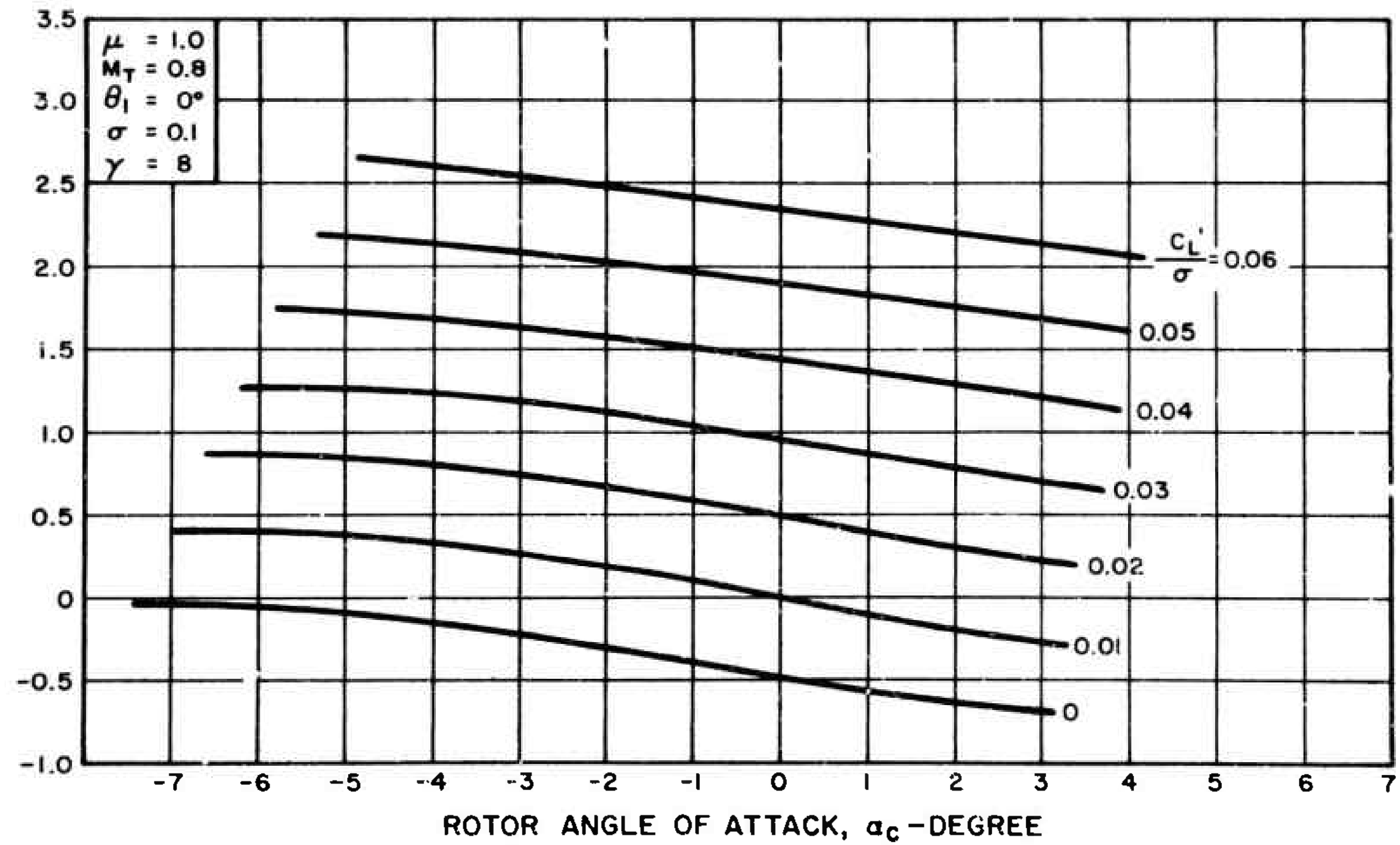

Figure 9. Calculated Characteristics of a Rotor With 0° Twist for μ = 1.0 and M_T = 0.8.

(a) a_0

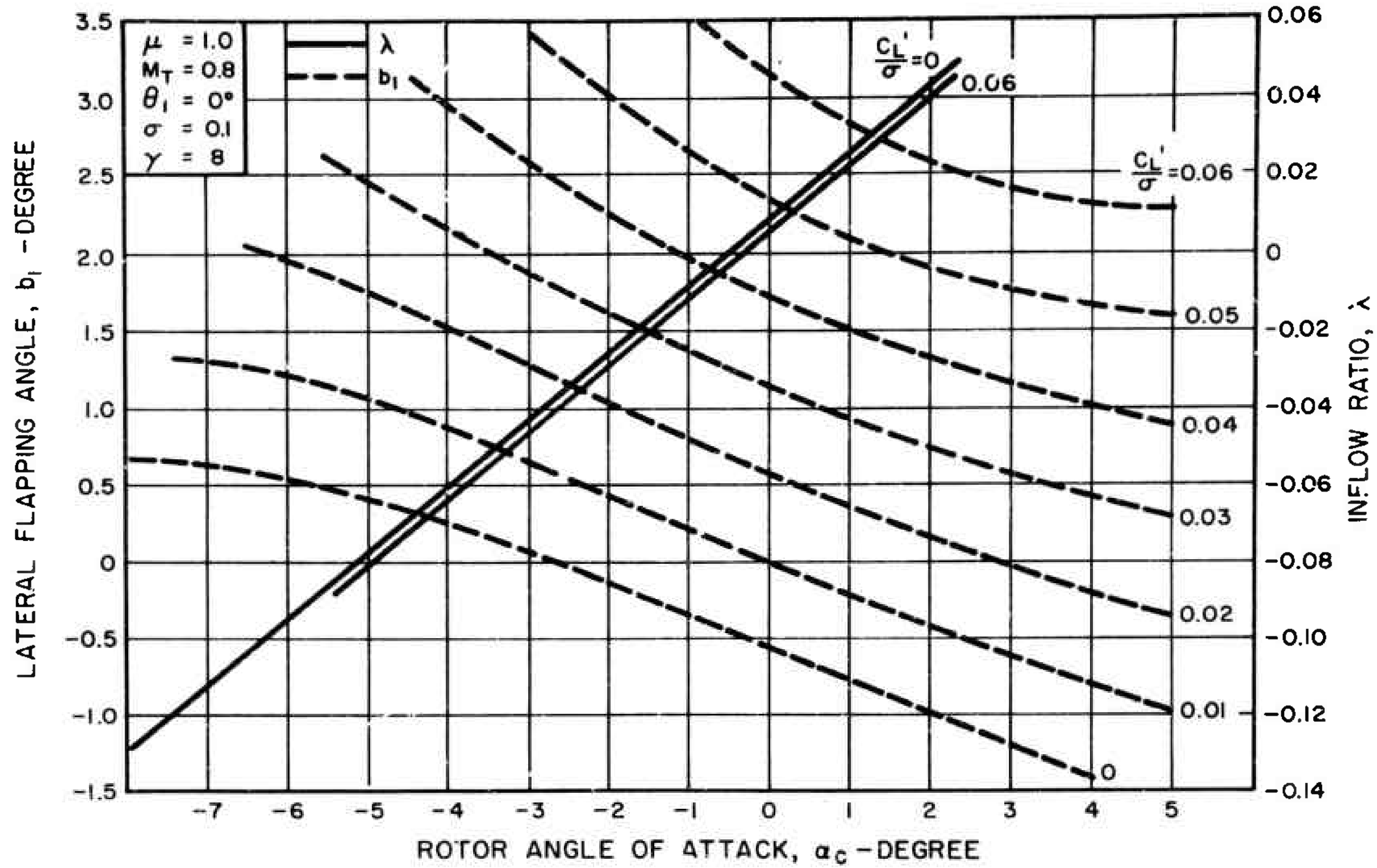

Figure 9. Concluded.

(b) b_1 and λ

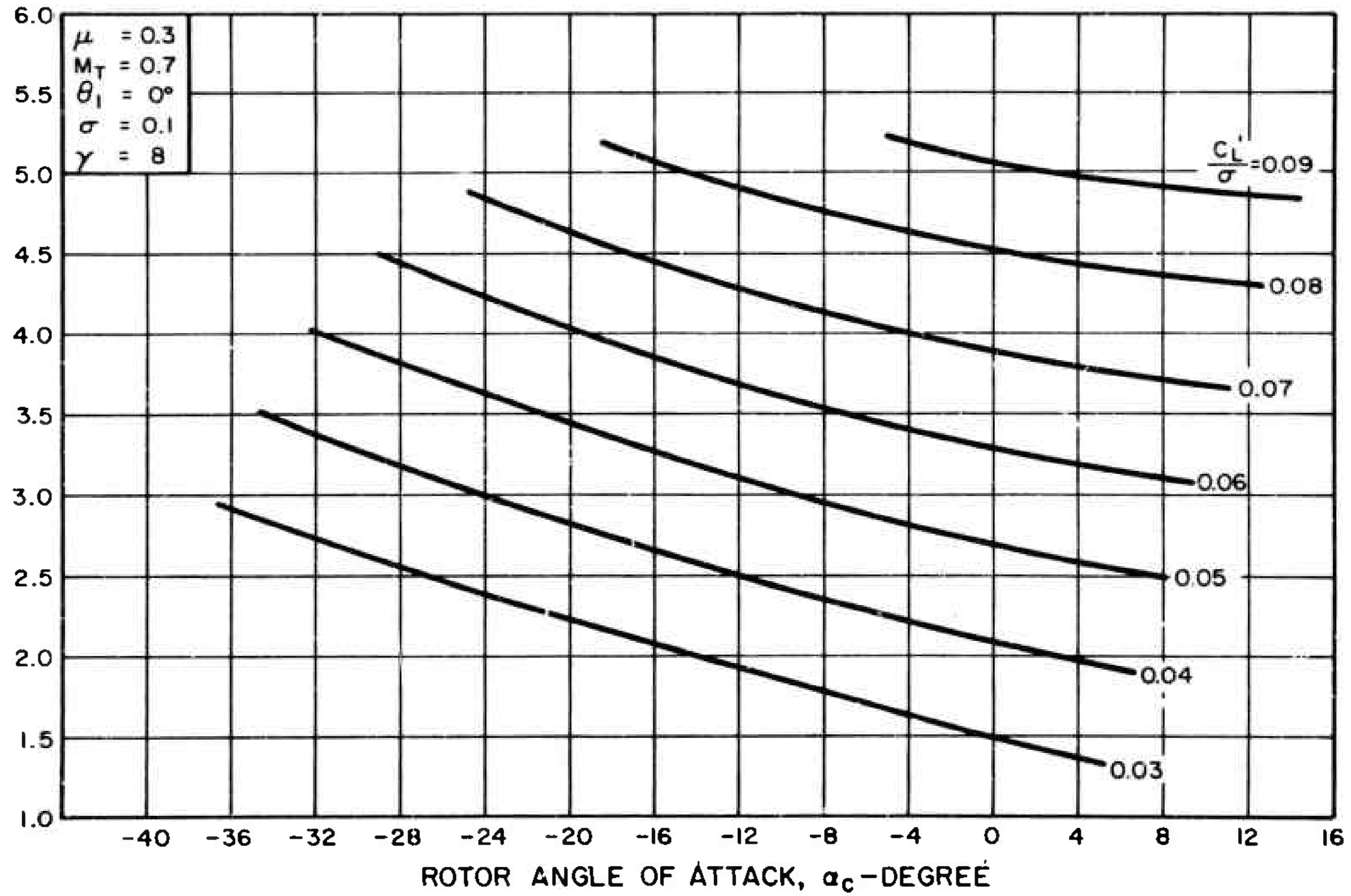

Figure 10. Calculated Characteristics of a Rotor With 0° Twist for $\mu = 0.3$ and $M_T = 0.7$.

(a) a_0

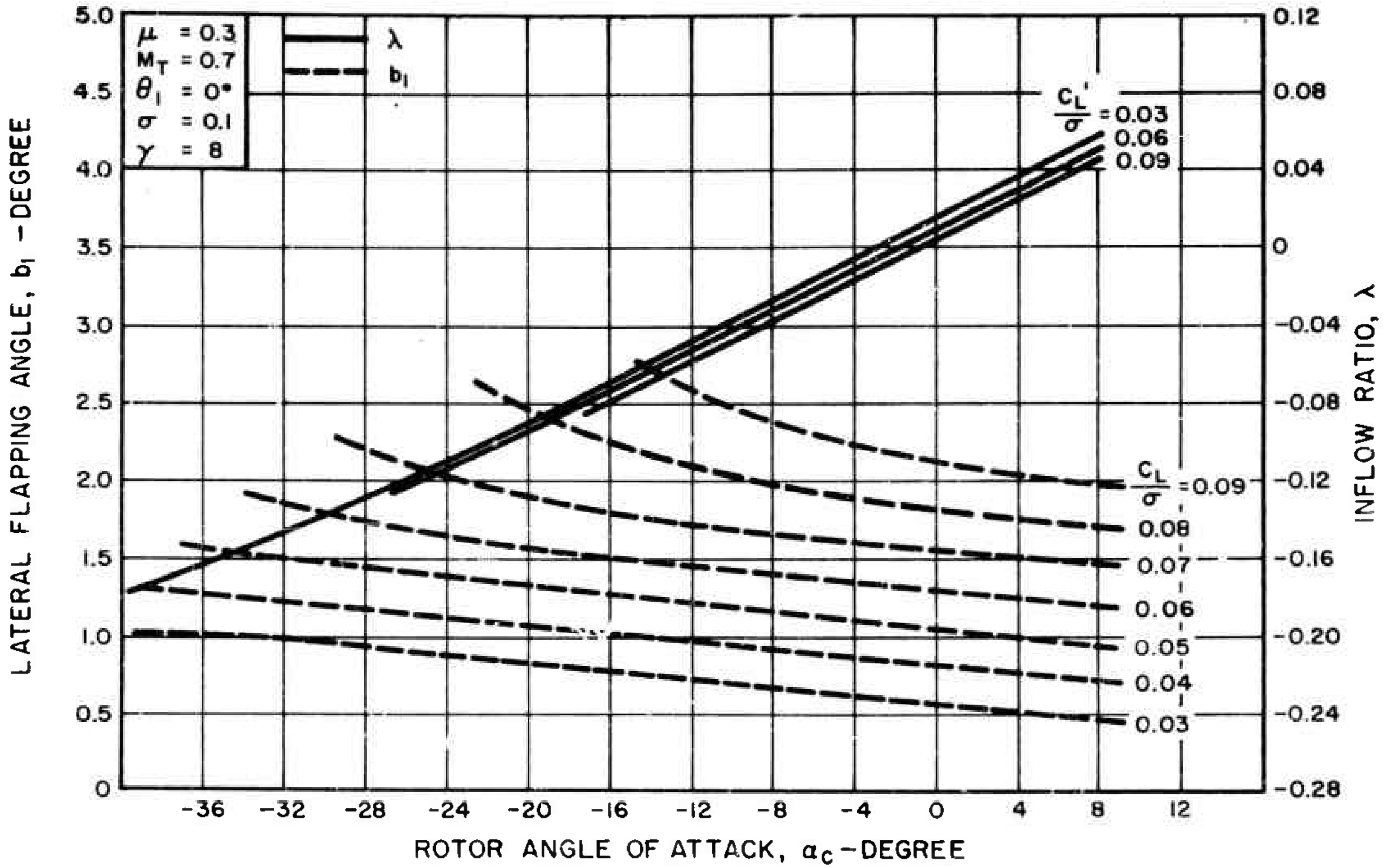

Figure 10. Concluded.

(b) b_1 and λ

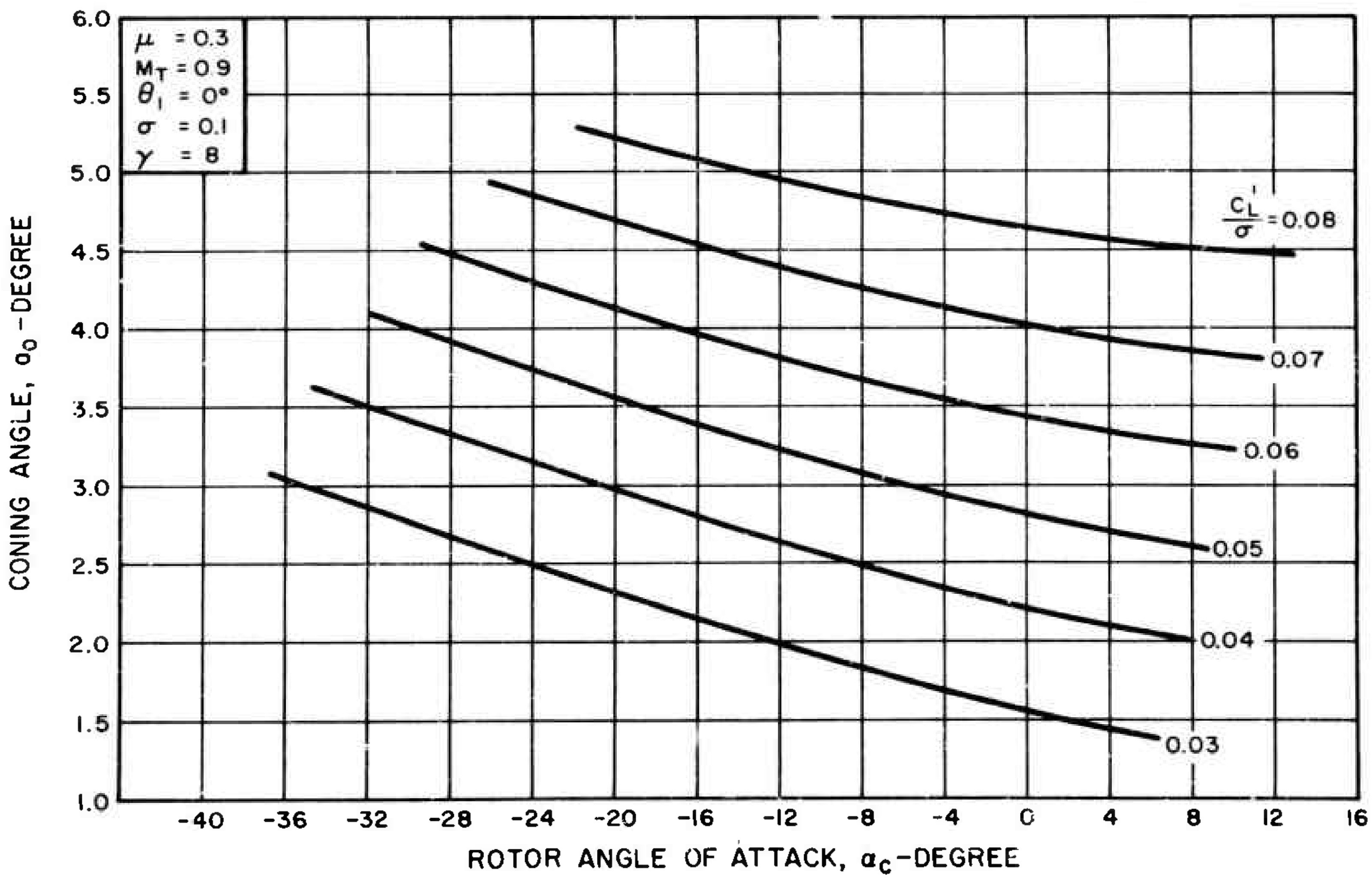

Figure 11. Calculated Characteristics of a Rotor With 0° Twist for μ = 0.3 and M_T = 0.9

(a) a_0

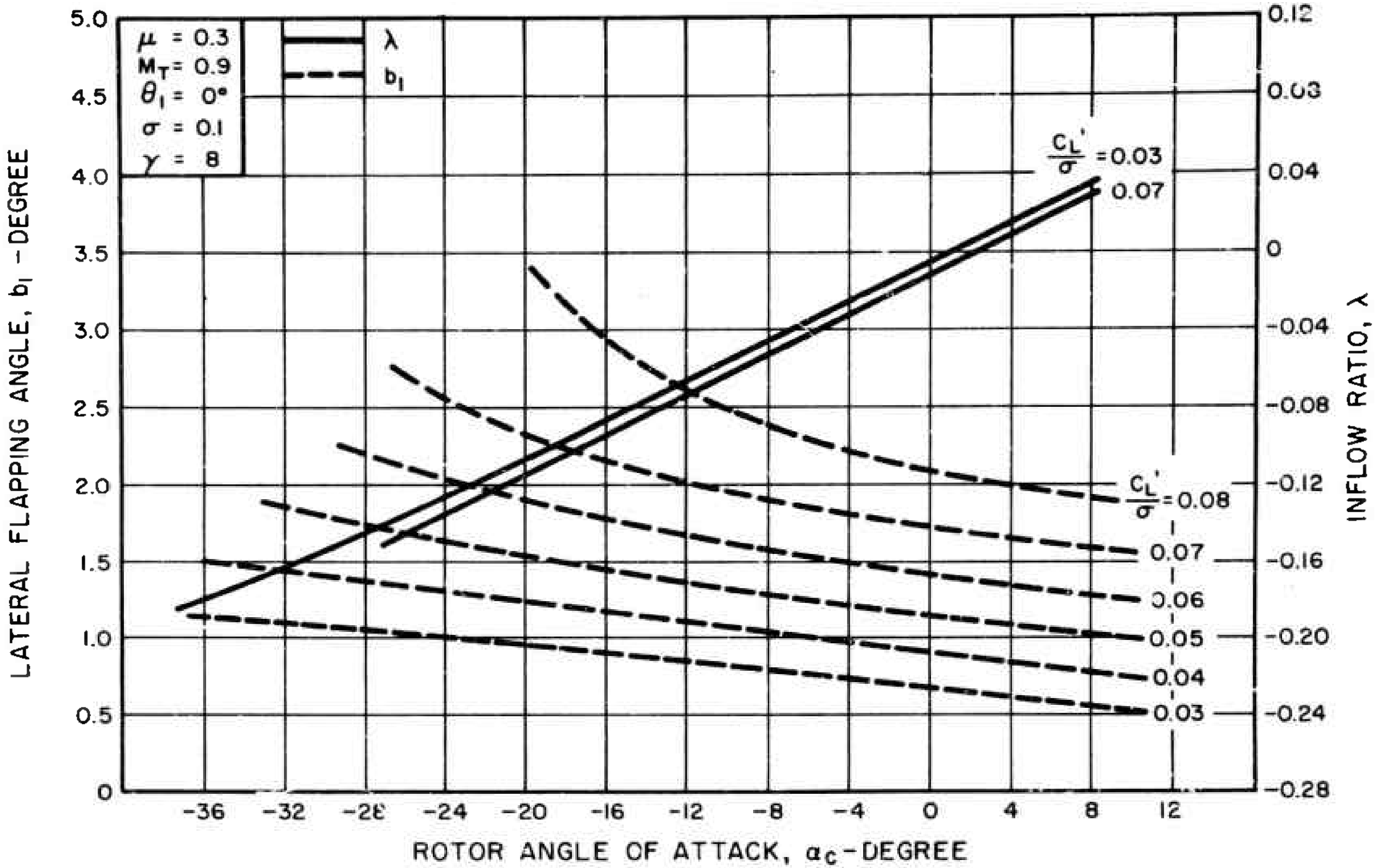

Figure 11. Concluded.

(b) b_1 and λ

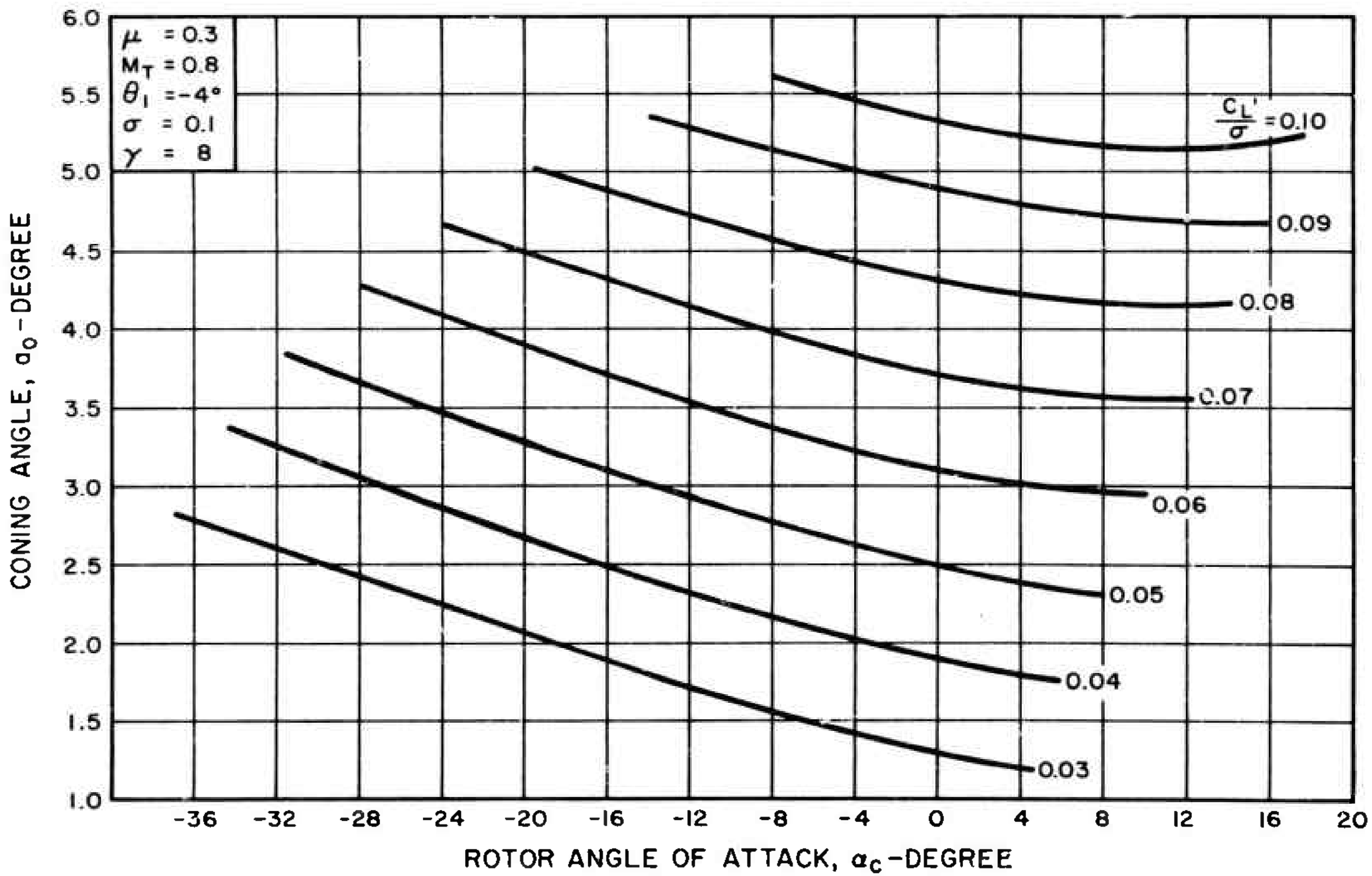

Figure 12. Calculated Characteristics of a Rotor With -4° Twist for μ = 0.3 and M_T = 0.8.

(a) a_0

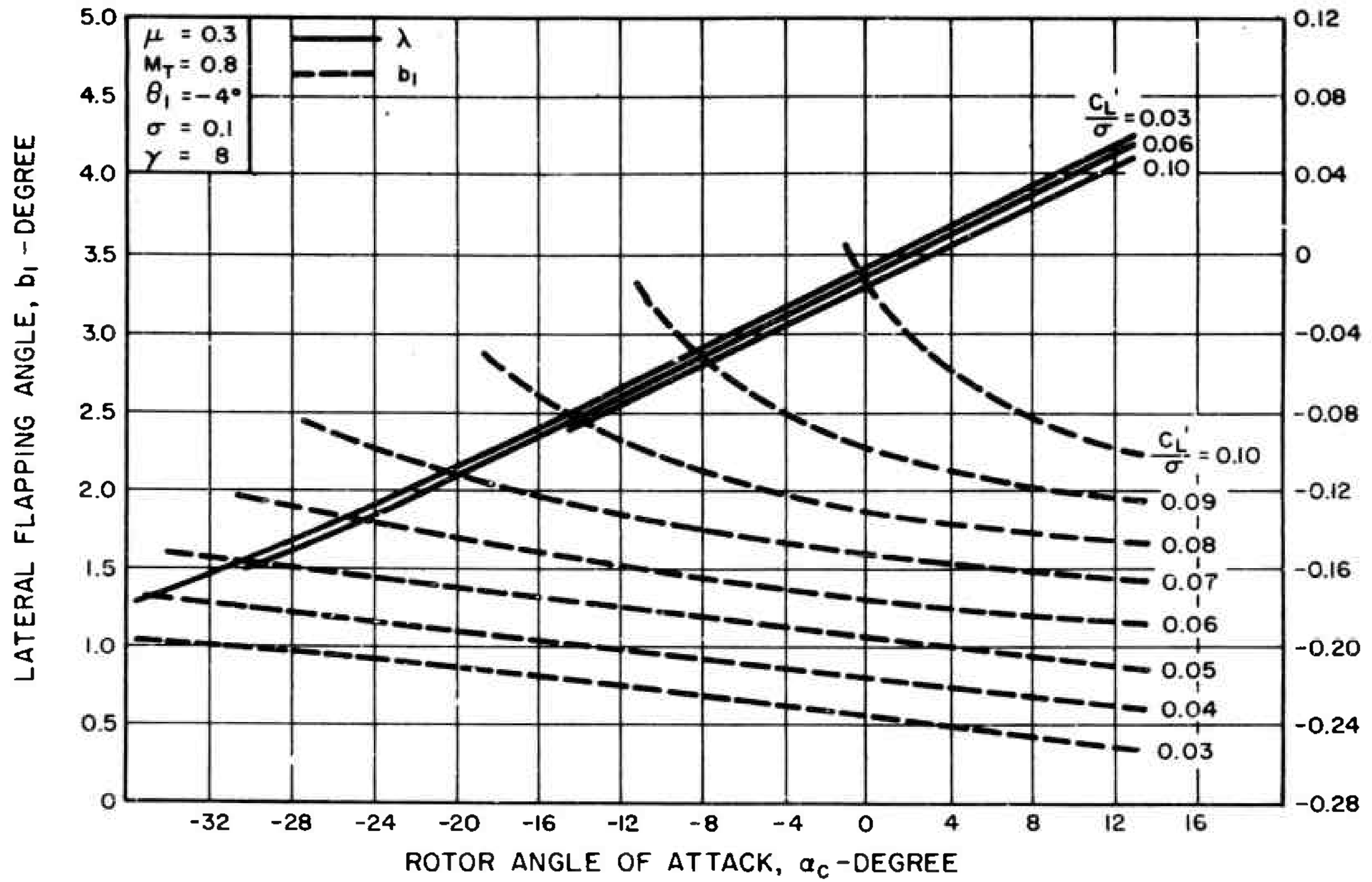

Figure 12. Concluded.

(b) b_1 and λ

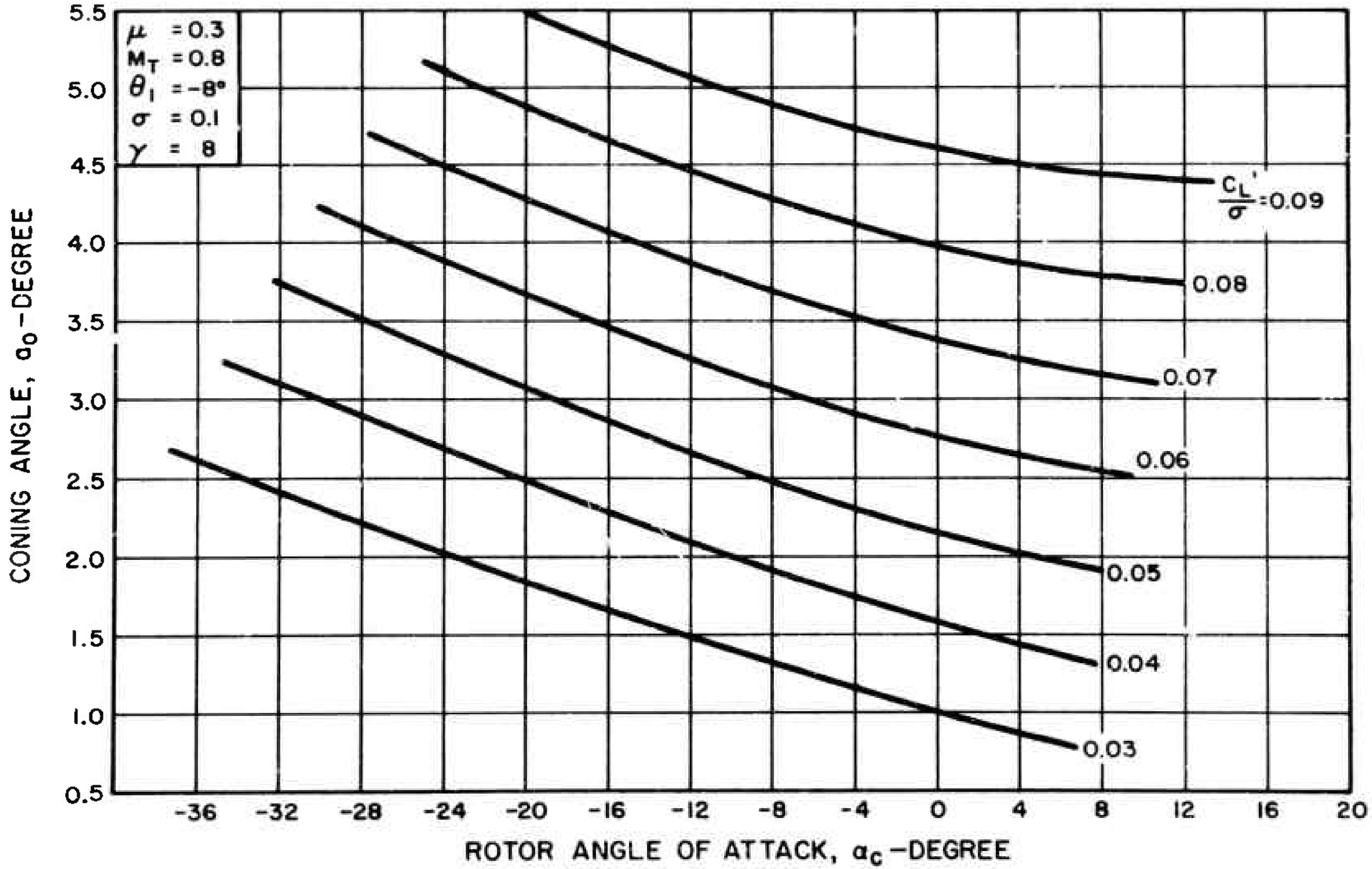

Figure 13. Calculated Characteristics of a Rotor With -8° Twist for $\mu = 0.3$ and $M_T = 0.8$.

(a) a_0

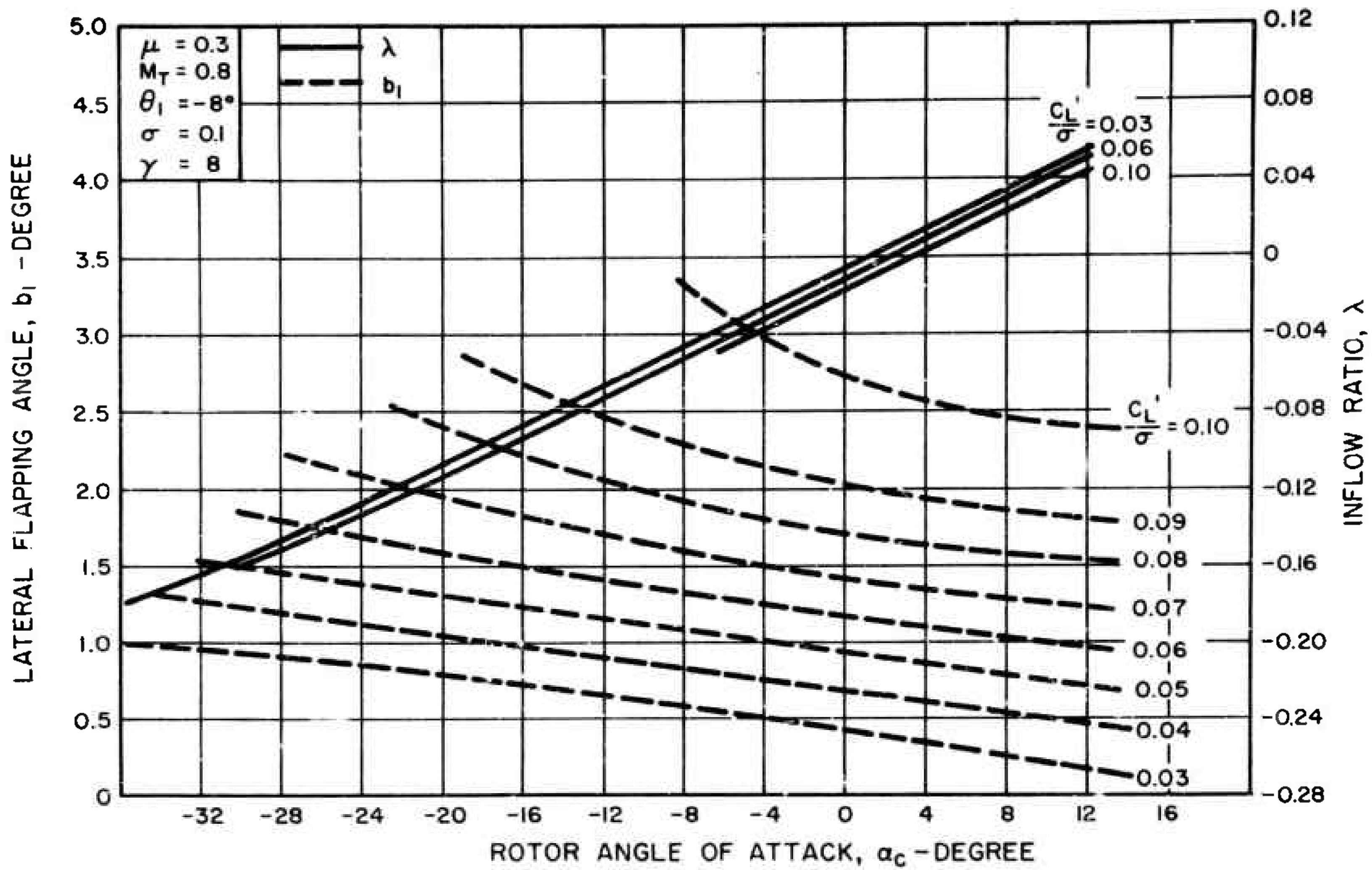

Figure 13. Concluded.

(b) b_1 and λ

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SECTION 6. PERTURBATION EQUATIONS OF MOTION

In accordance with the commonly used procedure of stability analysis, only small perturbations about the trim conditions are considered. This is accomplished by linearizing the equations of motion presented in Section 4. The variables θ, $\dot{\theta}$, etc., denote changes from trim conditions of aircraft pitch attitude, pitch rate, etc. The parameters A_{1_C} , B_{1_C} , θ_C , and θ_{TR} denote pilot control inputs in lateral and longitudinal cyclic pitch, main rotor collective pitch, and tail rotor collective pitch, respectively. J_1 and J_2 are the pilot's authority ratios in the longitudinal and lateral control, respectively. The terms X_θ, $X_{\dot{\theta}}$, etc., are the total or composite aircraft stability derivatives and denote the rate of change of forces or moments with respect to the subscript variable evaluated at the trim conditions. The composite or total stability derivatives are presented in Section 7.

The perturbation equations of motion can be expressed as follows:

(a) The X-Force Equation

$$X_u \bar{u} + X_{\dot{u}} \dot{\bar{u}} + X_v \bar{v} + X_w \bar{w} + X_{\dot{w}} \dot{\bar{w}} + X_\theta \bar{\theta} + X_{\dot{\theta}} \dot{\bar{\theta}} + X_{\dot{\phi}} \dot{\bar{\phi}} + X_{\dot{\psi}} \dot{\bar{\psi}}$$

$$+ J_1 (X_{B_{1_C}} \bar{B}_{1_C} + X_{\dot{B}_{1_C}} \dot{\bar{B}}_{1_C}) + X_{B_{1_S}} \bar{B}_{1_S} + X_{\dot{B}_{1_S}} \dot{\bar{B}}_{1_S}$$

$$+ J_2 (X_{A_{1_C}} \bar{A}_{1_C} + X_{\dot{A}_{1_C}} \dot{\bar{A}}_{1_C}) + X_{A_{1_S}} \bar{A}_{1_S} + X_{\dot{A}_{1_S}} \dot{\bar{A}}_{1_S}$$

$$+ J_3 (X_{\delta_{r_C}} \bar{\delta}_{r_C} + X_{\dot{\delta}_{r_C}} \dot{\bar{\delta}}_{r_C}) + X_{\delta_{r_S}} \bar{\delta}_{r_S} + X_{\dot{\delta}_{r_S}} \dot{\bar{\delta}}_{r_S}$$

$$+ J_4 (X_{\theta_C} \bar{\theta}_C + X_{\dot{\theta}_C} \dot{\bar{\theta}}_C) + X_{\theta_S} \bar{\theta}_S + X_{\dot{\theta}_S} \dot{\bar{\theta}}_S = 0$$

(b) <u>The Y-Force Equation</u>

$$Y_u \bar{u} + Y_v \bar{v} + Y_{\dot{v}} \dot{\bar{v}} + Y_w \bar{w} + Y_{\dot{\theta}} \dot{\bar{\theta}} + Y_\phi \bar{\phi} + Y_{\dot{\phi}} \dot{\bar{\phi}} + Y_\psi \bar{\psi} + Y_{\dot{\psi}} \dot{\bar{\psi}}$$

$$+ J_1 (Y_{B_{1_C}} \bar{B}_{1_C} + Y_{\dot{B}_{1_C}} \dot{\bar{B}}_{1_C}) + Y_{B_{1_S}} \bar{B}_{1_S} + Y_{\dot{B}_{1_S}} \dot{\bar{B}}_{1_S}$$

$$+ J_2 (Y_{A_{1_C}} \bar{A}_{1_C} + Y_{\dot{A}_{1_C}} \dot{\bar{A}}_{1_C}) + Y_{A_{1_S}} \bar{A}_{1_S} + Y_{\dot{A}_{1_S}} \dot{\bar{A}}_{1_S}$$

$$+ J_3 (Y_{\delta_{r_C}} \bar{\delta}_{r_C} + Y_{\dot{\delta}_{r_C}} \dot{\bar{\delta}}_{r_C}) + Y_{\delta_{r_S}} \bar{\delta}_{r_S} + Y_{\dot{\delta}_{r_S}} \dot{\bar{\delta}}_{r_S}$$

$$+ J_4 (Y_{\theta_C} \bar{\theta}_C + Y_{\dot{\theta}_C} \dot{\bar{\theta}}_C) + Y_{\theta_S} \bar{\theta}_S + Y_{\dot{\theta}_S} \dot{\bar{\theta}}_S = 0$$

(c) <u>The Z-Force Equation</u>

$$Z_u \bar{u} + Z_v \bar{v} + Z_w \bar{w} + Z_{\dot{w}} \dot{\bar{w}} + Z_\theta \bar{\theta} + Z_{\dot{\theta}} \dot{\bar{\theta}} + Z_{\dot{\phi}} \dot{\bar{\phi}} + Z_{\dot{\psi}} \dot{\bar{\psi}}$$

$$+ J_1 (Z_{B_{1_C}} \bar{B}_{1_C} + Z_{\dot{B}_{1_C}} \dot{\bar{B}}_{1_C}) + Z_{B_{1_S}} \bar{B}_{1_S} + Z_{\dot{B}_{1_S}} \dot{\bar{B}}_{1_S}$$

$$+ J_2 (Z_{A_{1_C}} \bar{A}_{1_C} + Z_{\dot{A}_{1_C}} \dot{\bar{A}}_{1_C}) + Z_{A_{1_S}} \bar{A}_{1_S} + Z_{\dot{A}_{1_S}} \dot{\bar{A}}_{1_S}$$

$$+ J_3 (Z_{\delta_{r_C}} \bar{\delta}_{r_C} + Z_{\dot{\delta}_{r_C}} \dot{\bar{\delta}}_{r_C}) + Z_{\delta_{r_S}} \bar{\delta}_{r_S} + Z_{\dot{\delta}_{r_S}} \dot{\bar{\delta}}_{r_S}$$

$$+ J_4 (Z_{\theta_C} \bar{\theta}_C + Z_{\dot{\theta}_C} \dot{\bar{\theta}}_C) + Z_{\theta_S} \bar{\theta}_S + Z_{\dot{\theta}_S} \dot{\bar{\theta}}_S = 0$$

(d) <u>The Rolling Moment Equation ($\mathcal{L}$)</u>

$$
\begin{aligned}
&\mathcal{L}_u \bar{u} + \mathcal{L}_v \bar{v} + \mathcal{L}_w \bar{w} + \mathcal{L}_{\dot{w}} \dot{\bar{w}} + \mathcal{L}_{\dot{\theta}} \dot{\bar{\theta}} + \mathcal{L}_{\ddot{\theta}} \ddot{\bar{\theta}} \\
&+ \mathcal{L}_{\dot{\phi}} \dot{\bar{\phi}} + \mathcal{L}_{\ddot{\phi}} \ddot{\bar{\phi}} + \mathcal{L}_{\dot{\psi}} \dot{\bar{\psi}} + \mathcal{L}_{\ddot{\psi}} \ddot{\bar{\psi}} \\
&+ J_1 (\mathcal{L}_{B_{1_c}} \bar{B}_{1_c} + \mathcal{L}_{\dot{B}_{1_c}} \dot{\bar{B}}_{1_c}) + \mathcal{L}_{B_{1_s}} \bar{B}_{1_s} + \mathcal{L}_{\dot{B}_{1_s}} \dot{\bar{B}}_{1_s} \\
&+ J_2 (\mathcal{L}_{A_{1_c}} \bar{A}_{1_c} + \mathcal{L}_{\dot{A}_{1_c}} \dot{\bar{A}}_{1_c}) + \mathcal{L}_{A_{1_s}} \bar{A}_{1_s} + \mathcal{L}_{\dot{A}_{1_s}} \dot{\bar{A}}_{1_s} \\
&+ J_3 (\mathcal{L}_{\delta_{r_c}} \bar{\delta}_{r_c} + \mathcal{L}_{\dot{\delta}_{r_c}} \dot{\bar{\delta}}_{r_c}) + \mathcal{L}_{\delta_{r_s}} \bar{\delta}_{r_s} + \mathcal{L}_{\dot{\delta}_{r_s}} \dot{\bar{\delta}}_{r_s} \\
&+ J_4 (\mathcal{L}_{\theta_c} \bar{\theta}_c + \mathcal{L}_{\dot{\theta}_c} \dot{\bar{\theta}}_c) + \mathcal{L}_{\theta_s} \bar{\theta}_s + \mathcal{L}_{\dot{\theta}_s} \dot{\bar{\theta}}_s = 0
\end{aligned}
$$

(e) <u>The Pitching Moment Equation (M)</u>

$$
\begin{aligned}
&M_u \bar{u} + M_v \bar{v} + M_w \bar{w} + M_{\dot{w}} \dot{\bar{w}} + M_{\dot{\theta}} \dot{\bar{\theta}} + M_{\ddot{\theta}} \ddot{\bar{\theta}} \\
&+ M_{\dot{\phi}} \dot{\bar{\phi}} + M_{\ddot{\phi}} \ddot{\bar{\phi}} + M_{\dot{\psi}} \dot{\bar{\psi}} + M_{\ddot{\psi}} \ddot{\bar{\psi}} \\
&+ J_1 (M_{B_{1_c}} \bar{B}_{1_c} + M_{\dot{B}_{1_c}} \dot{\bar{B}}_{1_c}) + M_{B_{1_s}} \bar{B}_{1_s} + M_{\dot{B}_{1_s}} \dot{\bar{B}}_{1_s} \\
&+ J_2 (M_{A_{1_c}} \bar{A}_{1_c} + M_{\dot{A}_{1_c}} \dot{\bar{A}}_{1_c}) + M_{A_{1_s}} \bar{A}_{1_s} + M_{\dot{A}_{1_s}} \dot{\bar{A}}_{1_s} \\
&+ J_3 (M_{\delta_{r_c}} \bar{\delta}_{r_c} + M_{\dot{\delta}_{r_c}} \dot{\bar{\delta}}_{r_c}) + M_{\delta_{r_s}} \bar{\delta}_{r_s} + M_{\dot{\delta}_{r_s}} \dot{\bar{\delta}}_{r_s}
\end{aligned}
$$

$$+J_4(M_{\theta_c}\bar{\theta}_c+M_{\dot{\theta}_c}\dot{\bar{\theta}}_c)+M_{\theta_s}\bar{\theta}_s+M_{\dot{\theta}_s}\dot{\bar{\theta}}_s=0$$

(f) The Yawing Moment Equation (N)

$$N_u\bar{u}+N_v\bar{v}+N_w\bar{w}+N_{\dot{w}}\dot{\bar{w}}+N_{\dot{\theta}}\dot{\bar{\theta}}+N_{\ddot{\theta}}\ddot{\bar{\theta}}$$

$$+N_{\dot{\phi}}\dot{\bar{\Phi}}+N_{\ddot{\phi}}\ddot{\bar{\Phi}}+N_{\dot{\psi}}\dot{\bar{\Psi}}+N_{\ddot{\psi}}\ddot{\bar{\Psi}}$$

$$+J_1(N_{B_{1_C}}\bar{B}_{1_C}+N_{\dot{B}_{1_C}}\dot{\bar{B}}_{1_C})+N_{B_{1_S}}\bar{B}_{1_S}+N_{\dot{B}_{1_S}}\dot{\bar{B}}_{1_S}$$

$$+J_2(N_{A_{1_C}}\bar{A}_{1_C}+N_{\dot{A}_{1_C}}\dot{\bar{A}}_{1_C})+N_{A_{1_S}}\bar{A}_{1_S}+N_{\dot{A}_{1_S}}\dot{\bar{A}}_{1_S}$$

$$+J_3(N_{\delta_{r_C}}\bar{\delta}_{r_C}+N_{\dot{\delta}_{r_C}}\dot{\bar{\delta}}_{r_C})+N_{\delta_{r_S}}\bar{\delta}_{r_S}+N_{\dot{\delta}_{r_S}}\dot{\bar{\delta}}_{r_S}$$

$$+J_4(N_{\theta_c}\bar{\theta}_c+N_{\dot{\theta}_c}\dot{\bar{\theta}}_c)+N_{\theta_s}\bar{\theta}_s+N_{\dot{\theta}_s}\dot{\bar{\theta}}_s=0$$

(g) The Stabilization Equations

Generalized stability augmentation system equations are as follows:

$$\dot{\bar{B}}_{1_S}=-\bar{B}_{1_S}(D_1+D_2\bar{B}_{1_S})+k_1\bar{\theta}+k_2\dot{\bar{\theta}}+k_3\bar{u}+k_4\bar{w}$$

$$\dot{\bar{A}}_{1_S}=-\bar{A}_{1_S}(D_1+D_2\bar{A}_{1_S})+k_5\bar{\Phi}-k_6\dot{\bar{\Phi}}-k_7\bar{v}$$

$$\dot{\bar{\delta}}_{r_S}=-k_8\bar{\delta}_{r_S}-k_9\bar{v}-k_{10}\dot{\bar{\Psi}}$$

The simplified "lagged rate" stabilization system can be represented by

$$\dot{\overline{B}}_{1_s} = -D_1 \overline{B}_{1_s} + k_1 \dot{\overline{\theta}}$$

$$\dot{\overline{A}}_{1_s} = -D_1 \overline{A}_{1_s} + k_5 \dot{\overline{\phi}}$$

$$\dot{\overline{\delta}}_{r_s} = -k_8 \overline{\delta}_{r_s} + k_{10} \dot{\overline{\psi}}$$

where

D_1 and D_2 are damping constants

k_1 , k_2 ....etc. are the linkage ratios.

SECTION 7. STABILITY DERIVATIVES

7.1 TOTAL STABILITY DERIVATIVES

The total aircraft stability derivatives are obtained by differentiating the equations of motion (presented in Section 4) with respect to the appropriate stability variables. The derivatives are obtained here for the following initial conditions:

(a) The aircraft is in level flight, $\gamma_C = 0$
(b) The roll attitude, sideslip angle, and heading is zero ($\phi = \beta_S = \psi = 0$)
(c) Fuselage angular rates are zero ($p = q = r = 0$)

7.1.1 The X-Force Derivatives

7.1.1.1 X_u

$$X_u = (X_u)_F + (X_u)_R + (X_u)_{FUS} + (X_u)_W + (X_u)_T + (X_u)_{VT} + (X_u)_{TR} + \sum_{i=1}^{n} (X_u)_{P_i}$$

where

$$(X_u)_F = \frac{\partial (X)_F}{\partial u} = X_{u_F} + X_{\alpha_F} \frac{\partial \alpha_F}{\partial u}$$

$$X_{u_F} = \frac{\partial (X)_F}{\partial u_F} = \left(\frac{\partial L_F}{\partial u_F} \cos A_{I_F} - \frac{\partial Y_F}{\partial u_F} \sin A_{I_F}\right) \sin(\alpha - \epsilon_F) - \frac{\partial D_F}{\partial u_F} \cos(\alpha - \epsilon_F)$$

$$X_{\alpha_F} = \frac{\partial (X)_F}{\partial \alpha_F} = \left(\frac{\partial L_F}{\partial \alpha_F} \cos A_{I_F} - \frac{\partial Y_F}{\partial \alpha_F} \sin A_{I_F}\right) \sin(\alpha - \epsilon_F) - \frac{\partial D_F}{\partial \alpha_F} \cos(\alpha - \epsilon_F)$$

$$+ (L_F \cos A_{I_F} - Y_F \sin A_{I_F}) \cos(\alpha - \epsilon_F) + D_F \sin(\alpha - \epsilon_F)$$

$$\frac{\partial \alpha_F}{\partial u} = - \frac{\partial \epsilon_F}{\partial u} = - \frac{K_{RF}}{V_0} \left(\frac{\lambda}{\mu} - \frac{\partial \lambda}{\partial \mu}\right)_R$$

$$(X_u)_R = \frac{\partial (X)_R}{\partial u} = X_{u_R} + X_{\alpha_R} \frac{\partial \alpha_R}{\partial u}$$

$$X_{u_R} = \frac{\partial (X)_R}{\partial u_R} = \left(\frac{\partial L_R}{\partial u_R} \cos A_{I_R} + \frac{\partial Y_R}{\partial u_R} \sin A_{I_R}\right) \sin(\alpha - \epsilon_R) - \frac{\partial D_R}{\partial u_R} \cos(\alpha - \epsilon_R)$$

$$X_{\alpha_R} = \frac{\partial (X)_R}{\partial \alpha_R} = \left(\frac{\partial L_R}{\partial \alpha_R} \cos A_{I_R} + \frac{\partial Y_R}{\partial \alpha_R} \sin A_{I_R}\right) \sin(\alpha - \epsilon_R) - \frac{\partial D_R}{\partial \alpha_R} \cos(\alpha - \epsilon_R)$$

$$+ (L_R \cos A_{I_R} + Y_R \sin A_{I_R}) \cos(\alpha - \epsilon_R) + D_R \sin(\alpha - \epsilon_R)$$

$$\frac{\partial \alpha_R}{\partial u} = - \frac{\partial \epsilon_R}{\partial u} = - \frac{K_{FR}}{V_0} \left(\frac{\lambda}{\mu} - \frac{\partial \lambda}{\partial \mu}\right)_F$$

$$(X_u)_{FUS} = \frac{\partial (X)_{FUS}}{\partial u} = X_{u_{FUS}} + X_{\alpha_{FUS}} \frac{\partial \alpha_{FUS}}{\partial u}$$

$$X_{u_{FUS}} = \frac{\partial (X)_{FUS}}{\partial u_{FUS}} = \frac{\partial L_{FUS}}{\partial u_{FUS}} \sin(\alpha - \epsilon_{FUS}) - \frac{\partial D_{FUS}}{\partial u_{FUS}} \cos(\alpha - \epsilon_{FUS})$$

$$X_{\alpha_{FUS}} = \frac{\partial (X)_{FUS}}{\partial \alpha_{FUS}} = \frac{\partial L_{FUS}}{\partial \alpha_{FUS}} \sin(\alpha - \epsilon_{FUS}) - \frac{\partial D_{FUS}}{\partial \alpha_{FUS}} \cos(\alpha - \epsilon_{FUS})$$

$$+ L_{FUS} \cos(\alpha - \epsilon_{FUS}) + D_{FUS} \sin(\alpha - \epsilon_{FUS})$$

$$\frac{\partial \alpha_{FUS}}{\partial u} = - \frac{\partial \epsilon_{FUS}}{\partial u} = - \frac{K_{F\,FUS}}{V_0} \left(\frac{\lambda}{\mu} - \frac{\partial \lambda}{\partial \mu}\right)_F - \frac{K_{R\,FUS}}{V_0} \left(\frac{\lambda}{\mu} - \frac{\partial \lambda}{\partial \mu}\right)_R$$

$$(X_u)_W = \frac{\partial (X)_W}{\partial u} = X_{u_W} + X_{\alpha_W} \frac{\partial \alpha_W}{\partial u}$$

$$X_{u_W} = \frac{\partial (X)_W}{\partial u_W} = \frac{\partial L_W}{\partial u_W} \sin(\alpha - \epsilon_W) - \frac{\partial D_W}{\partial u_W} \cos(\alpha - \epsilon_W)$$

$$X_{\alpha_W} = \frac{\partial (X)_W}{\partial \alpha_W} = \frac{\partial L_W}{\partial \alpha_W} \sin(\alpha - \epsilon_W) - \frac{\partial D_W}{\partial \alpha_W} \cos(\alpha - \epsilon_W)$$

$$+ L_W \cos(\alpha - \epsilon_W) + D_W \sin(\alpha - \epsilon_W)$$

$$\frac{\partial \alpha_W}{\partial u} = - \frac{\partial \epsilon_W}{\partial u} = - \frac{K_{F\,W}}{V_0} \left(\frac{\lambda}{\mu} - \frac{\partial \lambda}{\partial \mu}\right)_F - \frac{K_{R\,W}}{V_0} \left(\frac{\lambda}{\mu} - \frac{\partial \lambda}{\partial \mu}\right)_R$$

$$(X_u)_T = \frac{\partial (X)_T}{\partial u} = X_{u_T} + X_{\alpha_T}\frac{\partial \alpha_T}{\partial u}$$

$$X_{u_T} = \frac{\partial (X)_T}{\partial u_T} = \frac{\partial L_T}{\partial u_T}\sin(\alpha-\epsilon_T) - \frac{\partial D_T}{\partial u_T}\cos(\alpha-\epsilon_T)$$

$$X_{\alpha_T} = \frac{\partial (X)_T}{\partial \alpha_T} = \frac{\partial L_T}{\partial \alpha_T}\sin(\alpha-\epsilon_T) - \frac{\partial D_T}{\partial \alpha_T}\cos(\alpha-\epsilon_T)$$

$$+ L_T\cos(\alpha-\epsilon_T) + D_T\sin(\alpha-\epsilon_T)$$

$$\frac{\partial \alpha_T}{\partial u} = -\frac{\partial \epsilon_T}{\partial u} = -\frac{K_{FT}}{V_0}\left(\frac{\lambda}{\mu} - \frac{\partial \lambda}{\partial \mu}\right)_F - \frac{K_{RT}}{V_0}\left(\frac{\lambda}{\mu} - \frac{\partial \lambda}{\partial \mu}\right)_R$$

$$(X_u)_{VT} = \frac{\partial (X)_{VT}}{\partial u} = X_{u_{VT}} + X_{\alpha_{VT}}\frac{\partial \alpha_{VT}}{\partial u}$$

$$X_{u_{VT}} = \frac{\partial (X)_{VT}}{\partial u_{VT}} = -\frac{\partial D_{VT}}{\partial u_{VT}}\cos(\alpha-\epsilon_{VT})$$

$$X_{\alpha_{VT}} = \frac{\partial (X)_{VT}}{\partial \alpha_{VT}} = -\frac{\partial D_{VT}}{\partial \alpha_{VT}}\cos(\alpha-\epsilon_{VT}) + D_{VT}\sin(\alpha-\epsilon_{VT})$$

$$\frac{\partial \alpha_{VT}}{\partial u} = -\frac{\partial \epsilon_{VT}}{\partial u} = -\frac{K_{F\,VT}}{V_0}\left(\frac{\lambda}{\mu} - \frac{\partial \lambda}{\partial \mu}\right)_F - \frac{K_{R\,VT}}{V_0}\left(\frac{\lambda}{\mu} - \frac{\partial \lambda}{\partial \mu}\right)_R$$

$$(X_u)_{TR} = \frac{\partial (X)_{TR}}{\partial u_{TR}} = X_{u_{TR}} + X_{\alpha_{TR}}\frac{\partial \alpha_{TR}}{\partial u}$$

$$X_{u_{TR}} = \frac{\partial (X)_{TR}}{\partial u_{TR}} = \frac{\partial Y_{TR}}{\partial u_{TR}}\sin(\alpha-\epsilon_{TR}) - \frac{\partial D_{TR}}{\partial u_{TR}}\cos(\alpha-\epsilon_{TR})$$

$$X_{\alpha_{TR}} = \frac{\partial (X)_{TR}}{\partial \alpha_{TR}} = \frac{\partial Y_{TR}}{\partial \alpha_{TR}}\sin(\alpha-\epsilon_{TR}) - \frac{\partial D_{TR}}{\partial \alpha_{TR}}\cos(\alpha-\epsilon_{TR})$$

$$+ Y_{TR}\cos(\alpha-\epsilon_{TR}) + D_{TR}\sin(\alpha-\epsilon_{TR})$$

$$\frac{\partial \alpha_{TR}}{\partial u} = -\frac{\partial \epsilon_{TR}}{\partial u} = -\frac{K_{F\,TR}}{V_0}\left(\frac{\lambda}{\mu} - \frac{\partial \lambda}{\partial \mu}\right)_F - \frac{K_{R\,TR}}{V_0}\left(\frac{\lambda}{\mu} - \frac{\partial \lambda}{\partial \mu}\right)_R$$

$$(X_u)_{P_i} = \frac{\partial (X)_{P_i}}{\partial u} = X_{u_{P_i}} + X_{\alpha_{P_i}} \frac{\partial \alpha_{P_i}}{\partial u}$$

$$X_{u_{P_i}} = \frac{\partial (X)_{P_i}}{\partial u_{P_i}} = \frac{\partial T_{P_i}}{\partial u_{P_i}} \cos i_{P_i} - \frac{\partial N_{P_i}}{\partial u_{P_i}} \sin i_{P_i}$$

$$X_{\alpha_{P_i}} = \frac{\partial (X)_{P_i}}{\partial \alpha_{P_i}} = \frac{\partial T_{P_i}}{\partial \alpha_{P_i}} \cos i_{P_i} - \frac{\partial N_{P_i}}{\partial \alpha_{P_i}} \sin i_{P_i}$$

$$\frac{\partial \alpha_{P_i}}{\partial u} = -\frac{\partial \epsilon_{P_i}}{\partial u} = -\frac{K_{FP_i}}{V_0}\left(\frac{\lambda}{\mu} - \frac{\partial \lambda}{\partial \mu}\right)_F - \frac{K_{RP_i}}{V_0}\left(\frac{\lambda}{\mu} - \frac{\partial \lambda}{\partial \mu}\right)_R$$

7.1.1.2 $X_{\dot{u}}$

$$X_{\dot{u}} = -\frac{W}{g}$$

7.1.1.3 X_v

$$X_v = (X_v)_F + (X_v)_R + (X_v)_{FUS} + (X_v)_W + (X_v)_T + (X_v)_{VT} + (X_v)_{TR} + \sum_{i=1}^{n} (X_v)_{P_i}$$

where

$$(X_v)_F = \frac{\partial (X)_F}{\partial v} = \frac{1}{V_0} \frac{\partial (X)_F}{\partial \beta_s} = \frac{-1}{V_0}(L_F \sin A_{1_F} + Y_F \cos A_{1_F})$$

$$(X_v)_R = \frac{\partial (X)_R}{\partial v} = \frac{1}{V_0} \frac{\partial (X)_R}{\partial \beta_s} = \frac{-1}{V_0}(L_R \sin A_{1_R} - Y_R \cos A_{1_R})$$

$$(X_v)_{FUS} = \frac{\partial (X)_{FUS}}{\partial v} = \frac{1}{V_0} \frac{\partial (X)_{FUS}}{\partial \beta_s} = \frac{1}{V_0}\left[\frac{\partial L_{FUS}}{\partial \beta_s} \sin(\alpha - \epsilon_{FUS}) - \frac{\partial D_{FUS}}{\partial \beta_s} \cos(\alpha - \epsilon_{FUS}) - Y_{FUS}\right]$$

$$(X_v)_W = (X_v)_T = 0$$

$$(X_v)_{VT} = \frac{\partial(X)_{VT}}{\partial v} = \frac{1}{V_o}\frac{\partial(X)_{VT}}{\partial \beta_s} = \frac{1}{V_o}\left[L_{VT} - \frac{\partial D_{VT}}{\partial \beta_s}\cos(\alpha - \epsilon_{VT})\right]$$

$$(X_v)_{TR} = \frac{\partial(X)_{TR}}{\partial v} = \frac{1}{V_o}\frac{\partial(X)_{TR}}{\partial \beta_s} = \frac{1}{V_o}\left[-\frac{\partial Y_{TR}}{\partial \beta_s}\sin(\alpha - \epsilon_{TR}) - \frac{\partial D_{TR}}{\partial \beta_s}\cos(\alpha - \epsilon_{TR}) - T_{TR}\right]$$

$$(X_v)_{P_i} = \frac{\partial(X)_{P_i}}{\partial v} = \frac{1}{V_o}\frac{\partial(X)_{P_i}}{\partial \beta_s} = \frac{1}{V_o}\left(\frac{\partial T_{P_i}}{\partial \beta_s}\cos i_{P_i} - \frac{\partial N_{P_i}}{\partial \beta_s}\sin i_{P_i}\right)$$

7.1.1.4 X_w

$$X_w = (X_w)_F + (X_w)_R + (X_w)_{FUS} + (X_w)_W + (X_w)_T + (X_w)_{VT} + (X_w)_{TR} + \sum_{i=1}^{n}(X_w)_{P_i}$$

where

$$(X_w)_F = \frac{\partial(X)_F}{\partial w} = X_{w_F}\frac{\partial \alpha_F}{\partial \alpha}$$

$$X_{w_F} = \frac{\partial(X)_F}{\partial w_F} = \frac{1}{V_o}\frac{\partial(X)_F}{\partial \alpha_F} = \frac{1}{V_o}X_{\alpha_F}$$

$$\frac{\partial \alpha_F}{\partial \alpha} = 1 - \frac{\partial \epsilon_F}{\partial \alpha} = 1 - K_{RF}\left[1 - \frac{1}{\mu}\left(\frac{\partial \lambda}{\partial \alpha_c}\right)\right]_R$$

$$(X_w)_R = \frac{\partial(X)_R}{\partial w} = X_{w_R}\frac{\partial \alpha_R}{\partial \alpha}$$

$$X_{w_R} = \frac{\partial(X)_R}{\partial w_R} = \frac{1}{V_o}\frac{\partial(X)_R}{\partial \alpha_R} = \frac{1}{V_o}X_{\alpha_R}$$

$$\frac{\partial \alpha_R}{\partial \alpha} = 1 - \frac{\partial \epsilon_R}{\partial \alpha} = 1 - K_{FR}\left[1 - \frac{1}{\mu}\left(\frac{\partial \lambda}{\partial \alpha_c}\right)\right]_F$$

$$(X_w)_{FUS} = \frac{\partial (X)_{FUS}}{\partial w} = X_{w_{FUS}} \frac{\partial \alpha_{FUS}}{\partial \alpha}$$

$$X_{w_{FUS}} = \frac{\partial (X)_{FUS}}{\partial w_{FUS}} = \frac{1}{V_0} \frac{\partial (X)_{FUS}}{\partial \alpha_{FUS}} = \frac{1}{V_0} X_{\alpha_{FUS}}$$

$$\frac{\partial \alpha_{FUS}}{\partial \alpha} = 1 - \frac{\partial \epsilon_{FUS}}{\partial \alpha} = 1 - K_{F\,FUS}\left(1 - \frac{1}{\mu}\frac{\partial \lambda}{\partial \alpha_C}\right)_F - K_{R\,FUS}\left(1 - \frac{1}{\mu}\frac{\partial \lambda}{\partial \alpha_C}\right)_R$$

$$(X_w)_W = \frac{\partial (X)_W}{\partial w} = X_{w_W} \frac{\partial \alpha_W}{\partial \alpha}$$

$$X_{w_W} = \frac{\partial (X)_W}{\partial w_W} = \frac{1}{V_0} \frac{\partial (X)_W}{\partial \alpha_W} = \frac{1}{V_0} X_{\alpha_W}$$

$$\frac{\partial \alpha_W}{\partial \alpha} = 1 - \frac{\partial \epsilon_W}{\partial \alpha} = 1 - K_{FW}\left(1 - \frac{1}{\mu}\frac{\partial \lambda}{\partial \alpha_C}\right)_F - K_{RW}\left(1 - \frac{1}{\mu}\frac{\partial \lambda}{\partial \alpha_C}\right)_R$$

$$(X_w)_T = \frac{\partial (X)_T}{\partial w} = X_{w_T} \frac{\partial \alpha_T}{\partial \alpha}$$

$$X_{w_T} = \frac{\partial (X)_T}{\partial w_T} = \frac{1}{V_0} \frac{\partial (X)_T}{\partial \alpha_T} = \frac{1}{V_0} X_{\alpha_T}$$

$$\frac{\partial \alpha_T}{\partial \alpha} = 1 - \frac{\partial \epsilon_T}{\partial \alpha} = 1 - K_{FT}\left(1 - \frac{1}{\mu}\frac{\partial \lambda}{\partial \alpha_C}\right)_F - K_{RT}\left(1 - \frac{1}{\mu}\frac{\partial \lambda}{\partial \alpha_C}\right)_R$$

$$(X_w)_{VT} = \frac{\partial (X)_{VT}}{\partial w} = \frac{1}{V_0} X_{w_{VT}} \frac{\partial \alpha_{VT}}{\partial \alpha}$$

$$X_{w_{VT}} = \frac{\partial (X)_{VT}}{\partial w_{VT}} = \frac{1}{V_0} \frac{\partial (X)_{VT}}{\partial \alpha_{VT}} = \frac{1}{V_0} X_{\alpha_{VT}}$$

$$\frac{\partial \alpha_{VT}}{\partial \alpha} = 1 - \frac{\partial \epsilon_{VT}}{\partial \alpha} = 1 - K_{FVT}\left(1 - \frac{1}{\mu}\frac{\partial \lambda}{\partial \alpha_C}\right)_F - K_{RVT}\left(1 - \frac{1}{\mu}\frac{\partial \lambda}{\partial \alpha_C}\right)_R$$

$$(X_w)_{TR} = \frac{\partial (X)_{TR}}{\partial w} = \frac{1}{V_0} X_{w_{TR}} \frac{\partial \alpha_{TR}}{\partial \alpha}$$

$$X_{w_{TR}} = \frac{\partial (X)_{TR}}{\partial w_{TR}} = \frac{1}{V_0} \frac{\partial (X)_{TR}}{\partial \alpha_{TR}} = \frac{1}{V_0} X_{\alpha_{TR}}$$

$$\frac{\partial \alpha_{TR}}{\partial \alpha} = 1 - \frac{\partial \epsilon_{TR}}{\partial \alpha} = 1 - K_{FTR}\left(1 - \frac{1}{\mu}\frac{\partial \lambda}{\partial \alpha_C}_F\right) - K_{RTR}\left(1 - \frac{1}{\mu}\frac{\partial \lambda}{\partial \alpha_C}_R\right)$$

$$(X_{\dot{w}})_{P_i} = \frac{\partial (X)_{P_i}}{\partial \dot{w}} = \frac{1}{V_o} X_{w_{P_i}} \frac{\partial \alpha_{P_i}}{\partial \alpha}$$

$$X_{w_{P_i}} = \frac{\partial (X)_{P_i}}{\partial w_{P_i}} = \frac{1}{V_o}\frac{\partial (X)_{P_i}}{\partial \alpha_{P_i}} = \frac{1}{V_o} X_{\alpha_{P_i}}$$

$$\frac{\partial \alpha_{P_i}}{\partial \alpha} = 1 - \frac{\partial \epsilon_{P_i}}{\partial \alpha} = 1 - K_{FP_i}\left(1 - \frac{1}{\mu}\frac{\partial \lambda}{\partial \alpha_C}_F\right) - K_{RP_i}\left(1 - \frac{1}{\mu}\frac{\partial \lambda}{\partial \alpha_C}_R\right)$$

7.1.1.5 $X_{\dot{w}}$

$$X_{\dot{w}} = (X_{\dot{w}})_R + (X_{\dot{w}})_T + (X_{\dot{w}})_{VT} + (X_{\dot{w}})_{TR}$$

where

$$(X_{\dot{w}})_R = \frac{\partial (X)_R}{\partial \dot{w}} = \frac{1}{V_o}\frac{\partial (X)_R}{\partial \dot{\alpha}} = \frac{1}{V_o} X_{\alpha_R} \frac{\partial \alpha_R}{\partial \dot{\alpha}}$$

$$\frac{\partial \alpha_R}{\partial \dot{\alpha}} = -\frac{\partial \epsilon_R}{\partial \dot{\alpha}} = -\frac{\partial \epsilon_R}{\partial \alpha}\left(\frac{\ell x_F - \ell x_R}{V_o}\right) = -K_{FR}\left(1 - \frac{1}{\mu}\frac{\partial \lambda}{\partial \alpha_{C_F}}\right)\left(\frac{\ell x_F - \ell x_R}{V_o}\right)$$

$$(X_{\dot{w}})_T = \frac{\partial (X)_T}{\partial \dot{w}} = \frac{1}{V_o}\frac{\partial (X)_T}{\partial \dot{\alpha}} = \frac{1}{V_o} X_{\alpha_T} \frac{\partial \alpha_T}{\partial \dot{\alpha}}$$

$$\frac{\partial \alpha_T}{\partial \dot{\alpha}} = -\frac{\partial \epsilon_T}{\partial \dot{\alpha}} = -\frac{\partial \epsilon_T}{\partial \alpha}\left(\frac{\ell x_F - \ell x_T}{V_o}\right) = -\left[K_{FT}\left(1 - \frac{1}{\mu}\frac{\partial \lambda}{\partial \alpha_C}_F\right)\right.$$

$$\left. + K_{RT}\left(1 - \frac{1}{\mu}\frac{\partial \lambda}{\partial \alpha_C}_R\right)\right]\left(\frac{\ell x_F - \ell x_T}{V_o}\right)$$

$$(X_{\dot{w}})_{VT} = \frac{\partial (X)_{VT}}{\partial \dot{w}} = \frac{1}{V_o}\frac{\partial (X)_{VT}}{\partial \dot{\alpha}} = \frac{1}{V_o} X_{\alpha_{VT}} \frac{\partial \alpha_{VT}}{\partial \dot{\alpha}}$$

$$\frac{\partial \alpha_{VT}}{\partial \dot{\alpha}} = -\frac{\partial \epsilon_{VT}}{\partial \dot{\alpha}} = -\left[K_{FVT}\left(1-\frac{1}{\mu}\frac{\partial \lambda}{\partial \alpha_{C_F}}\right)+K_{RVT}\left(1-\frac{1}{\mu}\frac{\partial \lambda}{\partial \alpha_{C_R}}\right)\right]\left(\frac{\ell_{x_F}-\ell_{x_{VT}}}{V_0}\right)$$

$$(X_{\dot{w}})_{TR} = \frac{\partial (X)_{TR}}{\partial \dot{w}} = \frac{1}{V_0}\frac{\partial (X)_{TR}}{\partial \dot{\alpha}} = \frac{1}{V_0} X_{\alpha_{TR}} \frac{\partial \alpha_{TR}}{\partial \dot{\alpha}}$$

$$\frac{\partial \alpha_{TR}}{\partial \dot{\alpha}} = -\frac{\partial \epsilon_{TR}}{\partial \dot{\alpha}} = -\left[K_{FTR}\left(1-\frac{1}{\mu}\frac{\partial \lambda}{\partial \alpha_{C_F}}\right)+K_{RTR}\left(1-\frac{1}{\mu}\frac{\partial \lambda}{\partial \alpha_{C_R}}\right)\right]\left(\frac{\ell_{x_F}-\ell_{x_{TR}}}{V_0}\right)$$

7.1.1.6 X_θ

$$X_\theta = -W \cos \theta$$

7.1.1.7 $X_{\dot{\theta}}$

$$X_{\dot{\theta}} = (X_{\dot{\theta}})_F + (X_{\dot{\theta}})_R + (X_{\dot{\theta}})_W + (X_{\dot{\theta}})_T + (X_{\dot{\theta}})_{VT} + (X_{\dot{\theta}})_{TR} + \sum_{i=1}^{n} (X_{\dot{\theta}})_{P_i}$$

$$- \frac{W}{g} V_0 \sin \alpha$$

where

$$(X_{\dot{\theta}})_F = \frac{\partial (X)_F}{\partial \dot{\theta}} = X_{u_F}\ell_{z_F} - X_{w_F}\ell_{x_F} + \left(\frac{\partial X}{\partial a_1}\right)_F \frac{\partial a_{1_F}}{\partial q} + \left(\frac{\partial X}{\partial b_1}\right)_F \frac{\partial b_{1_F}}{\partial q}$$

$$\left(\frac{\partial X}{\partial a_1}\right)_F = \left(\frac{\partial L_F}{\partial a_{1_F}}\cos A_{1_F} - \frac{\partial Y_F}{\partial a_{1_F}}\sin A_{1_F}\right)\sin(\alpha-\epsilon_F) - \frac{\partial D_F}{\partial a_{1_F}}\cos(\alpha-\epsilon_F)$$

$$\left(\frac{\partial X}{\partial b_1}\right)_F = \left(\frac{\partial L_F}{\partial b_{1_F}}\cos A_{1_F} - \frac{\partial Y_F}{\partial b_{1_F}}\sin A_{1_F}\right)\sin(\alpha-\epsilon_F) - \frac{\partial D_F}{\partial b_{1_F}}\cos(\alpha-\epsilon_F)$$

$$(X_{\dot\theta})_R = X_{u_R}\ell_{z_R} - X_{w_R}\ell_{x_R} + \left(\frac{\partial X}{\partial a_1}\right)_R \frac{\partial a_{1_R}}{\partial q} + \left(\frac{\partial X}{\partial b_1}\right)_R \frac{\partial b_{1_R}}{\partial q}$$

$$\left(\frac{\partial X}{\partial a_1}\right)_R = \left(\frac{\partial L_R}{\partial a_{1_R}}\cos A_{1_R} + \frac{\partial Y_R}{\partial a_{1_R}}\sin A_{1_R}\right)\sin(\alpha-\epsilon_R) - \frac{\partial D_R}{\partial a_{1_R}}\cos(\alpha-\epsilon_R)$$

$$\left(\frac{\partial X}{\partial b_1}\right)_R = \left(\frac{\partial L_R}{\partial b_{1_R}}\cos A_{1_R} + \frac{\partial Y_R}{\partial b_{1_R}}\sin A_{1_R}\right)\sin(\alpha-\epsilon_R) - \frac{\partial D_R}{\partial a_{1_R}}\cos(\alpha-\epsilon_R)$$

$$(X_{\dot\theta})_W = \frac{\partial (X)_W}{\partial \dot\theta} = X_{u_W}\ell_{z_W} - X_{w_W}\ell_{x_W}$$

$$(X_{\dot\theta})_T = \frac{\partial (X)_T}{\partial \dot\theta} = X_{u_T}\ell_{z_T} - X_{w_T}\ell_{x_T}$$

$$(X_{\dot\theta})_{VT} = \frac{\partial (X)_{VT}}{\partial \dot\theta} = X_{u_{VT}}\ell_{z_{VT}} - X_{w_{VT}}\ell_{x_{VT}}$$

$$(X_{\dot\theta})_{TR} = \frac{\partial (X)_{TR}}{\partial \dot\theta} = X_{u_{TR}}\ell_{z_{TR}} - X_{w_{TR}}\ell_{x_{TR}}$$

$$(X_{\dot\theta})_{P_i} = \frac{\partial (X)_{P_i}}{\partial \dot\theta} = X_{u_{P_i}}\ell_{z_{P_i}} - X_{w_{P_i}}\ell_{x_{P_i}}$$

7.1.1.8 $X_{\dot\phi}$

$$X_{\dot\phi} = (X_{\dot\phi})_F + (X_{\dot\phi})_R + (X_{\dot\phi})_W + (X_{\dot\phi})_T + (X_{\dot\phi})_{VT} + (X_{\dot\phi})_{TR} + \sum_{i=1}^{n}(X_{\dot\phi})_{P_i}$$

where

$$(X_{\dot\phi})_F = \frac{\partial (X)_F}{\partial \dot\phi} = X_{w_F}\ell_{Y_F} - X_{v_F}\ell_{z_F} + \left(\frac{\partial X}{\partial a_1}\right)_F \frac{\partial a_{1_F}}{\partial p} + \left(\frac{\partial X}{\partial b_1}\right)_F \frac{\partial b_{1_F}}{\partial p}$$

$$(X_{\dot\phi})_R = \frac{\partial (X)_R}{\partial \dot\phi} = X_{w_R}\ell_{Y_R} - X_{v_R}\ell_{z_R} + \left(\frac{\partial X}{\partial a_1}\right)_R \frac{\partial a_{1_R}}{\partial p} + \left(\frac{\partial X}{\partial b_1}\right)_R \frac{\partial b_{1_R}}{\partial p}$$

$$(X_{\dot\phi})_W = \frac{\partial (X)_W}{\partial \dot\phi} = X_{w_W} \ell_{Y_W} - X_{v_W} \ell_{Z_W}$$

$$(X_{\dot\phi})_T = \frac{\partial (X)_T}{\partial \dot\phi} = X_{w_T} \ell_{Y_T} - X_{v_T} \ell_{Z_T}$$

$$(X_{\dot\phi})_{VT} = \frac{\partial (X)_{VT}}{\partial \dot\phi} = X_{w_{VT}} \ell_{Y_{VT}} - X_{v_{VT}} \ell_{Z_{VT}}$$

$$(X_{\dot\phi})_{TR} = \frac{\partial (X)_{TR}}{\partial \dot\phi} = X_{w_{TR}} \ell_{Y_{TR}} - X_{v_{TR}} \ell_{Z_{TR}}$$

$$(X_{\dot\phi})_{P_i} = \frac{\partial (X)_{P_i}}{\partial \dot\phi} = X_{w_{P_i}} \ell_{Y_{P_i}} - X_{v_{P_i}} \ell_{Z_{P_i}}$$

<u>7.1.1.9 $X_{\dot\psi}$</u>

$$X_{\dot\psi} = (X_{\dot\psi})_F + (X_{\dot\psi})_R + (X_{\dot\psi})_W + (X_{\dot\psi})_T + (X_{\dot\psi})_{VT} + (X_{\dot\psi})_{TR} + \sum_{i=1}^{n} (X_{\dot\psi})_{P_i}$$

where

$$(X_{\dot\psi})_F = \frac{\partial (X)_F}{\partial \dot\psi} = X_{v_F} \ell_{X_F} - X_{u_F} \ell_{Y_F} + \left(\frac{\partial X}{\partial a_1}\right)_F \frac{\partial a_{1_F}}{\partial r} + \left(\frac{\partial X}{\partial b_1}\right)_F \frac{\partial b_{1_F}}{\partial r}$$

$$(X_{\dot\psi})_R = \frac{\partial (X)_R}{\partial \dot\psi} = X_{v_R} \ell_{X_R} - X_{u_R} \ell_{Y_R} + \left(\frac{\partial X}{\partial a_1}\right)_R \frac{\partial a_{1_R}}{\partial r} + \left(\frac{\partial X}{\partial b_1}\right)_R \frac{\partial b_{1_R}}{\partial r}$$

$$(X_{\dot\psi})_W = \frac{\partial (X)_W}{\partial \dot\psi} = X_{v_W} \ell_{X_W} - X_{u_W} \ell_{Y_W}$$

$$(X_{\dot\psi})_T = \frac{\partial (X)_T}{\partial \dot\psi} = X_{v_T} \ell_{X_T} - X_{u_T} \ell_{Y_T}$$

$$(X_{\dot\psi})_{VT} = \frac{\partial (X)_{VT}}{\partial \dot\psi} = X_{v_{VT}} \ell_{X_{VT}} - X_{u_{VT}} \ell_{Y_{VT}}$$

$$(X_{\dot{\psi}})_{TR} = \frac{\partial(X)_{TR}}{\partial\dot{\psi}} = X_{V_{TR}}\ell_{X_{TR}} - X_{U_{TR}}\ell_{Y_{TR}}$$

$$(X_{\dot{\psi}})_{P_i} = \frac{\partial(X)_{P_i}}{\partial\dot{\psi}} = X_{V_{P_i}}\ell_{X_{P_i}} - X_{U_{P_i}}\ell_{Y_{P_i}}$$

7.1.2 The Y-Force Derivatives

7.1.2.1 Y_u

$$Y_u = (Y_u)_F + (Y_u)_R + (Y_u)_{FUS} + (Y_u)_W + (Y_u)_T + (Y_u)_{VT} + (Y_u)_{TR} + \sum_{i=1}^{n}(Y_u)_{P_i}$$

where

$$(Y_u)_F = \frac{\partial(X)_F}{\partial u} = Y_{u_F} + Y_{\alpha_F}\frac{\partial\alpha_F}{\partial u}$$

$$Y_{u_F} = \frac{\partial(Y)_F}{\partial u_F} = \frac{\partial L_F}{\partial u_F}\sin A_{1_F} + \frac{\partial Y_F}{\partial u_F}\cos A_{1_F}$$

$$Y_{\alpha_F} = \frac{\partial(Y)_F}{\partial\alpha_F} = \frac{\partial L_F}{\partial\alpha_F}\sin A_{1_F} + \frac{\partial Y_F}{\partial\alpha_F}\cos A_{1_F}$$

$$(Y_v)_W = \frac{\partial(Y)_W}{\partial v} = \frac{1}{V_0}\frac{\partial(Y)_W}{\partial \beta_s} = \frac{1}{V_0}\left[L_W \sin(\alpha-\epsilon_W) - D_W\cos(\alpha-\epsilon_W)\right]$$

$$(Y_v)_T = \frac{\partial(Y)_T}{\partial v} = \frac{1}{V_0}\frac{\partial(Y)_T}{\partial \beta_s} = \frac{1}{V_0}\left[L_T \sin(\alpha-\epsilon_T) - D_T\cos(\alpha-\epsilon_T)\right]$$

$$(Y_v)_{VT} = \frac{\partial(Y)_{VT}}{\partial v} = \frac{1}{V_0}\frac{\partial(Y)_{VT}}{\partial \beta_s} = \frac{1}{V_0}\left[-D_{VT}\cos(\alpha-\epsilon_{VT}) - \frac{\partial L_{VT}}{\partial \beta_s}\right]$$

$$(Y_v)_{TR} = \frac{\partial(Y)_{TR}}{\partial v} = \frac{1}{V_0}\frac{\partial(Y)_{TR}}{\partial \beta_s} = \frac{1}{V_0}\left[Y_{TR}\sin(\alpha-\epsilon_{TR}) - D_{TR}\cos(\alpha-\epsilon_{TR}) + \frac{\partial T_{TR}}{\partial \beta_s}\right]$$

$$(Y_v)_{P_i} = \frac{\partial(Y)_{P_i}}{\partial v} = \frac{1}{V_0}\frac{\partial(Y)_{P_i}}{\partial \beta_s} = \frac{1}{V_0}\frac{\partial Y_{P_i}}{\partial \beta_s}$$

7.1.2.3 $Y_{\dot{v}}$

$$Y_{\dot{v}} = -\frac{W}{g}$$

7.1.2.4 Y_w

$$Y_w = (Y_w)_F + (Y_w)_R + (Y_w)_{FUS} + (Y_w)_W + (Y_w)_T + (Y_w)_{VT} + (Y_w)_{TR} + \sum_{i=1}^{n}(Y_w)_{P_i}$$

where

$$(Y_w)_F = \frac{\partial(Y)_F}{\partial w} = Y_{w_F}\frac{\partial \alpha_F}{\partial \alpha}$$

$$Y_{w_F} = \frac{\partial(Y)_F}{\partial w_F} = \frac{1}{V_0}\frac{\partial(Y)_F}{\partial \alpha_F} = \frac{1}{V_0}Y_{\alpha_F}$$

$$(Y_w)_R = \frac{\partial(Y)_R}{\partial w} = Y_{w_R}\frac{\partial \alpha_R}{\partial \alpha}$$

$$Y_{w_R} = \frac{\partial (Y)_R}{\partial w_R} = \frac{1}{V_0} \frac{\partial (Y)_R}{\partial \alpha_R} = \frac{1}{V_0} Y_{\alpha_R}$$

$$(Y_w)_{FUS} = \frac{\partial (Y)_{FUS}}{\partial w} = Y_{w_{FUS}} \frac{\partial \alpha_{FUS}}{\partial \alpha}$$

$$Y_{w_{FUS}} = \frac{\partial (Y)_{FUS}}{\partial w_{FUS}} = \frac{1}{V_0} \frac{\partial (Y)_{FUS}}{\partial \alpha_{FUS}} = \frac{1}{V_0} Y_{\alpha_{FUS}}$$

$$(Y_w)_W = \frac{\partial (Y)_W}{\partial w} = Y_{w_W} \frac{\partial \alpha_W}{\partial \alpha}$$

$$Y_{w_W} = \frac{\partial (Y)_W}{\partial w_W} = \frac{1}{V_0} \frac{\partial (Y)_W}{\partial \alpha_W} = \frac{1}{V_0} Y_{\alpha_W}$$

$$(Y_w)_T = \frac{\partial (Y)_T}{\partial w} = Y_{w_T} \frac{\partial \alpha_T}{\partial \alpha}$$

$$Y_{w_T} = \frac{\partial (Y)_T}{\partial w_T} = \frac{1}{V_0} \frac{\partial (Y)_T}{\partial \alpha_T} = \frac{1}{V_0} Y_{\alpha_T}$$

$$(Y_w)_{VT} = \frac{\partial (Y)_{VT}}{\partial w} = Y_{w_{VT}} \frac{\partial \alpha_{VT}}{\partial \alpha}$$

$$Y_{w_{VT}} = \frac{\partial (Y)_{VT}}{\partial w_{VT}} = \frac{1}{V_0} \frac{\partial (Y)_{VT}}{\partial \alpha_{VT}} = \frac{1}{V_0} Y_{\alpha_{VT}}$$

$$(Y_w)_{TR} = \frac{\partial (Y)_{TR}}{\partial w} = Y_{w_{TR}} \frac{\partial \alpha_{TR}}{\partial \alpha}$$

$$Y_{w_{TR}} = \frac{\partial (Y)_{TR}}{\partial w_{TR}} = \frac{1}{V_0} \frac{\partial (Y)_{TR}}{\partial \alpha_{TR}} = \frac{1}{V_0} Y_{\alpha_{TR}}$$

$$(Y_w)_{P_i} = \frac{\partial (Y)_{P_i}}{\partial w} = Y_{w_{P_i}} \frac{\partial \alpha_{P_i}}{\partial \alpha}$$

$$Y_{w_{P_i}} = \frac{\partial (Y)_{P_i}}{\partial w_{P_i}} = \frac{1}{V_0} \frac{\partial (Y)_{P_i}}{\partial \alpha_{P_i}} = \frac{1}{V_0} Y_{\alpha_{P_i}}$$

7.1.2.5 $Y_{\dot{\theta}}$

$$Y_{\dot\theta} = (Y_{\dot\theta})_F + (Y_{\dot\theta})_R + (Y_{\dot\theta})_W + (Y_{\dot\theta})_T + (Y_{\dot\theta})_{VT} + (Y_{\dot\partial})_{TR} + \sum_{i=1}^{n} (Y_{\dot\theta})_{P_i}$$

where

$$(Y_{\dot\theta})_F = \frac{\partial(Y)_F}{\partial\dot\theta} = Y_{u_F}\,\ell_{z_F} - Y_{w_F}\,\ell_{x_F} + (\frac{\partial Y}{\partial a_1})_F \frac{\partial a_{1_F}}{\partial q} + (\frac{\partial Y}{\partial b_1})_F \frac{\partial b_{1_F}}{\partial q}$$

$$(\frac{\partial Y}{\partial a_1})_F = \frac{\partial L_F}{\partial a_{1_F}} \sin A_{1_F} + \frac{\partial Y_F}{\partial a_{1_F}} \cos A_{1_F}$$

$$(\frac{\partial Y}{\partial b_1})_F = \frac{\partial L_F}{\partial b_{1_F}} \sin A_{1_F} + \frac{\partial Y_F}{\partial b_{1_F}} \cos A_{1_F}$$

$$(Y_{\dot\theta})_R = \frac{\partial(Y)_R}{\partial\dot\theta} = Y_{u_R}\,\ell_{z_R} - Y_{w_R}\,\ell_{x_R} + (\frac{\partial Y}{\partial a_1})_R \frac{\partial a_{1_R}}{\partial q} + (\frac{\partial Y}{\partial b_1})_R \frac{\partial b_{1_R}}{\partial q}$$

$$(\frac{\partial Y}{\partial a_1})_R = \frac{\partial L_R}{\partial a_{1_R}} \sin A_{1_R} - \frac{\partial Y_R}{\partial a_{1_R}} \cos A_{1_R}$$

$$(\frac{\partial Y}{\partial b_1})_R = \frac{\partial L_R}{\partial b_{1_R}} \sin A_{1_R} - \frac{\partial Y_R}{\partial b_{1_R}} \cos A_{1_R}$$

$$(Y_{\dot\theta})_W = \frac{\partial(Y)_W}{\partial\dot\theta} = Y_{u_W}\ell_{z_W} - Y_{w_W}\,\ell_{x_W}$$

$$(Y_{\dot\theta})_T = \frac{\partial(Y)_T}{\partial\dot\theta} = Y_{u_T}\,\ell_{z_T} - Y_{w_T}\,\ell_{x_T}$$

$$(Y_{\dot\theta})_{VT} = \frac{\partial(Y)_{VT}}{\partial\dot\theta} = Y_{u_{VT}}\ell_{z_{VT}} - Y_{w_{VT}}\ell_{x_{VT}}$$

$$(Y_{\dot\theta})_{TR} = \frac{\partial(Y)_{TR}}{\partial\dot\theta} = Y_{u_{TR}}\ell_{z_{TR}} - Y_{w_{TR}}\ell_{x_{TR}}$$

$$(Y_{\dot\theta})_{P_i} = \frac{\partial(Y)_{P_i}}{\partial\dot\theta} = Y_{u_{P_i}}\,\ell_{z_{P_i}} - Y_{w_{P_i}}\ell_{x_{P_i}}$$

7.1.2.6 Y_ϕ

$$Y_\phi = W$$

7.1.2.7 $Y_{\dot\phi}$

$$Y_{\dot\phi} = (Y_{\dot\phi})_F + (Y_{\dot\phi})_R + (Y_{\dot\phi})_W + (Y_{\dot\phi})_T + (Y_{\dot\phi})_{VT} + (Y_{\dot\phi})_{TR} + \sum_{i=1}^{n} (Y_{\dot\phi})_{P_i} + \frac{W}{g} V_0 \sin\alpha$$

where

$$(Y_{\dot\phi})_F = \frac{\partial(Y)_F}{\partial\dot\phi} = Y_{w_F}\ \ell_{Y_F} - Y_{v_F}\ \ell_{Z_F} + \left(\frac{\partial Y}{\partial a_1}\right)_F \frac{\partial a_{1_F}}{\partial p} + \left(\frac{\partial Y}{\partial b_1}\right)_F \frac{\partial b_{1_F}}{\partial p}$$

$$(Y_{\dot\phi})_R = \frac{\partial(Y)_R}{\partial\dot\phi} = Y_{w_R}\ \ell_{Y_R} - Y_{v_R}\ \ell_{Z_R} + \left(\frac{\partial Y}{\partial a_1}\right)_R \frac{\partial a_{1_R}}{\partial p} + \left(\frac{\partial Y}{\partial b_1}\right)_R \frac{\partial b_{1_R}}{\partial p}$$

$$(Y_{\dot\phi})_W = \frac{\partial(Y)_W}{\partial\dot\phi} = Y_{w_W}\ \ell_{Y_W} - Y_{v_W}\ \ell_{Z_W}$$

$$(Y_{\dot\phi})_T = \frac{\partial(Y)_T}{\partial\dot\phi} = Y_{w_T}\ \ell_{Y_T} - Y_{v_T}\ \ell_{Z_T}$$

$$(Y_{\dot\phi})_{VT} = \frac{\partial(Y)_{VT}}{\partial\dot\phi} = Y_{w_{VT}}\ \ell_{Y_{VT}} - Y_{v_{VT}}\ \ell_{Z_{VT}}$$

$$(Y_{\dot\phi})_{TR} = \frac{\partial(Y)_{TR}}{\partial\dot\phi} = Y_{w_{TR}}\ \ell_{Y_{TR}} - Y_{v_{TR}}\ \ell_{Z_{TR}}$$

$$(Y_{\dot\phi})_{P_i} = \frac{\partial(Y)_{P_i}}{\partial\dot\phi} = Y_{w_{P_i}}\ \ell_{Y_{P_i}} - Y_{v_{P_i}}\ \ell_{Z_{P_i}}$$

7.1.2.8 Y_ψ

$$Y_{\psi} = W \sin \theta$$

<u>7.1.2.9 $Y_{\dot{\psi}}$</u>

$$Y_{\dot{\psi}} = (Y_{\dot{\psi}})_F + (Y_{\dot{\psi}})_R + (Y_{\dot{\psi}})_W + (Y_{\dot{\psi}})_T + (Y_{\dot{\psi}})_{VT} + (Y_{\dot{\psi}})_{TR} + \sum_{i=1}^{n} (Y_{\dot{\psi}})_{P_i} - \frac{W}{g} V_a$$

where

$$(Y_{\dot{\psi}})_F = \frac{\partial (Y)_F}{\partial \dot{\psi}} = Y_{V_F} \ell_{X_F} - Y_{U_F} \ell_{Y_F} + \left(\frac{\partial Y}{\partial a_1}\right)_F \frac{\partial a_{1_F}}{\partial r} + \left(\frac{\partial Y}{\partial b_1}\right)_F \frac{\partial b_{1_F}}{\partial r}$$

$$(Y_{\dot{\psi}})_R = \frac{\partial (Y)_R}{\partial \dot{\psi}} = Y_{V_R} \ell_{X_R} - Y_{U_R} \ell_{Y_R} + \left(\frac{\partial Y}{\partial a_1}\right)_R \frac{\partial a_{1_R}}{\partial r} + \left(\frac{\partial Y}{\partial b_1}\right)_R \frac{\partial b_{1_R}}{\partial r}$$

$$(Y_{\dot{\psi}})_W = \frac{\partial (Y)_W}{\partial \dot{\psi}} = Y_{V_W} \ell_{X_W} - Y_{U_W} \ell_{Y_W}$$

$$(Y_{\dot{\psi}})_T = \frac{\partial (Y)_T}{\partial \dot{\psi}} = Y_{V_T} \ell_{X_T} - Y_{U_T} \ell_{Y_T}$$

$$(Y_{\dot{\psi}})_{VT} = \frac{\partial (Y)_{VT}}{\partial \dot{\psi}} = Y_{V_{VT}} \ell_{X_{VT}} - Y_{U_{VT}} \ell_{Y_{VT}}$$

$$(Y_{\dot{\psi}})_{TR} = \frac{\partial (Y)_{TR}}{\partial \dot{\psi}} = Y_{V_{TR}} \ell_{X_{TR}} - Y_{U_{TR}} \ell_{Y_{TR}}$$

$$(Y_{\dot{\psi}})_{P_i} = \frac{\partial (Y)_{P_i}}{\partial \dot{\psi}} = Y_{V_{P_i}} \ell_{X_{P_i}} - Y_{U_{P_i}} \ell_{Y_{P_i}}$$

7.1.3 The Z-Force Derivatives

7.1.3.1 Z_u

$$Z_u = (Z_u)_F + (Z_u)_R + (Z_u)_{FUS} + (Z_u)_W + (Z_u)_T + (Z_u)_{VT} + (Z_u)_{TR} + \sum_{i=1}^{n} (Z_u)_{P_i}$$

where

$$(Z_u)_F = \frac{\partial (Z)_F}{\partial u} = Z_{u_F} + Z_{\alpha_F} \frac{\partial \alpha_F}{\partial u}$$

$$Z_{u_F} = \frac{\partial (Z)_F}{\partial u_F} = -\left[\frac{\partial D_F}{\partial u_F} \sin(\alpha - \epsilon_F) + \left(\frac{\partial L_F}{\partial u_F} \cos A_{1_F} - \frac{\partial Y_F}{\partial u_F} \sin A_{1_F}\right)\cos(\alpha - \epsilon_F)\right]$$

$$Z_{\alpha_F} = \frac{\partial (Z)_F}{\partial \alpha_F} = -\left[\frac{\partial D_F}{\partial \alpha_F} \sin(\alpha - \epsilon_F) + \left(\frac{\partial L_F}{\partial \alpha_F} \cos A_{1_F} - \frac{\partial Y_F}{\partial \alpha_F} \sin A_{1_F}\right)\cos(\alpha - \epsilon_F)\right]$$

$$-\left[D_F \cos(\alpha - \epsilon_F) - (L_F \cos A_{1_F} - Y_F \sin A_{1_F}) \sin(\alpha - \epsilon_F)\right]$$

$$(Z_u)_R = \frac{\partial (Z)_R}{\partial u} = Z_{u_R} + Z_{\alpha_R} \frac{\partial \alpha_R}{\partial u}$$

$$Z_{u_R} = \frac{\partial (Z)_R}{\partial u_R} = -\left[\frac{\partial D_R}{\partial u_R} \sin(\alpha - \epsilon_R) + \left(\frac{\partial L_R}{\partial u_R} \cos A_{1_R} + \frac{\partial Y_R}{\partial u_R} \sin A_{1_R}\right)\cos(\alpha - \epsilon_R)\right]$$

$$Z_{\alpha_R} = \frac{\partial (Z)_R}{\partial \alpha_R} = -\left[\frac{\partial D_R}{\partial \alpha_R} \sin(\alpha - \epsilon_R) + \left(\frac{\partial L_R}{\partial \alpha_R} \cos A_{1_R} + \frac{\partial Y_R}{\partial \alpha_R} \sin A_{1_R}\right)\cos(\alpha - \epsilon_R)\right]$$

$$-\left[D_R \cos(\alpha - \epsilon_R) - (L_R \cos A_{1_R} + Y_R \sin A_{1_R}) \sin(\alpha - \epsilon_R)\right]$$

$$(Z_u)_{FUS} = \frac{\partial (Z)_{FUS}}{\partial u} = Z_{u_{FUS}} + Z_{\alpha_{FUS}} \frac{\partial \alpha_{FUS}}{\partial u}$$

$$Z_{u_{FUS}} = \frac{\partial (Z)_{FUS}}{\partial u_{FUS}} = -\left[\frac{\partial D_{FUS}}{\partial u_{FUS}} \sin(\alpha - \epsilon_{FUS}) + \frac{\partial L_{FUS}}{\partial u_{FUS}} \cos(\alpha - \epsilon_{FUS})\right]$$

$$Z_{\alpha_{FUS}} = \frac{\partial (Z)_{FUS}}{\partial \alpha_{FUS}} = -\left[\frac{\partial D_{FUS}}{\partial \alpha_{FUS}} \sin(\alpha - \epsilon_{FUS}) + \frac{\partial L_{FUS}}{\partial \alpha_{FUS}} \cos(\alpha - \epsilon_{FUS})\right]$$

$$-\left[D_{FUS} \cos(\alpha - \epsilon_{FUS}) - L_{FUS} \sin(\alpha - \epsilon_{FUS})\right]$$

$$(Z_u)_W = \frac{\partial (Z)_W}{\partial u} = Z_{u_W} + Z_{\alpha_W} \frac{\partial \alpha_W}{\partial u}$$

$$Z_{u_W} = \frac{\partial (Z)_W}{\partial u_W} = -\left[\frac{\partial D_W}{\partial u_W} \sin(\alpha - \epsilon_W) + \frac{\partial L_W}{\partial u_W} \cos(\alpha - \epsilon_W)\right]$$

$$Z_{\alpha_W} = \frac{\partial (Z)_W}{\partial \alpha_W} = -\left[\frac{\partial D_W}{\partial \alpha_W} \sin(\alpha - \epsilon_W) + \frac{\partial L_W}{\partial \alpha_W} \cos(\alpha - \epsilon_W)\right]$$

$$-\left[D_W \cos(\alpha - \epsilon_W) - L_W \sin(\alpha - \epsilon_W)\right]$$

$$(Z_u)_T = \frac{\partial (Z)_T}{\partial u} = Z_{u_T} + Z_{\alpha_T} \frac{\partial \alpha_T}{\partial u}$$

$$Z_{u_T} = \frac{\partial (Z)_T}{\partial u_T} = -\left[\frac{\partial D_T}{\partial u_T} \sin(\alpha - \epsilon_T) + \frac{\partial L_T}{\partial u_T} \cos(\alpha - \epsilon_T)\right]$$

$$Z_{\alpha_T} = \frac{\partial (Z)_T}{\partial \alpha_T} = -\left[\frac{\partial D_T}{\partial \alpha_T} \sin(\alpha - \epsilon_T) + \frac{\partial L_T}{\partial \alpha_T} \cos(\alpha - \epsilon_T)\right]$$

$$-\left[D_T \cos(\alpha-\epsilon_T) - L_T \sin(\alpha-\epsilon_T)\right]$$

$$(Z_u)_{VT} = \frac{\partial (Z)_{VT}}{\partial u} = Z_{u_{VT}} + Z_{\alpha_{VT}} \frac{\partial \alpha_{VT}}{\partial u}$$

$$Z_{u_{VT}} = \frac{\partial (Z)_{VT}}{\partial u_{VT}} = -\frac{\partial D_{VT}}{\partial u_{VT}} \sin(\alpha-\epsilon_{VT})$$

$$Z_{\alpha_{VT}} = \frac{\partial (Z)_{VT}}{\partial \alpha_{VT}} = -\frac{\partial D_{VT}}{\partial \alpha_{VT}} \sin(\alpha-\epsilon_{VT}) - D_{VT} \cos(\alpha-\epsilon_{VT})$$

$$(Z_u)_{TR} = \frac{\partial (Z)_{TR}}{\partial u} = Z_{u_{TR}} + Z_{\alpha_{TR}} \frac{\partial \alpha_{TR}}{\partial u}$$

$$Z_{u_{TR}} = \frac{\partial (Z)_{TR}}{\partial u_{TR}} = -\left[\frac{\partial D_{TR}}{\partial u_{TR}} \sin(\alpha-\epsilon_{TR}) + \frac{\partial Y_{TR}}{\partial u_{TR}} \cos(\alpha-\epsilon_{TR})\right]$$

$$Z_{\alpha_{TR}} = \frac{\partial (Z)_{TR}}{\partial \alpha_{TR}} = -\left[\frac{\partial D_{TR}}{\partial \alpha_{TR}} \sin(\alpha-\epsilon_{TR}) + \frac{\partial Y_{TR}}{\partial \alpha_{TR}} \cos(\alpha-\epsilon_{TR})\right]$$

$$-\left[D_{TR} \cos(\alpha-\epsilon_{TR}) - Y_{TR} \sin(\alpha-\epsilon_{TR})\right]$$

$$(Z_u)_{P_i} = \frac{\partial (Z)_{P_i}}{\partial u} = Z_{u_{P_i}} + Z_{\alpha_{P_i}} \frac{\partial \alpha_{P_i}}{\partial u}$$

$$Z_{u_{P_i}} = \frac{\partial (Z)_{P_i}}{\partial u_{P_i}} = -\left[\frac{\partial T_{P_i}}{\partial u_{P_i}} \sin i_{P_i} + \frac{\partial N_{P_i}}{\partial u_{P_i}} \cos i_{P_i}\right]$$

$$Z_{\alpha_{P_i}} = \frac{\partial (Z)_{P_i}}{\partial \alpha_{P_i}} = -\left[\frac{\partial T_{P_i}}{\partial \alpha_{P_i}} \sin i_{P_i} + \frac{\partial N_{P_i}}{\partial \alpha_{P_i}} \cos i_{P_i}\right]$$

7.1.3.2 Z_V

$$Z_V = (Z_V)_F + (Z_V)_R + (Z_V)_{FUS} + (Z_V)_W + (Z_V)_T + (Z_V)_{VT} + (Z_V)_{TR} + \sum_{i=1}^{n} (Z_V)_{P_i}$$

where

$$(Z_v)_F = 0$$

$$(Z_v)_R = 0$$

$$(Z_v)_{FUS} = \frac{\partial(Z)_{FUS}}{\partial v} = \frac{1}{V_0}\frac{\partial(Z)_{FUS}}{\partial\beta_s} = -\frac{1}{V_0}\left[\frac{\partial D_{FUS}}{\partial\beta_s}\sin(\alpha-\epsilon_{FUS}) + \frac{\partial L_{FUS}}{\partial\beta_s}\cos(\alpha-\epsilon_{FUS})\right]$$

$$(Z_v)_W = 0$$

$$(Z_v)_T = 0$$

$$(Z_v)_{VT} = \frac{\partial(Z)_{VT}}{\partial v} = \frac{1}{V_0}\frac{\partial(Z)_{VT}}{\partial\beta_s} = -\frac{1}{V_0}\left[\frac{\partial D_{VT}}{\partial\beta_s}\sin(\alpha-\epsilon_{VT})\right]$$

$$(Z_v)_{TR} = \frac{\partial(Z)_{TR}}{\partial v} = \frac{1}{V_0}\frac{\partial(Z)_{TR}}{\partial\beta_s} = -\frac{1}{V_0}\left[\frac{\partial D_{TR}}{\partial\beta_s}\sin(\alpha-\epsilon_{TR}) + \frac{\partial Y_{TR}}{\partial\beta_s}\cos(\alpha-\epsilon_{TR})\right]$$

$$(Z_v)_{P_i} = \frac{\partial(Z)_{P_i}}{\partial v} = \frac{1}{V_0}\frac{\partial(Z)_{P_i}}{\partial\beta_s} = -\frac{1}{V_0}\left[-\frac{\partial T_{P_i}}{\partial\beta_s}\sin i_{P_i} + \frac{\partial N_{P_i}}{\partial\beta_s}\cos i_{P_i}\right]$$

7.1.3.3 Z_w

$$Z_w = (Z_w)_F + (Z_w)_R + (Z_w)_{FUS} + (Z_w)_W + (Z_w)_T + (Z_w)_{VT} + (Z_w)_{TR} + \sum_{i=1}^{n}(Z_w)_{P_i}$$

where

$$(Z_w)_F = \frac{\partial(Z)_F}{\partial w} = Z_{w_F}\frac{\partial\alpha_F}{\partial\alpha}$$

$$Z_{w_F} = \frac{\partial (Z)_F}{\partial w_F} = \frac{1}{V_0} \frac{\partial (Z)_F}{\partial \alpha_F} = \frac{1}{V_0} Z_{\alpha_F}$$

$$(Z_w)_R = \frac{\partial (Z)_R}{\partial w} = Z_{w_R} \frac{\partial \alpha_R}{\partial \alpha}$$

$$Z_{w_R} = \frac{\partial (Z)_R}{\partial w_R} = \frac{1}{V_0} \frac{\partial (Z)_R}{\partial \alpha_R} = \frac{1}{V_0} Z_{\alpha_R}$$

$$(Z_w)_{FUS} = \frac{\partial (Z)_{FUS}}{\partial w} = Z_{w_{FUS}} \frac{\partial \alpha_{FUS}}{\partial \alpha}$$

$$Z_{w_{FUS}} = \frac{\partial (Z)_{FUS}}{\partial w_{FUS}} = \frac{1}{V_0} \frac{\partial (Z)_{FUS}}{\partial \alpha_{FUS}} = \frac{1}{V_0} Z_{\alpha_{FUS}}$$

$$(Z_w)_W = \frac{\partial (Z)_W}{\partial w} = Z_{w_W} \frac{\partial \alpha_W}{\partial \alpha}$$

$$Z_{w_W} = \frac{\partial (Z)_W}{\partial w_W} = \frac{1}{V_0} \frac{\partial (Z)_W}{\partial \alpha_W} = \frac{1}{V_0} Z_{\alpha_W}$$

$$(Z_w)_T = \frac{\partial (Z)_T}{\partial w} = Z_{w_T} \frac{\partial \alpha_T}{\partial \alpha}$$

$$Z_{w_T} = \frac{\partial (Z)_T}{\partial w_T} = \frac{1}{V_0} \frac{\partial (Z)_T}{\partial \alpha_T} = \frac{1}{V_0} Z_{\alpha_T}$$

$$(Z_w)_{VT} = \frac{\partial (Z)_{VT}}{\partial w} = Z_{w_{VT}} \frac{\partial \alpha_{VT}}{\partial \alpha}$$

$$Z_{w_{VT}} = \frac{\partial (Z)_{VT}}{\partial w_{VT}} = \frac{1}{V_0} \frac{\partial (Z)_{VT}}{\partial \alpha_{VT}} = \frac{1}{V_0} Z_{\alpha_{VT}}$$

$$(Z_w)_{TR} = \frac{\partial (Z)_{TR}}{\partial w} = Z_{w_{TR}} \frac{\partial \alpha_{TR}}{\partial \alpha}$$

$$Z_{w_{TR}} = \frac{\partial (Z)_{TR}}{\partial w_{TR}} = \frac{1}{V_0} \frac{\partial (Z)_{TR}}{\partial \alpha_{TR}} = \frac{1}{V_0} Z_{\alpha_{TR}}$$

$$(Z_w)_{P_i} = \frac{\partial (Z)_{P_i}}{\partial w} = Z_{w_{P_i}} \frac{\partial \alpha_{P_i}}{\partial \alpha}$$

$$Z_{w_{P_i}} = \frac{\partial (Z)_{P_i}}{\partial w_{P_i}} = \frac{1}{V_o} \frac{\partial (Z)_{P_i}}{\partial \alpha_{P_i}} = \frac{1}{V_o} Z_{\alpha_{P_i}}$$

7.1.3.4 $Z_{\dot{w}}$

$$Z_{\dot{w}} = (Z_{\dot{w}})_R + (Z_{\dot{w}})_T + (Z_{\dot{w}})_{VT} + (Z_{\dot{w}})_{TR} - \frac{W}{g}$$

where

$$(Z_{\dot{w}})_R = \frac{\partial (Z)_R}{\partial \dot{w}} = \frac{1}{V_o} \frac{\partial (Z)_R}{\partial \dot{\alpha}} = \frac{1}{V_o} Z_{\alpha_R} \frac{\partial \alpha_R}{\partial \dot{\alpha}}$$

$$(Z_{\dot{w}})_T = \frac{\partial (Z)_T}{\partial \dot{w}} = \frac{1}{V_o} \frac{\partial (Z)_T}{\partial \dot{\alpha}} = \frac{1}{V_o} Z_{\alpha_T} \frac{\partial \alpha_T}{\partial \dot{\alpha}}$$

$$(Z_{\dot{w}})_{VT} = \frac{\partial (Z)_{VT}}{\partial \dot{w}} = \frac{1}{V_o} \frac{\partial (Z)_{VT}}{\partial \dot{\alpha}} = \frac{1}{V_o} Z_{\alpha_{VT}} \frac{\partial \alpha_{VT}}{\partial \dot{\alpha}}$$

$$(Z_{\dot{w}})_{TR} = \frac{\partial (Z)_{TR}}{\partial \dot{w}} = \frac{1}{V_o} \frac{\partial (Z)_{TR}}{\partial \dot{\alpha}} = \frac{1}{V_o} Z_{\alpha_{TR}} \frac{\partial \alpha_{TR}}{\partial \dot{\alpha}}$$

7.1.3.5 Z_{θ}

$$Z_{\theta} = -W \sin \theta$$

7.1.3.6 $Z_{\dot{\theta}}$

$$Z_{\dot\theta} = (Z_{\dot\theta})_F + (Z_{\dot\theta})_R + (Z_{\dot\theta})_W + (Z_{\dot\theta})_T + (Z_{\dot\theta})_{VT} + (Z_{\dot\theta})_{TR} + \sum_{i=1}^{n} (Z_{\dot\theta})_{P_i} + \frac{W}{g} V_o$$

where

$$(Z_{\dot\theta})_F = \frac{\partial (Z)_F}{\partial \dot\theta} = Z_{u_F} \ell_{z_F} - Z_{w_F} \ell_{x_F} + \left(\frac{\partial Z}{\partial a_1}\right)_F \frac{\partial a_{1_F}}{\partial q} + \left(\frac{\partial Z}{\partial b_1}\right)_F \frac{\partial b_{1_F}}{\partial q}$$

$$\left(\frac{\partial Z}{\partial a_1}\right)_F = -\left[\frac{\partial D_F}{\partial a_{1_F}} \sin(\alpha - \epsilon_F) + \left(\frac{\partial L_F}{\partial a_{1_F}} \cos A_{1_F} - \frac{\partial Y_F}{\partial a_{1_F}} \sin A_{1_F}\right) \cos(\alpha - \epsilon_F)\right]$$

$$\left(\frac{\partial Z}{\partial b_1}\right)_F = -\left[\frac{\partial D_F}{\partial b_{1_F}} \sin(\alpha - \epsilon_F) + \left(\frac{\partial L_F}{\partial b_{1_F}} \cos A_{1_F} - \frac{\partial Y_F}{\partial b_{1_F}} \sin A_{1_F}\right) \cos(\alpha - \epsilon_F)\right]$$

$$(Z_{\dot\theta})_R = \frac{\partial (Z)_R}{\partial \dot\theta} = Z_{u_R} \ell_{z_R} - Z_{w_R} \ell_{x_R} + \left(\frac{\partial Z}{\partial a_1}\right)_R \frac{\partial a_{1_R}}{\partial q} + \left(\frac{\partial Z}{\partial b_1}\right)_R \frac{\partial b_{1_R}}{\partial q}$$

$$\left(\frac{\partial Z}{\partial a_1}\right)_R = -\left[\frac{\partial D_R}{\partial a_{1_R}} \sin(\alpha - \epsilon_R) + \left(\frac{\partial L_R}{\partial a_{1_R}} \cos A_{1_R} + \frac{\partial Y_R}{\partial a_{1_R}} \sin A_{1_R}\right) \cos(\alpha - \epsilon_R)\right]$$

$$\left(\frac{\partial Z}{\partial b_1}\right)_R = -\left[\frac{\partial D_R}{\partial b_{1_R}} \sin(\alpha - \epsilon_R) + \left(\frac{\partial L_R}{\partial b_{1_R}} \cos A_{1_R} + \frac{\partial Y_R}{\partial b_{1_R}} \sin A_{1_R}\right) \cos(\alpha - \epsilon_R)\right]$$

$$(Z_{\dot\theta})_W = \frac{\partial (Z)_W}{\partial \dot\theta} = Z_{u_W} \ell_{z_W} - Z_{w_W} \ell_{x_W}$$

$$(Z_{\dot\theta})_T = \frac{\partial (Z)_T}{\partial \dot\theta} = Z_{u_T} \ell_{z_T} - Z_{w_T} \ell_{x_T}$$

$$(Z_{\dot\theta})_{VT} = \frac{\partial (Z)_{VT}}{\partial \dot\theta} = Z_{u_{VT}} \ell_{z_{VT}} - Z_{w_{VT}} \ell_{x_{VT}}$$

$$(Z_{\dot\theta})_{TR} = \frac{\partial (Z)_{TR}}{\partial \dot\theta} = Z_{u_{TR}} \ell_{z_{TR}} - Z_{w_{TR}} \ell_{x_{TR}}$$

$$(Z_{\dot\theta})_{P_i} = \frac{\partial (Z)_{P_i}}{\partial \dot\theta} = Z_{u_{P_i}} \ell_{z_{P_i}} - Z_{w_{P_i}} \ell_{x_{P_i}}$$

7.1.3.7 $Z_{\dot\phi}$

$$Z_{\dot\phi} = (Z_{\dot\phi})_F + (Z_{\dot\phi})_R + (Z_{\dot\phi})_W + (Z_{\dot\phi})_T + (Z_{\dot\phi})_{VT} + (Z_{\dot\phi})_{TR} + \sum_{i=1}^{n} (Z_{\dot\phi})_{P_i}$$

where

$$(Z_{\dot\phi})_F = \frac{\partial (Z)_F}{\partial \dot\phi} = Z_{w_F}\, \ell_{Y_F} - Z_{v_F}\, \ell_{Z_F} + \left(\frac{\partial Z}{\partial a_1}\right)_F \frac{\partial a_{1_F}}{\partial p} + \left(\frac{\partial Z}{\partial b_1}\right)_F \frac{\partial b_{1_F}}{\partial p}$$

$$(Z_{\dot\phi})_R = \frac{\partial (Z)_R}{\partial \dot\phi} = Z_{w_R}\, \ell_{Y_R} - Z_{v_R}\, \ell_{Z_R} + \left(\frac{\partial Z}{\partial a_1}\right)_R \frac{\partial a_{1_R}}{\partial p} + \left(\frac{\partial Z}{\partial b_1}\right)_R \frac{\partial b_{1_R}}{\partial p}$$

$$(Z_{\dot\phi})_W = \frac{\partial (Z)_W}{\partial \dot\phi} = Z_{w_W}\, \ell_{Y_W} - Z_{v_W}\, \ell_{Z_W}$$

$$(Z_{\dot\phi})_T = \frac{\partial (Z)_T}{\partial \dot\phi} = Z_{w_T}\, \ell_{Y_T} - Z_{v_T}\, \ell_{Z_T}$$

$$(Z_{\dot\phi})_{VT} = \frac{\partial (Z)_{VT}}{\partial \dot\phi} = Z_{w_{VT}}\, \ell_{Y_{VT}} - Z_{v_{VT}}\, \ell_{Z_{VT}}$$

$$(Z_{\dot\phi})_{TR} = \frac{\partial (Z)_{TR}}{\partial \dot\phi} = Z_{w_{TR}}\, \ell_{Y_{TR}} - Z_{v_{TR}}\, \ell_{Z_{TR}}$$

$$(Z_{\dot\phi})_{P_i} = \frac{\partial (Z)_{P_i}}{\partial \dot\phi} = Z_{w_{P_i}}\, \ell_{Y_{P_i}} - Z_{v_{P_i}}\, \ell_{Z_{P_i}}$$

7.1.3.8 $Z_{\dot\psi}$

$$Z_{\dot\psi} = (Z_{\dot\psi})_F + (Z_{\dot\psi})_R + (Z_{\dot\psi})_W + (Z_{\dot\psi})_T + (Z_{\dot\psi})_{VT} + (Z_{\dot\psi})_{TR} + \sum_{i=1}^{n} (Z_{\dot\psi})_{P_i}$$

where

$$(Z_{\dot\psi})_F = \frac{\partial (Z)_F}{\partial \dot\psi} = Z_{v_F}\, \ell_{X_F} - Z_{u_F}\, \ell_{Y_F} + \left(\frac{\partial Z}{\partial a_1}\right)_F \frac{\partial a_{1_F}}{\partial r} + \left(\frac{\partial Z}{\partial b_1}\right)_F \frac{\partial b_{1_F}}{\partial r}$$

$$(Z_{\dot\psi})_R = \frac{\partial (Z)_R}{\partial \dot\psi} = Z_{V_R}\, \ell_{X_R} - Z_{U_R}\, \ell_{Y_R} + \left(\frac{\partial Z}{\partial a_1}\right)_R \frac{\partial a_{1_R}}{\partial r} + \left(\frac{\partial Z}{\partial b_1}\right)_R \frac{\partial b_{1_R}}{\partial r}$$

$$(Z_{\dot\psi})_W = \frac{\partial (Z)_W}{\partial \dot\psi} = Z_{V_W}\, \ell_{X_W} - Z_{U_W}\, \ell_{Y_W}$$

$$(Z_{\dot\psi})_T = \frac{\partial (Z)_T}{\partial \dot\psi} = Z_{V_T}\, \ell_{X_T} - Z_{U_T}\, \ell_{Y_T}$$

$$(Z_{\dot\psi})_{VT} = \frac{\partial (Z)_{VT}}{\partial \dot\psi} = Z_{V_{VT}}\, \ell_{X_{VT}} - Z_{U_{VT}}\, \ell_{Y_{VT}}$$

$$(Z_{\dot\psi})_{TR} = \frac{\partial (Z)_{TR}}{\partial \dot\psi} = Z_{V_{TR}}\, \ell_{X_{TR}} - Z_{U_{TR}}\, \ell_{Y_{TR}}$$

$$(Z_{\dot\psi})_{P_i} = \frac{\partial (Z)_{P_i}}{\partial \dot\psi} = Z_{V_{P_i}}\, \ell_{X_{P_i}} - Z_{U_{P_i}}\, \ell_{Y_{P_i}}$$

7.1.4 The Rolling Moment ($\mathcal{L}$) Derivatives

The rolling moment (about body X-axis) can be written in its abbreviated form as:

$$\mathcal{L} = \sum_{i=1}^{n} (\mathcal{L})_i = \sum_{i=1}^{n} \left[(Z)_i\, \ell_{Y_i} - (Y)_i\, \ell_{Z_i} + (\mathcal{L}_0)_i \right] + \mathcal{L}_I$$

where

$(Y)_i$ and $(Z)_i$ are forces in the Y and Z directions of body axes, respectively, due to i^{th} aircraft components, and ℓ_{Y_i} and ℓ_{Z_i} are respective moment arms.

$(\mathcal{L}_0)_i$ is the steady aerodynamic rolling moment about aircraft C.G. due to i^{th} aircraft components, and $\mathcal{L}_I$ is the inertia rolling moment.

7.1.4.1 $\mathcal{L}_u$

$$\mathcal{L}_U = (Z_U)_F \ell_{Y_F} - (Y_U)_F \ell_{Z_F} + (Z_U)_R \ell_{Y_R} - (Y_U)_R \ell_{Z_R}$$

$$+ (Z_U)_W \ell_{Y_W} - (Y_U)_W \ell_{Z_W} + (Z_U)_T \ell_{Y_T} - (Y_U)_T \ell_{Z_T}$$

$$+ (Z_U)_{VT} \ell_{Y_{VT}} - (Y_U)_{VT} \ell_{Z_{VT}} + (Z_U)_{TR} \ell_{Y_{TR}} - (Y_U)_{TR} \ell_{Z_{TR}}$$

$$+ \sum_{i=1}^{n} \left[(Z_U)_{P_i} \ell_{Y_{P_i}} - (Y_U)_{P_i} \ell_{Z_{P_i}} + \frac{\partial Q_{P_i}}{\partial u} \right] + \frac{\partial \mathcal{L}_{FUS}}{\partial u} + \frac{\partial \mathcal{L}_{HUB_F}}{\partial u} - \frac{\partial \mathcal{L}_{HUB_R}}{\partial u}$$

where

$$\frac{\partial \mathcal{L}_{FUS}}{\partial u} = \frac{\partial \mathcal{L}_{FUS}}{\partial u_{FUS}} + \frac{\partial \mathcal{L}_{FUS}}{\partial \alpha_{FUS}} \frac{\partial \alpha_{FUS}}{\partial u}$$

$$\frac{\partial \mathcal{L}_{HUB_F}}{\partial u} = \frac{\partial \mathcal{L}_{HUB_F}}{\partial u_F} + \frac{\partial \mathcal{L}_{HUB_F}}{\partial \alpha_F} \frac{\partial \alpha_F}{\partial u}$$

$$\frac{\partial \mathcal{L}_{HUB_R}}{\partial u} = \frac{\partial \mathcal{L}_{HUB_R}}{\partial u_R} + \frac{\partial \mathcal{L}_{HUB_R}}{\partial \alpha_R} \frac{\partial \alpha_R}{\partial u}$$

7.1.4.2 $\mathcal{L}_V$

$$\mathcal{L}_V = (Z_V)_F \ell_{Y_F} - (Y_V)_F \ell_{Z_F} + (Z_V)_R \ell_{Y_R} - (Y_V)_R \ell_{Z_R}$$

$$+ (Z_V)_W \ell_{Y_W} - (Y_V)_W \ell_{Z_W} + (Z_V)_T \ell_{Y_T} - (Y_V)_T \ell_{Z_T}$$

$$+ (Z_V)_{VT} \ell_{Y_{VT}} - (Y_V)_{VT} \ell_{Z_{VT}} + (Z_V)_{TR} \ell_{Y_{TR}} - (Y_V)_{TR} \ell_{Z_{TR}}$$

$$+ \sum_{i=1}^{n} \left[(Z_V)_{P_i} \ell_{Y_{P_i}} - (Y_V)_{P_i} \ell_{Z_{P_i}} + \frac{1}{V_0} \frac{\partial Q_{P_i}}{\partial \beta_S} \right]$$

$$+\frac{1}{V_0}\left(\frac{\partial \mathcal{L}_{FUS}}{\partial \beta_S}+\frac{\partial \mathcal{L}_{HUB_F}}{\partial \beta_S}-\frac{\partial \mathcal{L}_{HUB_R}}{\partial \beta_S}\right)$$

7.1.4.3 $\mathcal{L}_w$

$$\mathcal{L}_w=(Z_w)_F \ell_{Y_F}-(Y_w)_F \ell_{Z_F}+(Z_w)_R \ell_{Y_R}-(Y_w)_R \ell_{Z_R}$$

$$+(Z_w)_W \ell_{Y_W}-(Y_w)_W \ell_{Z_W}+(Z_w)_T \ell_{Y_T}-(Y_w)_T \ell_{Z_T}$$

$$+(Z_w)_{VT} \ell_{Y_{VT}}-(Y_w)_{VT} \ell_{Z_{VT}}+(Z_w)_{TR} \ell_{Y_{TR}}-(Y_w)_{TR} \ell_{Z_{TR}}$$

$$+\sum_{i=1}^{n}\left[(Z_w)_{P_i} \ell_{Y_{P_i}}-(Y_w)_{P_i} \ell_{Z_{P_i}}+\frac{1}{V_0}\frac{\partial Q_{P_i}}{\partial \alpha}\right]$$

$$+\frac{1}{V_0}\left[\frac{\partial \mathcal{L}_{FUS}}{\partial \alpha}+\frac{\partial \mathcal{L}_{HUB_F}}{\partial \alpha}-\frac{\partial \mathcal{L}_{HUB_R}}{\partial \alpha}\right]$$

where

$$\frac{\partial \mathcal{L}_{FUS}}{\partial \alpha}=\frac{\partial \mathcal{L}_{FUS}}{\partial \alpha_{FUS}}\frac{\partial \alpha_{FUS}}{\partial \alpha}$$

$$\frac{\partial \mathcal{L}_{HUB_F}}{\partial \alpha}=\frac{\partial \mathcal{L}_{HUB_F}}{\partial \alpha_F}\frac{\partial \alpha_F}{\partial \alpha}$$

$$\frac{\partial \mathcal{L}_{HUB_R}}{\partial \alpha}=\frac{\partial \mathcal{L}_{HUB_R}}{\partial \alpha_R}\frac{\partial \alpha_R}{\partial \alpha}$$

7.1.4.4 $\mathcal{L}_{\dot{w}}$

$$\mathcal{L}_{\dot{w}} = (Z_{\dot{w}})_R \ell_{Y_R} + (Z_{\dot{w}})_T \ell_{Y_T} + (Z_{\dot{w}})_{VT} \ell_{Y_{VT}} + (Z_{\dot{w}})_{TR} \ell_{Y_{TR}}$$

7.1.4.5 $\mathcal{L}_{\dot{\theta}}$

$$\mathcal{L}_{\dot{\theta}} = (Z_{\dot{\theta}})_F \ell_{Y_F} - (Y_{\dot{\theta}})_F \ell_{Z_F} + (Z_{\dot{\theta}})_R \ell_{Y_R} - (Y_{\dot{\theta}})_R \ell_{Z_R}$$

$$+ (Z_{\dot{\theta}})_W \ell_{Y_W} - (Y_{\dot{\theta}})_W \ell_{Z_W} + (Z_{\dot{\theta}})_T \ell_{Y_T} - (Y_{\dot{\theta}})_T \ell_{Z_T}$$

$$+ (Z_{\dot{\theta}})_{VT} \ell_{Y_{VT}} - (Y_{\dot{\theta}})_{VT} \ell_{Z_{VT}} + (Z_{\dot{\theta}})_{TR} \ell_{Y_{TR}} - (Y_{\dot{\theta}})_{TR} \ell_{Z_{TR}}$$

$$+ \sum_{i=1}^{n} \left[(Z_{\dot{\theta}})_{P_i} \ell_{Y_{P_i}} - (Y_{\dot{\theta}})_{P_i} \ell_{Z_{P_i}} + \frac{\partial Q_{P_i}}{\partial q} \right] + \frac{\partial \mathcal{L}_{HUB_F}}{\partial q} - \frac{\partial \mathcal{L}_{HUB_R}}{\partial q}$$

where

$$\frac{\partial \mathcal{L}_{HUB_F}}{\partial q} = \frac{\partial \mathcal{L}_{HUB_F}}{\partial u} \ell_{Z_F} - \frac{1}{V_0} \frac{\partial \mathcal{L}_{HUB_F}}{\partial \alpha} \ell_{X_F} + \frac{eb\Omega^2 M_S}{2} \left(\frac{\partial b_{1_F}}{\partial q} \right)$$

$$\frac{\partial \mathcal{L}_{HUB_R}}{\partial q} = \frac{\partial \mathcal{L}_{HUB_R}}{\partial u} \ell_{Z_R} - \frac{1}{V_0} \frac{\partial \mathcal{L}_{HUB_R}}{\partial \alpha} \ell_{X_R} + \frac{eb\Omega^2 M_S}{2} \left(\frac{\partial b_{1_R}}{\partial q} \right)$$

7.1.4.6 $\mathcal{L}_{\ddot{\theta}}$

$$\mathcal{L}_{\ddot{\theta}} = I_{XY}$$

7.1.4.7 $\mathcal{L}_{\dot{\phi}}$

$$\mathcal{L}_{\dot\phi} = (Z_{\dot\phi})_F \ell_{Y_F} - (Y_{\dot\phi})_F \ell_{Z_F} + (Z_{\dot\phi})_R \ell_{Y_R} - (Y_{\dot\phi})_R \ell_{Z_R}$$

$$+ (Z_{\dot\phi})_W \ell_{Y_W} - (Y_{\dot\phi})_W \ell_{Z_W} + (Z_{\dot\phi})_T \ell_{Y_T} - (Y_{\dot\phi})_T \ell_{Z_T}$$

$$+ (Z_{\dot\phi})_{VT} \ell_{Y_{VT}} - (Y_{\dot\phi})_{VT} \ell_{Z_{VT}} + (Z_{\dot\phi})_{TR} \ell_{Y_{TR}} - (Y_{\dot\phi})_{TR} \ell_{Z_{TR}}$$

$$+ \sum_{i=1}^{n} \left[(Z_{\dot\phi})_{P_i} \ell_{Y_{P_i}} - (Y_{\dot\phi})_{P_i} \ell_{Z_{P_i}} + \frac{\partial Q_{P_i}}{\partial p} \right] + \frac{\partial \mathcal{L}_{HUB_F}}{\partial p} - \frac{\partial \mathcal{L}_{HUB_R}}{\partial p}$$

where

$$\frac{\partial \mathcal{L}_{HUB_F}}{\partial p} = \frac{1}{V_0} \left[\frac{\partial \mathcal{L}_{HUB_F}}{\partial \alpha} \ell_{Y_F} - \frac{\partial \mathcal{L}_{HUB_F}}{\partial \beta_S} \ell_{Z_F} \right] + \frac{eb\Omega^2 M_S}{2} \left(\frac{\partial b_{1_F}}{\partial p} \right)$$

$$\frac{\partial \mathcal{L}_{HUB_R}}{\partial p} = \frac{1}{V_0} \left[\frac{\partial \mathcal{L}_{HUB_R}}{\partial \alpha} \ell_{Y_R} - \frac{\partial \mathcal{L}_{HUB_R}}{\partial \beta_S} \ell_{Z_R} \right] + \frac{eb\Omega^2 M_S}{2} \left(\frac{\partial b_{1_R}}{\partial p} \right)$$

7.1.4.8 $\mathcal{L}_{\ddot\phi}$

$$\mathcal{L}_{\ddot\phi} = -I_{xx}$$

7.1.4.9 $\mathcal{L}_{\dot\psi}$

$$\mathcal{L}_{\dot\psi} = (Z_{\dot\psi})_F \ell_{Y_F} - (Y_{\dot\psi})_F \ell_{Z_F} + (Z_{\dot\psi})_R \ell_{Y_R} - (Y_{\dot\psi})_R \ell_{Z_R}$$

$$+ (Z_{\dot\psi})_W \ell_{Y_W} - (Y_{\dot\psi})_W \ell_{Z_W} + (Z_{\dot\psi})_T \ell_{Y_T} - (Y_{\dot\psi})_T \ell_{Z_T}$$

$$+(Z_{\dot\psi})_{VT}\ell_{Y_{VT}}-(Y_{\dot\psi})_{VT}\ell_{Z_{VT}}+(Z_{\dot\psi})_{TR}\ell_{Y_{TR}}-(Y_{\dot\psi})_{TR}\ell_{Z_{TR}}$$

$$+\sum_{i=1}^{n}\left[(Z_{\dot\psi})_{P_i}\ell_{YP_i}-(Y_{\dot\psi})_{P_i}\ell_{ZP_i}+\frac{\partial Q_{P_i}}{\partial r}\right]+\frac{\partial \mathcal{L}_{HUB_F}}{\partial r}-\frac{\partial \mathcal{L}_{HUB_R}}{\partial r}$$

where

$$\frac{\partial \mathcal{L}_{HUB_F}}{\partial r}=\frac{1}{V_0}\frac{\partial \mathcal{L}_{HUB_F}}{\partial \beta_S}\ell_{X_F}-\frac{\partial \mathcal{L}_{HUB_F}}{\partial u}\ell_{Y_F}+\frac{eb\Omega^2 M_S}{2}\left(\frac{\partial b_{1_F}}{\partial r}\right)$$

$$\frac{\partial \mathcal{L}_{HUB_R}}{\partial r}=\frac{1}{V_0}\frac{\partial \mathcal{L}_{HUB_R}}{\partial \beta_S}\ell_{X_R}-\frac{\partial \mathcal{L}_{HUB_R}}{\partial u}\ell_{Y_R}+\frac{eb\Omega^2 M_S}{2}\left(\frac{\partial b_{1_R}}{\partial r}\right)$$

7.1.4.10 $\mathcal{L}_{\ddot\psi}$

$\mathcal{L}_{\ddot\psi}=I_{XZ}$

7.1.5 The Pitching Moment (M) Derivatives

The pitching moment (about body Y-axis) can be written in its abbreviated form as:

$$M=\sum_{i=1}^{n}(M)_i=\sum_{i=1}^{n}\left[(X)_i\ell_{Z_i}-(Z)_i\ell_{X_i}+(M_0)_i\right]+M_I$$

where

$(X)_i$ and $(Z)_i$ are forces in the X and Z directions of body axes, respectively, due to i^{th} aircraft components, and ℓ_{X_i} and ℓ_{Z_i} are their respective moment arms.

$(M_o)_i$ is the steady aerodynamic pitching moment about aircraft C. G. due to i^{th} aircraft components, and M_I is the inertia pitching moment.

7.1.5.1 M_U

$$M_U = (X_U)_F \ell_{Z_F} - (Z_U)_F \ell_{X_F} + (X_U)_R \ell_{Z_R} - (Z_U)_R \ell_{X_R}$$

$$+ (X_U)_W \ell_{Z_W} - (Z_U)_W \ell_{X_W} + (X_U)_T \ell_{Z_T} - (Z_U)_T \ell_{X_T}$$

$$+ (X_U)_{VT} \ell_{Z_{VT}} - (Z_U)_{VT} \ell_{X_{VT}} + (X_U)_{TR} \ell_{Z_{TR}} - (Z_U)_{TR} \ell_{X_{TR}}$$

$$+ \sum_{i=1}^{n} \left[(X_U)_{P_i} \ell_{Z_{P_i}} - (Z_U)_{P_i} \ell_{X_{P_i}} + \frac{\partial M_{P_i}}{\partial u} \right] + \frac{\partial M_{FUS}}{\partial u} + \frac{\partial M_{HUB_F}}{\partial u}$$

$$+ \frac{\partial M_{HUB_R}}{\partial u} + \frac{\partial Q_{TR}}{\partial u}$$

where

$$\frac{\partial M_{FUS}}{\partial u} = \frac{\partial M_{FUS}}{\partial u_{FUS}} + \frac{\partial M_{FUS}}{\partial \alpha_{FUS}} \frac{\partial \alpha_{FUS}}{\partial u}$$

$$\frac{\partial M_{HUB_F}}{\partial u} = \frac{\partial M_{HUB_F}}{\partial u_F} + \frac{\partial M_{HUB_F}}{\partial \alpha_F} \frac{\partial \alpha_F}{\partial u}$$

$$\frac{\partial M_{HUB_R}}{\partial u} = \frac{\partial M_{HUB_R}}{\partial u_R} + \frac{\partial M_{HUB_R}}{\partial \alpha_R} \frac{\partial \alpha_R}{\partial u}$$

$$\frac{\partial Q_{TR}}{\partial u} = \frac{\partial Q_{TR}}{\partial u_{TR}} + \frac{\partial Q_{TR}}{\partial \alpha_{TR}} \frac{\partial \alpha_{TR}}{\partial u}$$

7.1.5.2 M_V

$$M_V = (X_V)_F \ell_{z_F} - (Z_V)_F \ell_{x_F} + (X_V)_R \ell_{z_R} - (Z_V)_R \ell_{x_R}$$

$$+ (X_V)_W \ell_{z_W} - (Z_V)_W \ell_{x_W} + (X_V)_T \ell_{z_T} - (Z_V)_T \ell_{x_T}$$

$$+ (X_V)_{VT} \ell_{z_{VT}} - (Z_V)_{VT} \ell_{x_{VT}} + (X_V)_{TR} \ell_{z_{TR}} - (Z_V)_{TR} \ell_{x_{TR}}$$

$$+ \sum_{i=1}^{n} \left[(X_V)_{P_i} \ell_{z_{P_i}} - (Z_V)_{P_i} \ell_{x_{P_i}} + \frac{1}{V_0} \frac{\partial M_{P_i}}{\partial \beta_S} \right]$$

$$+ \frac{1}{V_0} \left(\frac{\partial M_{FUS}}{\partial \beta_S} + \frac{\partial M_{HUB_F}}{\partial \beta_S} + \frac{\partial M_{HUB_R}}{\partial \beta_S} + \frac{\partial Q_{TR}}{\partial \beta_S} \right)$$

7.1.5.3 M_W

$$M_W = (X_W)_F \ell_{z_F} - (Z_W)_F \ell_{x_F} + (X_W)_R \ell_{z_R} - (Z_W)_R \ell_{x_R}$$

$$+ (X_W)_W \ell_{z_W} - (Z_W)_W \ell_{x_W} + (X_W)_T \ell_{z_T} - (Z_W)_T \ell_{x_T}$$

$$+ (X_W)_{VT} \ell_{z_{VT}} - (Z_W)_{VT} \ell_{x_{VT}} + (X_W)_{TR} \ell_{z_{TR}} - (Z_W)_{TR} \ell_{x_{TR}}$$

$$+ \sum_{i=1}^{n} \left[(X_W)_{P_i} \ell_{z_{P_i}} - (Z_W)_{P_i} \ell_{x_{P_i}} + \frac{1}{V_0} \frac{\partial M_{P_i}}{\partial \alpha} \right]$$

$$+ \frac{1}{V_0} \left(\frac{\partial M_{FUS}}{\partial \alpha} + \frac{\partial M_{HUB_F}}{\partial \alpha} + \frac{\partial M_{HUB_R}}{\partial \alpha} + \frac{\partial Q_{TR}}{\partial \alpha} \right)$$

where

$$\frac{\partial M_{FUS}}{\partial \alpha} = \frac{\partial M_{FUS}}{\partial \alpha_{FUS}} \frac{\partial \alpha_{FUS}}{\partial \alpha}$$

$$\frac{\partial M_{HUB_F}}{\partial \alpha} = \frac{\partial M_{HUB_F}}{\partial \alpha_F} \frac{\partial \alpha_F}{\partial \alpha}$$

$$\frac{\partial M_{HUB_R}}{\partial \alpha} = \frac{\partial M_{HUB_R}}{\partial \alpha_R} \frac{\partial \alpha_R}{\partial \alpha}$$

$$\frac{\partial Q_{TR}}{\partial \alpha} = \frac{\partial Q_{TR}}{\partial \alpha_{TR}} \frac{\partial \alpha_{TR}}{\partial \alpha}$$

7.1.5.4 $M_{\dot{w}}$

$$M_{\dot{w}} = (X_{\dot{w}})_R \ell_{z_R} - (Z_{\dot{w}})_R \ell_{x_R} + (X_{\dot{w}})_T \ell_{z_T} - (Z_{\dot{w}})_T \ell_{x_T}$$

$$+ (X_{\dot{w}})_{VT} \ell_{z_{VT}} - (Z_{\dot{w}})_{VT} \ell_{x_{VT}} + (X_{\dot{w}})_{TR} \ell_{z_{TR}} - (Z_{\dot{w}})_{TR} \ell_{x_{TR}}$$

7.1.5.5 $M_{\dot{\theta}}$

$$M_{\dot{\theta}} = (X_{\dot{\theta}})_F \ell_{z_F} - (Z_{\dot{\theta}})_F \ell_{x_F} + (X_{\dot{\theta}})_R \ell_{z_R} - (Z_{\dot{\theta}})_R \ell_{x_R}$$

$$+ (X_{\dot{\theta}})_W \ell_{z_W} - (Z_{\dot{\theta}})_W \ell_{x_W} + (X_{\dot{\theta}})_T \ell_{z_T} - (Z_{\dot{\theta}})_T \ell_{x_T}$$

$$+ (X_{\dot{\theta}})_{VT} \ell_{z_{VT}} - (Z_{\dot{\theta}})_{VT} \ell_{x_{VT}} + (X_{\dot{\theta}})_{TR} \ell_{z_{TR}} - (Z_{\dot{\theta}})_{TR} \ell_{x_{TR}}$$

$$+ \sum_{i=1}^{n} \left[(X_{\dot{\theta}})_{P_i} \ell_{z_{P_i}} - (Z_{\dot{\theta}})_{P_i} \ell_{x_{P_i}} + \frac{\partial M_{P_i}}{\partial q} \right] + \frac{\partial M_{HUB_F}}{\partial q} + \frac{\partial M_{HUB_R}}{\partial q}$$

where

$$\frac{\partial M_{HUB_F}}{\partial q} = \frac{\partial M_{HUB_F}}{\partial u} \ell_{Z_F} - \frac{1}{V_o} \frac{\partial M_{HUB_F}}{\partial \alpha} \ell_{X_F} + \frac{eb\Omega^2 M_S}{2} \left(\frac{\partial a_{1_F}}{\partial q}\right)$$

$$\frac{\partial M_{HUB_R}}{\partial q} = \frac{\partial M_{HUB_R}}{\partial u} \ell_{Z_R} - \frac{1}{V_o} \frac{\partial M_{HUB_R}}{\partial \alpha} \ell_{X_R} + \frac{eb\Omega^2 M_S}{2} \left(\frac{\partial a_{1_R}}{\partial q}\right)$$

7.1.5.6 $M_{\ddot{\theta}}$

$$M_{\ddot{\theta}} = -I_{YY}$$

7.1.5.7 $M_{\dot{\phi}}$

$$M_{\dot{\phi}} = (X_{\dot{\phi}})_F \ell_{Z_F} - (Z_{\dot{\phi}})_F \ell_{X_F} + (X_{\dot{\phi}})_R \ell_{Z_R} - (Z_{\dot{\phi}})_R \ell_{X_R}$$

$$+ (X_{\dot{\phi}})_W \ell_{Z_W} - (Z_{\dot{\phi}})_W \ell_{X_W} + (X_{\dot{\phi}})_T \ell_{Z_T} - (Z_{\dot{\phi}})_T \ell_{X_T}$$

$$+ (X_{\dot{\phi}})_{VT} \ell_{Z_{VT}} - (Z_{\dot{\phi}})_{VT} \ell_{X_{VT}} + (X_{\dot{\phi}})_{TR} \ell_{Z_{TR}} - (Z_{\dot{\phi}})_{TR} \ell_{X_{TR}}$$

$$+ \sum_{i=1}^{n} \left[(X_{\dot{\phi}})_{P_i} \ell_{Z_{P_i}} - (Z_{\dot{\phi}})_{P_i} \ell_{X_{P_i}} + \frac{\partial M_{P_i}}{\partial p} \right] + \frac{\partial M_{HUB_F}}{\partial p} + \frac{\partial M_{HUB_R}}{\partial p}$$

where

$$\frac{\partial M_{HUB_F}}{\partial p} = \frac{1}{V_o} \left[\frac{\partial M_{HUB_F}}{\partial \alpha} \ell_{Y_F} - \frac{\partial M_{HUB_F}}{\partial \beta_S} \ell_{Z_F} \right] + \frac{eb\Omega^2 M_S}{2} \left(\frac{\partial a_{1_F}}{\partial p}\right)$$

$$\frac{\partial M_{HUB_R}}{\partial p} = \frac{1}{V_0}\left[\frac{\partial M_{HUB_R}}{\partial \alpha}\ell_{Y_R} - \frac{\partial M_{HUB_R}}{\partial \beta_S}\ell_{Z_R}\right] + \frac{eb\Omega^2 M_S}{2}\left(\frac{\partial a_{1_R}}{\partial p}\right)$$

7.1.5.8 $M_{\ddot{\phi}}$

$$M_{\ddot{\phi}} = I_{XY}$$

7.1.5.9 $M_{\dot{\psi}}$

$$\begin{aligned} M_{\dot{\psi}} = {} & (X_{\dot{\psi}})_F \ell_{Z_F} - (Z_{\dot{\psi}})_F \ell_{X_F} + (X_{\dot{\psi}})_R \ell_{Z_R} - (Z_{\dot{\psi}})_R \ell_{X_R} \\ & + (X_{\dot{\psi}})_W \ell_{Z_W} - (Z_{\dot{\psi}})_W \ell_{X_W} + (X_{\dot{\psi}})_T \ell_{Z_T} - (Z_{\dot{\psi}})_T \ell_{X_T} \\ & + (X_{\dot{\psi}})_{VT} \ell_{Z_{VT}} - (Z_{\dot{\psi}})_{VT} \ell_{X_{VT}} + (X_{\dot{\psi}})_{TR} \ell_{Z_{TR}} - (Z_{\dot{\psi}})_{TR} \ell_{X_{TR}} \\ & + \sum_{i=1}^{n}\left[(X_{\dot{\psi}})_{P_i} \ell_{Z_{P_i}} - (Z_{\dot{\psi}})_{P_i} \ell_{X_{P_i}} + \frac{\partial M_{P_i}}{\partial r}\right] + \frac{\partial M_{HUB_F}}{\partial r} + \frac{\partial M_{HUB_R}}{\partial r} \end{aligned}$$

where

$$\frac{\partial M_{HUB_F}}{\partial r} = \frac{1}{V_0}\frac{\partial M_{HUB_F}}{\partial \beta_S}\ell_{X_F} - \frac{\partial M_{HUB_F}}{\partial u}\ell_{Y_F} + \frac{eb\Omega^2 M_S}{2}\left(\frac{\partial a_{1_F}}{\partial r}\right)$$

$$\frac{\partial M_{HUB_R}}{\partial r} = \frac{1}{V_0}\frac{\partial M_{HUB_R}}{\partial \beta_S}\ell_{X_R} - \frac{\partial M_{HUB_R}}{\partial u}\ell_{Y_R} + \frac{eb\Omega^2 M_S}{2}\left(\frac{\partial a_{1_R}}{\partial r}\right)$$

7.1.5.10 $M_{\ddot{\psi}}$

$$M_{\ddot{\psi}} = I_{YZ}$$

7.1.6 The Yawing Moment (N) Derivatives

The yawing moment (about body Z-axis) can be written in its abbreviated form as:

$$N = \sum_{i=1}^{n} (N)_i = \sum_{i=1}^{n} \left[(Y)_i \ell_{X_i} - (X)_i \ell_{Y_i} + (N_o)_i \right] + N_I$$

where

$(X)_i$ and $(Y)_i$ are forces in the X and Y directions of body axes, respectively, due to i^{th} aircraft components, and ℓ_{X_i} and ℓ_{Y_i} are their respective moment arms.

$(N_o)_i$ is the steady aerodynamic yawing moment about aircraft C.G. due to i^{th} aircraft components, and N_I is the inertia yawing moment.

7.1.6.1 N_u

$$\begin{aligned} N_u = {} & (Y_u)_F \ell_{X_F} - (X_u)_F \ell_{Y_F} + (Y_u)_R \ell_{X_R} - (X_u)_R \ell_{Y_R} \\ & + (Y_u)_W \ell_{X_W} - (X_u)_W \ell_{Y_W} + (Y_u)_T \ell_{X_T} - (X_u)_T \ell_{Y_T} \\ & + (Y_u)_{VT} \ell_{X_{VT}} - (X_u)_{VT} \ell_{Y_{VT}} + (Y_u)_{TR} \ell_{X_{TR}} - (X_u)_{TR} \ell_{Y_{TR}} \\ & + \sum_{i=1}^{n} \left[(Y_u)_{P_i} \ell_{X_{P_i}} - (X_u)_{P_i} \ell_{Y_{P_i}} \right] + \frac{\partial N_{FUS}}{\partial u} + \frac{\partial Q_F}{\partial u} - \frac{\partial Q_R}{\partial u} \end{aligned}$$

where

$$\frac{\partial N_{FUS}}{\partial u} = \frac{\partial N_{FUS}}{\partial u_{FUS}} + \frac{\partial N_{FUS}}{\partial \alpha_{FUS}} \frac{\partial \alpha_{FUS}}{\partial u}$$

$$\frac{\partial Q_F}{\partial u} = \frac{\partial Q_F}{\partial u_F} + \frac{\partial Q_F}{\partial \alpha_F} \frac{\partial \alpha_F}{\partial u}$$

$$\frac{\partial Q_R}{\partial u} = \frac{\partial Q_R}{\partial u_R} + \frac{\partial Q_R}{\partial \alpha_R} \frac{\partial \alpha_R}{\partial u}$$

7.1.6.2 N_V

$$N_V = (Y_V)_F \ell_{X_F} - (X_V)_F \ell_{Y_F} + (Y_V)_R \ell_{X_R} - (X_V)_R \ell_{Y_R}$$

$$+ (Y_V)_W \ell_{X_W} - (X_V)_W \ell_{Y_W} + (Y_V)_T \ell_{X_T} - (X_V)_T \ell_{Y_T}$$

$$+ (Y_V)_{VT} \ell_{X_{VT}} - (X_V)_{VT} \ell_{Y_{VT}} + (Y_V)_{TR} \ell_{X_{TR}} - (X_V)_{TR} \ell_{Y_{TR}}$$

$$+ \sum_{i=1}^{n} \left[(Y_V)_{P_i} \ell_{X_{P_i}} - (X_V)_{P_i} \ell_{Y_{P_i}} \right] + \frac{1}{V_0} \left(\frac{\partial N_{FUS}}{\partial \beta_S} + \frac{\partial Q_F}{\partial \beta_S} - \frac{\partial Q_R}{\partial \beta_S} \right)$$

7.1.6.3 N_W

$$N_W = (Y_W)_F \ell_{X_F} - (X_W)_F \ell_{Y_F} + (Y_W)_R \ell_{X_R} - (X_W)_R \ell_{Y_R}$$

$$+ (Y_W)_W \ell_{X_W} - (X_W)_W \ell_{Y_W} + (Y_W)_T \ell_{X_T} - (X_W)_T \ell_{Y_T}$$

$$+(Y_{\dot{w}})_{VT}\ell_{X_{VT}}-(X_{\dot{w}})_{VT}\ell_{Y_{VT}}+(Y_{\dot{w}})_{TR}\ell_{X_{TR}}-(X_{\dot{w}})_{TR}\ell_{Y_{TR}}$$

$$+\sum_{i=1}^{n}\left[(Y_{\dot{w}})_{P_i}\ell_{X_{P_i}}-(X_{\dot{w}})_{P_i}\ell_{Y_{P_i}}\right]+\frac{1}{V_0}\left(\frac{\partial N_{FUS}}{\partial\alpha}+\frac{\partial Q_F}{\partial\alpha}-\frac{\partial Q_R}{\partial\alpha}\right)$$

where

$$\frac{\partial N_{FUS}}{\partial\alpha}=\frac{\partial N_{FUS}}{\partial\alpha_{FUS}}\ \frac{\partial\alpha_{FUS}}{\partial\alpha}$$

$$\frac{\partial Q_F}{\partial\alpha}=\frac{\partial Q_F}{\partial\alpha_F}\ \frac{\partial\alpha_F}{\partial\alpha}$$

$$\frac{\partial Q_R}{\partial\alpha}=\frac{\partial Q_R}{\partial\alpha_R}\ \frac{\partial\alpha_R}{\partial\alpha}$$

7.1.6.4 $N_{\dot{w}}$

$$N_{\dot{w}}=-(X_{\dot{w}})_R\ell_{Y_R}-(X_{\dot{w}})_T\ell_{Y_T}-(X_{\dot{w}})_{VT}\ell_{Y_{VT}}-(X_{\dot{w}})_{TR}\ell_{Y_{TR}}$$

7.1.6.5 $N_{\dot{\theta}}$

$$N_{\dot{\theta}}=(Y_{\dot{\theta}})_F\ell_{X_F}-(X_{\dot{\theta}})_F\ell_{Y_F}+(Y_{\dot{\theta}})_R\ell_{X_R}-(X_{\dot{\theta}})_R\ell_{Y_R}$$

$$+(Y_{\dot{\theta}})_W\ell_{X_W}-(X_{\dot{\theta}})_W\ell_{Y_W}+(Y_{\dot{\theta}})_T\ell_{X_T}-(X_{\dot{\theta}})_T\ell_{Y_T}$$

$$+(Y_{\dot{\theta}})_{VT}\ell_{X_{VT}}-(X_{\dot{\theta}})_{VT}\ell_{Y_{VT}}+(Y_{\dot{\theta}})_{TR}\ell_{X_{TR}}-(X_{\dot{\theta}})_{TR}\ell_{Y_{TR}}$$

$$+\sum_{i=1}^{n}\left[(Y_{\dot\theta})_{P_i}\ell_{X_{P_i}} -(X_{\dot\theta})_{P_i}\ell_{Y_{P_i}}\right] + \frac{\partial Q_F}{\partial q} - \frac{\partial Q_R}{\partial q}$$

where

$$\frac{\partial Q_F}{\partial q} = \frac{\partial Q_F}{\partial u}\ell_{Z_F} - \frac{1}{V_0}\frac{\partial Q_F}{\partial \alpha}\ell_{X_F} + \left(\frac{\partial Q_F}{\partial a_{1_F}}\right)\left(\frac{\partial a_{1_F}}{\partial q}\right) + \left(\frac{\partial Q_F}{\partial b_{1_F}}\right)\left(\frac{\partial b_{1_F}}{\partial q}\right)$$

$$\frac{\partial Q_R}{\partial q} = \frac{\partial Q_R}{\partial u}\ell_{Z_R} - \frac{1}{V_0}\frac{\partial Q_R}{\partial \alpha}\ell_{X_R} + \left(\frac{\partial Q_R}{\partial a_{1_R}}\right)\left(\frac{\partial a_{1_R}}{\partial q}\right) + \left(\frac{\partial Q_R}{\partial b_{1_R}}\right)\left(\frac{\partial b_{1_R}}{\partial q}\right)$$

7.1.6.6 $N_{\ddot\theta}$

$$N_{\ddot\theta} = I_{YZ}$$

7.1.6.7 $N_{\dot\phi}$

$$N_{\dot\phi} = (Y_{\dot\phi})_F\ell_{X_F} - (X_{\dot\phi})_F\ell_{Y_F} + (Y_{\dot\phi})_R\ell_{X_R} - (X_{\dot\phi})_R\ell_{Y_R}$$

$$+ (Y_{\dot\phi})_W\ell_{X_W} - (X_{\dot\phi})_W\ell_{Y_W} + (Y_{\dot\phi})_T\ell_{X_T} - (X_{\dot\phi})_T\ell_{Y_T}$$

$$+ (Y_{\dot\phi})_{VT}\ell_{X_{VT}} - (X_{\dot\phi})_{VT}\ell_{Y_{VT}} + (Y_{\dot\phi})_{TR}\ell_{X_{TR}} - (X_{\dot\phi})_{TR}\ell_{Y_{TR}}$$

$$+\sum_{i=1}^{n}\left[(Y_{\dot\phi})_{P_i}\ell_{X_{P_i}} - (X_{\dot\phi})_{P_i}\ell_{Y_{P_i}}\right] + \frac{\partial Q_F}{\partial p} - \frac{\partial Q_R}{\partial p}$$

where

$$\frac{\partial Q_F}{\partial p} = \frac{1}{V_0}\left(\frac{\partial Q_F}{\partial \alpha}\ell_{Y_F} - \frac{\partial Q_F}{\partial \beta_S}\ell_{Z_F}\right) + \left(\frac{\partial Q_F}{\partial a_{1_F}}\right)\left(\frac{\partial a_{1_F}}{\partial p}\right) + \left(\frac{\partial Q_F}{\partial b_{1_F}}\right)\left(\frac{\partial b_{1_F}}{\partial p}\right)$$

$$\frac{\partial Q_R}{\partial p} = \frac{1}{V_0}\left(\frac{\partial Q_R}{\partial \alpha}\ell_{Y_R} - \frac{\partial Q_R}{\partial \beta_S}\ell_{Z_R}\right) + \left(\frac{\partial Q_R}{\partial a_{1_R}}\right)\left(\frac{\partial a_{1_R}}{\partial p}\right) + \left(\frac{\partial Q_R}{\partial b_{1_R}}\right)\left(\frac{\partial b_{1_R}}{\partial p}\right)$$

7.1.6.8 $N_{\ddot{\phi}}$

$N_{\ddot{\phi}} = I_{xz}$

7.1.6.9 $N_{\dot{\psi}}$

$$\begin{aligned} N_{\dot{\psi}} = {} & (Y_{\dot{\psi}})_F \ell_{X_F} - (X_{\dot{\psi}})_F \ell_{Y_F} + (Y_{\dot{\psi}})_R \ell_{X_R} - (X_{\dot{\psi}})_R \ell_{Y_R} \\ & + (Y_{\dot{\psi}})_W \ell_{X_W} - (X_{\dot{\psi}})_W \ell_{Y_W} + (Y_{\dot{\psi}})_T \ell_{X_T} - (X_{\dot{\psi}})_T \ell_{Y_T} \\ & + (Y_{\dot{\psi}})_{VT} \ell_{X_{VT}} - (X_{\dot{\psi}})_{VT} \ell_{Y_{VT}} + (Y_{\dot{\psi}})_{TR} \ell_{X_{TR}} - (X_{\dot{\psi}})_{TR} \ell_{Y_{TR}} \\ & + \sum_{i=1}^{n} \left[(Y_{\dot{\psi}})_{P_i} \ell_{X_{P_i}} - (X_{\dot{\psi}})_{P_i} \ell_{Y_{P_i}} \right] + \frac{\partial Q_F}{\partial r} - \frac{\partial Q_R}{\partial r} \end{aligned}$$

where

$$\frac{\partial Q_F}{\partial r} = \frac{1}{V_0}\frac{\partial Q_F}{\partial \beta_S}\ell_{X_F} - \frac{\partial Q_F}{\partial u}\ell_{Y_F} + \left(\frac{\partial Q_F}{\partial a_{1_F}}\right)\left(\frac{\partial a_{1_F}}{\partial r}\right) + \left(\frac{\partial Q_F}{\partial b_{1_F}}\right)\left(\frac{\partial b_{1_F}}{\partial r}\right)$$

$$\frac{\partial Q_R}{\partial r} = \frac{1}{V_0}\frac{\partial Q_R}{\partial \beta_S}\ell_{X_R} - \frac{\partial Q_R}{\partial u}\ell_{Y_R} + \left(\frac{\partial Q_R}{\partial a_{1_R}}\right)\left(\frac{\partial a_{1_R}}{\partial r}\right) + \left(\frac{\partial Q_R}{\partial b_{1_R}}\right)\left(\frac{\partial b_{1_R}}{\partial r}\right)$$

7.1.6.10 $N_{\ddot{\psi}}$

$$N_{\ddot{\psi}} = -I_{zz}$$

7.2 CONTROL DERIVATIVES

In response calculations, particularly when stability augmentation devices are used, it is necessary to determine the control derivatives.

For some high performance helicopters, especially the compound type, the control system may consist of a mixture of conventional helicopter and fixed wing aircraft controls. The analysis of this type of control system is beyond the scope of the present work. However, the mathematical procedures given below for the conventional helicopter controls can be readily modified to include such control systems.

The conventional helicopter controls consist of:

(a) Pilot Longitudinal Cyclic Control (B_{1_C})

The longitudinal control, B_{1_C} , is applied through a forward or aft control stick motion. This control gives rise to a pitch moment about the aircraft center of gravity. For single rotor helicopters, the stick motion actuates the longitudinal cyclic pitch, B_{1_F} . For tandem rotor helicopters this control, which in some cases may also activate the cyclic controls of both rotors, B_{1_F} and B_{1_R} , always applies differential collective pitch. Differential collective pitch is achieved by reducing the collective pitch on one rotor head and increasing it on the other.

On tandem rotor helicopters, the control moments due to longitudinal cyclic control are very small compared to those obtainable due to differential collective pitch. Hence, longitudinal cyclic pitch is utilized mainly to maintain the flapping of the rotors within reasonable limits. In some modern tandem rotor helicopters, longitudinal stick activates only the differential collective pitch, and the longitudinal cyclic pitch is a separate system which may be programmed automatically with speed.

Mathematically, the longitudinal control of a helicopter can be expressed as:

$$B_{1_C} = d_1 B_{1_F} + e_1 B_{1_R} - f_1 \Delta\theta_{0_F} + g_1 \Delta\theta_{0_R}$$

where d_1, e_1, f_1, and g_1 are the appropriate linkage ratios between the stick motion and the actual control motion. In the case of single rotor helicopters, the linkage ratios e_1, f_1, and g_1 are zero. For tandem rotor helicopters, generally $d_1 = e_1$ (or both d_1 and e_1 may be zero for some modern tandem helicopters), and $f_1 = g_1$. Thus, the longitudinal control for tandem rotor helicopters becomes:

$$B_{1_C} = d_1 (B_{1_F} + B_{1_R}) - f_1 (\Delta\theta_{0_F} - \Delta\theta_{0_R})$$

(b) Pilot Lateral Cyclic Control (A_{1_C})

The lateral control is applied through a lateral stick motion (right or left) which activates lateral cyclic pitch control at the front and rear rotor (A_{1_F} and A_{1_R}) in the same direction as the stick motion.

The lateral control can be expressed as:

$$A_{1_C} = d_2 A_{1_F} + e_2 A_{1_R}$$

where d_2 and e_2 are the control linkage ratios.

For single rotor helicopters, $e_2 = 0$ and, for tandem rotor helicopters, $d_2 = e_2$, thus

$$A_{1_C} = d_2 (A_{1_F} + A_{1_R})$$

(c) Pilot Directional Control (δ_{r_C})

Directional control is applied through a pedal movement. For a tandem rotor helicopter, the right pedal forward applies the lateral cyclic to the right on the front rotor head (A_{1_F}) and to the left on the rear rotor (A_{1_R}). In the case of a single rotor helicopter, the right pedal forward increases the thrust of the tail rotor to the left, through a change of tail rotor collective pitch ($\Delta\theta_{0_{TR}}$).

In general, the directional control can be expressed as:

$$\delta_{r_C} = d_3 A_{1_F} - e_3 A_{1_R} - f_3 \Delta\theta_{0_{TR}}$$

For single rotor configuration, the linkage ratios $d_3 = e_3 = 0$. For tandem rotor configuration, $f_3 = 0$ and $d_3 = e_3$. Hence, for tandem rotor helicopters,

$$\delta_{r_C} = d_3 (A_{1_F} - A_{1_R})$$

(d) Pilot Vertical Control (θ_C)

Vertical control which is achieved through a change in rotor thrust is applied through a collective pitch lever, which activates the collective pitch of the front and rear rotor in the same direction. Thus:

$$\theta_C = d_4 \Delta\theta_{0_F} + e_4 \Delta\theta_{0_R}$$

For a single rotor helicopter, the linkage ratio $e_4 = 0$, and in the case of a tandem configuration, $d_4 = e_4$. Thus:

$$\theta_C = d_4 (\Delta\theta_{0_F} + \Delta\theta_{0_R})$$

(e) Stability Augmentation Systems

Stability augmentation devices are utilized to introduce corrective control inputs automatically into the helicopter control system. These inputs are generally mixed with pilot control inputs. The total control motion can be written as a superposition of inputs from the pilot control and from the stability augmentation system as follows:

Longitudinal Control

$$B_1 = J_1 B_{1_C} + B_{1_S}$$

Lateral Control

$$A_1 = J_2 A_{1_C} + A_{1_S}$$

Directional Control

$$\delta_r = J_3 \delta_{r_C} + \delta_{r_S}$$

Vertical Control

$$\Theta = J_4 \theta_C + \theta_S$$

In the above expressions J_1 to J_4 are the pilot authority ratios for longitudinal, lateral, directional and vertical controls, respectively.

7.2.1 The Longitudinal Control (B_{1_C}) Derivatives

7.2.1.1 $X_{B_{1C}}$

$$X_{B_{1_C}} = \frac{\partial X}{\partial B_{1_F}}\frac{\partial B_{1_F}}{\partial B_{1_C}} + \frac{\partial X}{\partial \theta_{0_F}}\frac{\partial \theta_{0_F}}{\partial B_{1_C}} + \frac{\partial X}{\partial B_{1_R}}\frac{\partial B_{1_R}}{\partial B_{1_C}} + \frac{\partial X}{\partial \theta_{0_R}}\frac{\partial \theta_{0_R}}{\partial B_{1_C}}$$

$$= \frac{1}{d_1}\left[\frac{\partial X}{\partial B_{1_F}} + \frac{\partial X}{\partial B_{1_R}}\right] - \frac{1}{f_1}\left[\frac{\partial X}{\partial \theta_{0_F}} - \frac{\partial X}{\partial \theta_{0_R}}\right]$$

where

$$\frac{\partial X}{\partial B_{1_F}} = \frac{\partial D_F}{\partial \alpha_F}\cos(\alpha-\epsilon_F) - \left(\frac{\partial L_F}{\partial \alpha_F}\cos A_{1_F} - \frac{\partial Y_F}{\partial \alpha_F}\sin A_{1_F}\right)\sin(\alpha-\epsilon_F)$$

$$\frac{\partial X}{\partial \theta_{0_F}} = -\frac{\partial D_F}{\partial \theta_{0_F}}\cos(\alpha-\epsilon_F) + \left(\frac{\partial L_F}{\partial \theta_{C}}\cos A_{1_F} - \frac{\partial Y_F}{\partial \theta_{0_F}}\sin A_{1_F}\right)\sin(\alpha-\epsilon_F)$$

$$\frac{\partial X}{\partial B_{1_R}} = \frac{\partial D_R}{\partial \alpha_R}\cos(\alpha-\epsilon_R) - \left(\frac{\partial L_R}{\partial \alpha_R}\cos A_{1_R} + \frac{\partial Y_R}{\partial \alpha_R}\sin A_{1_R}\right)\sin(\alpha-\epsilon_R)$$

$$\frac{\partial X}{\partial \theta_{0_R}} = -\frac{\partial D_R}{\partial \theta_{0_R}}\cos(\alpha-\epsilon_R) + \left(\frac{\partial L_R}{\partial \theta_{0_R}}\cos A_{1_R} + \frac{\partial Y_R}{\partial \theta_{0_R}}\sin A_{1_R}\right)\sin(\alpha-\epsilon_R)$$

7.2.1.2 $Y_{B_{1_C}}$

$$Y_{B_{1_C}} = \frac{1}{d_1}\left[\frac{\partial Y}{\partial B_{1_F}} + \frac{\partial Y}{\partial B_{1_R}}\right] - \frac{1}{f_1}\left[\frac{\partial Y}{\partial \theta_{0_F}} - \frac{\partial Y}{\partial \theta_{0_R}}\right]$$

where

$$\frac{\partial Y}{\partial B_{1_F}} = -\left(\frac{\partial L_F}{\partial \alpha_F}\sin A_{1_F} + \frac{\partial Y_F}{\partial \alpha_F}\cos A_{1_F}\right)$$

$$\frac{\partial Y}{\partial \theta_{O_F}} = \frac{\partial L_F}{\partial \theta_{O_F}} \sin A_{I_F} + \frac{\partial Y_F}{\partial \theta_{O_F}} \cos A_{I_F}$$

$$\frac{\partial Y}{\partial B_{I_R}} = -\left(\frac{\partial L_R}{\partial \alpha_R} \sin A_{I_R} - \frac{\partial Y_R}{\partial \alpha_R} \cos A_{I_R}\right)$$

$$\frac{\partial Y}{\partial \theta_{O_R}} = \frac{\partial L_R}{\partial \theta_{O_R}} \sin A_{I_R} - \frac{\partial Y_R}{\partial \theta_{O_R}} \cos A_{I_R}$$

7.2.1.3 $Z_{B_{I_C}}$

$$Z_{B_{I_C}} = \frac{1}{d_I}\left[\frac{\partial Z}{\partial B_{I_F}} + \frac{\partial Z}{\partial B_{I_R}}\right] - \frac{1}{f_I}\left[-\frac{\partial Z}{\partial \theta_{O_F}} - \frac{\partial Z}{\partial \theta_{O_R}}\right]$$

where

$$\frac{\partial Z}{\partial B_{I_F}} = \frac{\partial D_F}{\partial \alpha_F} \sin(\alpha - \epsilon_F) + \left(-\frac{\partial L_F}{\partial \alpha_F} \cos A_{I_F} - \frac{\partial Y_F}{\partial \alpha_F} \sin A_{I_F}\right) \cos(\alpha - \epsilon_F)$$

$$\frac{\partial Z}{\partial \theta_{O_F}} = -\left[\frac{\partial D_F}{\partial \theta_{O_F}} \sin(\alpha - \epsilon_F) + \left(\frac{\partial L_F}{\partial \theta_{O_F}} \cos A_{I_F} - \frac{\partial Y_F}{\partial \theta_{O_F}} \sin A_{I_F}\right) \cos(\alpha - \epsilon_F)\right]$$

$$\frac{\partial Z}{\partial B_{I_R}} = \frac{\partial D_R}{\partial \alpha_R} \sin(\alpha - \epsilon_R) + \left(\frac{\partial L_R}{\partial \alpha_R} \cos A_{I_R} + \frac{\partial Y_R}{\partial \alpha_R} \sin A_{I_R}\right) \cos(\alpha - \epsilon_R)$$

$$\frac{\partial Z}{\partial \theta_{O_R}} = -\left[\frac{\partial D_R}{\partial \theta_{O_R}} \sin(\alpha - \epsilon_R) + \left(\frac{\partial L_R}{\partial \theta_{O_R}} \cos A_{I_R} + \frac{\partial Y_R}{\partial \theta_{O_R}} \sin A_{I_R}\right) \cos(\alpha - \epsilon_R)\right]$$

7.2.1.4 $\mathcal{L}_{B_{I_C}}$

$$\mathcal{L}_{B_{I_C}} = \frac{1}{d_I}\left[\frac{\partial \mathcal{L}}{\partial B_{I_F}} + \frac{\partial \mathcal{L}}{\partial B_{I_R}}\right] - \frac{1}{f_I}\left[\frac{\partial \mathcal{L}}{\partial \theta_{O_F}} - \frac{\partial \mathcal{L}}{\partial \theta_{O_F}}\right]$$

where

$$\frac{\partial \mathcal{L}}{\partial B_{1_F}} = \frac{\partial Z}{\partial B_{1_F}} \ell_{Y_F} - \frac{\partial Y}{\partial B_{1_F}} \ell_{Z_F} + \frac{\partial \mathcal{L}_{HUB_F}}{\partial B_{1_F}}$$

$$\frac{\partial \mathcal{L}}{\partial \theta_{0_F}} = \frac{\partial Z}{\partial \theta_{0_F}} \ell_{Y_F} - \frac{\partial Y}{\partial \theta_{0_F}} \ell_{Z_F} + \frac{\partial \mathcal{L}_{HUB_F}}{\partial \theta_{0_F}}$$

$$\frac{\partial \mathcal{L}}{\partial B_{1_R}} = \frac{\partial Z}{\partial B_{1_R}} \ell_{Y_R} - \frac{\partial Y}{\partial B_{1_R}} \ell_{Z_R} + \frac{\partial \mathcal{L}_{HUB_R}}{\partial B_{1_R}}$$

$$\frac{\partial \mathcal{L}}{\partial \theta_{0_R}} = \frac{\partial Z}{\partial \theta_{0_R}} \ell_{Y_R} - \frac{\partial Y}{\partial \theta_{0_R}} \ell_{Z_R} + \frac{\partial \mathcal{L}_{HUB_R}}{\partial \theta_{0_R}}$$

and

$$\frac{\partial \mathcal{L}_{HUB_F}}{\partial B_{1_F}} = -\left(\frac{eb\Omega^2 M_S}{2}\right)_F \frac{\partial b_{1_F}}{\partial \alpha_F}$$

$$\frac{\partial \mathcal{L}_{HUB_R}}{\partial B_{1_R}} = -\left(\frac{eb\Omega^2 M_S}{2}\right)_R \frac{\partial b_{1_R}}{\partial \alpha_R}$$

7.2.1.5 $M_{B_{1C}}$

$$M_{B_{1_C}} = \frac{1}{d_1}\left[\frac{\partial M}{\partial B_{1_F}} + \frac{\partial M}{\partial B_{1_R}}\right] - \frac{1}{f_1}\left[\frac{\partial M}{\partial \theta_{0_F}} - \frac{\partial M}{\partial \theta_{0_R}}\right]$$

where

$$\frac{\partial M}{\partial B_{1_F}} = \frac{\partial X}{\partial B_{1_F}} \ell_{Z_F} - \frac{\partial Z}{\partial B_{1_F}} \ell_{X_F} + \frac{\partial M_{HUB_F}}{\partial B_{1_F}}$$

$$\frac{\partial M}{\partial \theta_{0_F}} = \frac{\partial X}{\partial \theta_{0_F}} \ell_{Z_F} - \frac{\partial Z}{\partial \theta_{0_F}} \ell_{X_F} + \frac{\partial M_{HUB_F}}{\partial \theta_{0_F}}$$

$$\frac{\partial M}{\partial B_{1_R}} = \frac{\partial X}{\partial B_{1_R}} \ell_{Z_R} - \frac{\partial Z}{\partial B_{1_R}} \ell_{X_R} + \frac{\partial M_{HUB_R}}{\partial B_{1_R}}$$

$$\frac{\partial M}{\partial \theta_{0_R}} = \frac{\partial X}{\partial \theta_{0_R}} \ell_{Z_R} - \frac{\partial Z}{\partial \theta_{0_R}} \ell_{X_R} + \frac{\partial M_{HUB_R}}{\partial \theta_{0_R}}$$

and

$$\frac{\partial M_{HUB_F}}{\partial B_{1_F}} = \left(\frac{e b \Omega^2 M_S}{2}\right)_F \left(1 - \frac{\partial a_{1_F}}{\partial \alpha_F}\right)$$

$$\frac{\partial M_{HUB_R}}{\partial B_{1_R}} = \left(\frac{e b \Omega^2 M_S}{2}\right)_R \left(1 - \frac{\partial a_{1_R}}{\partial \alpha_R}\right)$$

7.2.1.6 $N_{B_{1_C}}$

$$N_{B_{1_C}} = \frac{1}{d_1}\left(\frac{\partial N}{\partial B_{1_F}} + \frac{\partial N}{\partial B_{1_R}}\right) - \frac{1}{f_1}\left(\frac{\partial N}{\partial \theta_{0_F}} - \frac{\partial N}{\partial \theta_{0_R}}\right)$$

where

$$\frac{\partial N}{\partial B_{1_F}} = \frac{\partial Y}{\partial B_{1_F}} \ell_{X_F} - \frac{\partial X}{\partial B_{1_F}} \ell_{Y_F} + \frac{\partial Q_F}{\partial B_{1_F}}$$

$$\frac{\partial N}{\partial \theta_{0_F}} = \frac{\partial Y}{\partial \theta_{0_F}} \ell_{X_F} - \frac{\partial X}{\partial \theta_{0_F}} \ell_{Y_F} + \frac{\partial Q_F}{\partial \theta_{0_F}}$$

$$\frac{\partial N}{\partial B_{1_R}} = \frac{\partial Y}{\partial B_{1_R}} \ell_{X_R} - \frac{\partial X}{\partial B_{1_R}} \ell_{Y_R} - \frac{\partial Q_R}{\partial B_{1_R}}$$

$$\frac{\partial N}{\partial \theta_{0_R}} = \frac{\partial Y}{\partial \theta_{0_R}} \ell_{X_R} - \frac{\partial X}{\partial \theta_{0_R}} \ell_{Y_R} - \frac{\partial Q_R}{\partial \theta_{0_R}}$$

and

$$\frac{\partial Q_F}{\partial B_{1_F}} = -\Big[(T.F.)\,\sigma R\Big]_F \left[\frac{\partial \frac{C_Q}{\sigma}}{\partial a_C}\right]_F$$

$$\frac{\partial Q}{\partial B_{1_R}} = -\Big[(T.F.)\,\sigma R\Big]_R \left[\frac{\partial \frac{C_Q}{\sigma}}{\partial a_C}\right]_R$$

7.2.1.7 Stability Augmentation System (B_{1_S}) Derivatives

All required (B_{1_S}) derivatives for stability augmentation systems are identical to the control derivatives (B_{1_C}) presented above. Thus:

$$X_{B_{1_S}} = X_{B_{1_C}} \qquad \mathcal{L}_{B_{1_S}} = \mathcal{L}_{B_{1_C}}$$

$$Y_{B_{1_S}} = Y_{B_{1_C}} \qquad M_{B_{1_S}} = M_{B_{1_C}}$$

$$Z_{B_{1_S}} = Z_{B_{1_C}} \qquad N_{B_{1_S}} = N_{B_{1_C}}$$

NOTE: In order to obtain the longitudinal control derivative (B_{1_C}) or (B_{1_S}) for a single rotor helicopter, all derivatives with respect to θ_{0_F}, B_{1_R}, and θ_{0_R} are eliminated.

7.2.1.8 Rate Derivatives $(\dot{B}_{1_C}$ and $\dot{B}_{1_S})$

The rate derivatives, $\dot{B}_{1_C}$ and $\dot{B}_{1_S}$, are considered to be small and are herein neglected.

7.2.2 The Lateral Control (A_{1_C}) Derivatives

7.2.2.1 $X_{A_{1C}}$

$$X_{A_{1_C}} = \frac{\partial X}{\partial A_{1_F}} \frac{\partial A_{1F}}{\partial A_{1_C}} + \frac{\partial X}{\partial A_{1_R}} \frac{\partial A_{1R}}{\partial A_{1_C}} = \frac{1}{d_2}\left(\frac{\partial X}{\partial A_{1_F}} + \frac{\partial X}{\partial A_{1_R}}\right)$$

where

$$\frac{\partial X}{\partial A_{1_F}} = -(L_F \sin A_{1_F} + Y_F \cos A_{1_F}) \sin(\alpha - \epsilon_F)$$

$$\frac{\partial X}{\partial A_{1_R}} = -(L_R \sin A_{1_R} - Y_R \cos A_{1_R}) \sin(\alpha - \epsilon_R)$$

7.2.2.2 $Y_{A_{1C}}$

$$Y_{A_{1_C}} = \frac{1}{d_2}\left(\frac{\partial Y}{\partial A_{1_F}} + \frac{\partial Y}{\partial A_{1_R}}\right)$$

where

$$\frac{\partial Y}{\partial A_{1_F}} = L_F \cos A_{1_F} - Y_F \sin A_{1_F}$$

$$\frac{\partial Y}{\partial A_{1_R}} = L_R \cos A_{1_R} + Y_R \sin A_{1_R}$$

7.2.2.3 $Z_{A_{1_C}}$

$$Z_{A_{1_C}} = \frac{1}{d_2}\left(\frac{\partial Z}{\partial A_{1_F}} + \frac{\partial Z}{\partial A_{1_R}}\right)$$

where

$$\frac{\partial Z}{\partial A_{1_F}} = (L_F \sin A_{1_F} + Y_F \cos A_{1_F}) \cos(\alpha - \epsilon_F)$$

$$\frac{\partial Z}{\partial A_{1_R}} = (L_R \sin A_{1_R} - Y_R \cos A_{1_R}) \cos(\alpha - \epsilon_R)$$

<u>7.2.2.4 $\mathcal{L}_{A_{1C}}$</u>

$$\mathcal{L}_{A_{1_C}} = \frac{1}{d_2}\left(\frac{\partial \mathcal{L}}{\partial A_{1_F}} + \frac{\partial \mathcal{L}}{\partial A_{1_R}}\right)$$

where

$$\frac{\partial \mathcal{L}}{\partial A_{1_F}} = \frac{\partial Z}{\partial A_{1_F}} \ell_{Y_F} - \frac{\partial Y}{\partial A_{1_F}} \ell_{Z_F} + \frac{\partial \mathcal{L}_{HUB_F}}{\partial A_{1_F}}$$

$$\frac{\partial \mathcal{L}}{\partial A_{1_R}} = \frac{\partial Z}{\partial A_{1_R}} \ell_{Y_R} - \frac{\partial Y}{\partial A_{1_R}} \ell_{Z_R} - \frac{\partial \mathcal{L}_{HUB_R}}{\partial A_{1_R}}$$

$$\frac{\partial \mathcal{L}_{HUB_F}}{\partial A_{1_F}} = -\left(\frac{eb\Omega^2 M_S}{2}\right)_F$$

$$\frac{\partial \mathcal{L}_{HUB_R}}{\partial A_{1_R}} = -\left(\frac{eb\Omega^2 M_S}{2}\right)_R$$

<u>7.2.2.5 $M_{A_{1C}}$</u>

$$M_{A_{1_C}} = \frac{1}{d_2}\left(\frac{\partial M}{\partial A_{1_F}} + \frac{\partial M}{\partial A_{1_R}}\right)$$

where

$$\frac{\partial M}{\partial A_{1_F}} = \frac{\partial X}{\partial A_{1_F}} \ell_{Z_F} - \frac{\partial Z}{\partial A_{1_F}} \ell_{X_F}$$

$$\frac{\partial M}{\partial A_{1_R}} = \frac{\partial X}{\partial A_{1_R}} \ell_{Z_R} - \frac{\partial Z}{\partial A_{1_R}} \ell_{X_R}$$

7.2.2.6 $N_{A_{1C}}$

$$N_{A_{1_C}} = \frac{1}{d_2}\left(\frac{\partial N}{\partial A_{1_F}} + \frac{\partial N}{\partial A_{1_R}}\right)$$

where

$$\frac{\partial N}{\partial A_{1_F}} = \frac{\partial Y}{\partial A_{1_F}} \ell_{X_F} - \frac{\partial X}{\partial A_{1_F}} \ell_{Y_F}$$

$$\frac{\partial N}{\partial A_{1_R}} = \frac{\partial Y}{\partial A_{1_R}} \ell_{X_R} - \frac{\partial X}{\partial A_{1_R}} \ell_{Y_R}$$

7.2.2.7 Stability Augmentation System (A_{1S}) Derivatives

All required (A_{1_S}) derivatives for stability augmentation systems are identical to the control derivatives (A_{1_C}) presented above. Thus:

$X_{A_{1_S}} = X_{A_{1_C}}$ $\qquad\qquad$ $\mathcal{L}_{A_{1_S}} = \mathcal{L}_{A_{1_C}}$

$Y_{A_{1_S}} = Y_{A_{1_A}}$ $\qquad\qquad$ $M_{A_{1_S}} = M_{A_{1_C}}$

$Z_{A_{1_S}} = Z_{A_{1_C}}$ $\qquad N_{A_{1_S}} = N_{A_{1_C}}$

NOTE: In order to obtain the lateral control derivatives (A_{1_C}) or (A_{1_S}) for a single rotor helicopter, all derivatives with respect to A_{1_R} are eliminated.

7.2.2.8 Rate Derivatives $(\dot{A}_{1C}$ and $\dot{A}_{1S})$

The rate derivatives $(\dot{A}_{1_C}$ and $\dot{A}_{1_S})$ are considered to be small and are herein neglected.

7.2.3 The Directional Control (δ_{rc}) Derivatives

7.2.3.1 $X_{\delta_{r_c}}$

$$X_{\delta_{r_c}} = \frac{\partial X}{\partial A_{1_F}} \frac{\partial A_{1_F}}{\partial \delta_{r_c}} + \frac{\partial X}{\partial A_{1_R}} \frac{\partial A_{1_R}}{\partial \delta_{r_c}} + \frac{\partial X}{\partial \theta_{0_{TR}}} \frac{\partial \theta_{0_{TR}}}{\partial \delta_{r_c}}$$

$$= \frac{1}{d_3}\left(\frac{\partial X}{\partial A_{1_F}} - \frac{\partial X}{\partial A_{1_R}}\right) - \frac{1}{f_3} \frac{\partial X}{\partial \theta_{0_{TR}}}$$

where

$$\frac{\partial X}{\partial \theta_{0_{TR}}} = \frac{\partial Y_{TR}}{\partial \theta_{0_{TR}}} \sin(\alpha - \epsilon_{TR}) - \frac{\partial D_{TR}}{\partial \theta_{0_{TR}}} \cos(\alpha - \epsilon_{TR})$$

7.2.3.2 $Y_{\delta_{r_c}}$

$$Y_{\delta_{r_c}} = \frac{1}{d_3}\left(\frac{\partial Y}{\partial A_{1_F}} - \frac{\partial Y}{\partial A_{1_R}}\right) - \frac{1}{f_3}\frac{\partial Y}{\partial \theta_{0_{TR}}}$$

where

$$\frac{\partial Y}{\partial \theta_{C_{TR}}} = \frac{\partial T_{TR}}{\partial \theta_{0_{TR}}}$$

7.2.3.3 $Z_{\delta_{r_c}}$

$$Z_{\delta_{r_c}} = \frac{1}{d_3}\left(\frac{\partial Z}{\partial A_{1_F}} - \frac{\partial Z}{\partial A_{1_R}}\right) - \frac{1}{f_3}\frac{\partial Z}{\partial \theta_{0_{TR}}}$$

where

$$\frac{\partial Z}{\partial \theta_{0_{TR}}} = -\left[\frac{\partial Y_{TR}}{\partial \theta_{0_{TR}}}\cos(\alpha-\epsilon_{TR}) + \frac{\partial D_{TR}}{\partial \theta_{0_{TR}}}\sin(\alpha-\epsilon_{TR})\right]$$

7.2.3.4 $\mathcal{L}_{\delta_{r_c}}$

$$\mathcal{L}_{\delta_{r_c}} = \frac{1}{d_3}\left(\frac{\partial \mathcal{L}}{\partial A_{1_F}} - \frac{\partial \mathcal{L}}{\partial A_{1_R}}\right) - \frac{1}{f_3}\frac{\partial \mathcal{L}}{\partial \theta_{0_{TR}}}$$

where

$$\frac{\partial \mathcal{L}}{\partial \theta_{0_{TR}}} = \frac{\partial Z}{\partial \theta_{0_{TR}}}\ell_{Y_{TR}} - \frac{\partial Y}{\partial \theta_{0_{TR}}}\ell_{Z_{TR}}$$

7.2.3.5 $M_{\delta_{r_c}}$

$$M_{\delta_{r_c}} = \frac{1}{d_3}\left(\frac{\partial M}{\partial A_{1_F}} - \frac{\partial M}{\partial A_{1_R}}\right) - \frac{1}{f_3}\frac{\partial M}{\partial \theta_{0_{TR}}}$$

where

$$\frac{\partial M}{\partial \theta_{0_{TR}}} = \frac{\partial X}{\partial \theta_{0_{TR}}}\ell_{Z_{TR}} - \frac{\partial Z}{\partial \theta_{0_{TR}}}\ell_{X_{TR}} + \frac{\partial Q_{TR}}{\partial \theta_{0_{TR}}}$$

7.2.3.6 $N_{\delta_{r_c}}$

$$N_{\delta_{r_c}} = \frac{1}{d_3}\left(\frac{\partial N}{\partial A_{1_F}} - \frac{\partial N}{\partial A_{1_R}}\right) - \frac{1}{f_3}\frac{\partial N}{\partial \theta_{0_{TR}}}$$

where

$$\frac{\partial N}{\partial \theta_{0_{TR}}} = \frac{\partial Y}{\partial \theta_{0_{TR}}}\ell_{X_{TR}} - \frac{\partial X}{\partial \theta_{0_{TR}}}\ell_{Y_{TR}}$$

7.2.3.7 Stability Augmentation System (δ_{r_s}) Derivatives

All required (δ_{r_s}) derivatives for stability augmentation systems are identical to the control derivatives (δ_{r_c}) presented above. Thus:

$X_{\delta_{r_s}} = X_{\delta_{r_c}}$ $\qquad$ $\mathcal{L}_{\delta_{r_s}} = \mathcal{L}_{\delta_{r_c}}$

$Y_{\delta_{r_s}} = Y_{\delta_{r_c}}$ $\qquad$ $M_{\delta_{r_s}} = M_{\delta_{r_c}}$

$Z_{\delta_{r_s}} = Z_{\delta_r}$ $\qquad$ $N_{\delta_{r_s}} = N_{\delta_{r_c}}$

NOTE: In order to obtain the directional control derivatives (δ_{r_c}) or (δ_{r_s}) for a tandem rotor helicopter, eliminate all derivatives with respect to $(\theta_{C_{TR}})$. In the case of a single rotor helicopter, eliminate all derivatives with respect to (A_{1_R}) .

7.2.3.8 Rate Derivatives $(\dot{\delta}_{r_c}$ and $\dot{\delta}_{r_s})$

The rate derivatives $\dot{\delta}_{r_c}$ and $\dot{\delta}_{r_s}$ are considered to be small and are herein neglected.

7.2.4 The Vertical Control (θ_C) Derivatives

7.2.4.1 X_{θ_C}

$$X_{\theta_C} = \frac{\partial X}{\partial \theta_{0_F}} \frac{\partial \theta_{0_F}}{\partial \theta_C} + \frac{\partial X}{\partial \theta_{0_R}} \frac{\partial \theta_{0_R}}{\partial \theta_C} = \frac{1}{d_4}\left(\frac{\partial X}{\partial \theta_{0_F}} + \frac{\partial X}{\partial \theta_{0_R}}\right)$$

where

$$\frac{\partial X}{\partial \theta_{0_F}} = -\frac{\partial D_F}{\partial \theta_{0_F}} \cos(\alpha - \epsilon_F) + \left(\frac{\partial L_F}{\partial \theta_{0_F}} \cos A_{1_F} - \frac{\partial Y_F}{\partial \theta_{0_F}} \sin A_{1_F}\right) \sin(\alpha - \epsilon_F)$$

$$\frac{\partial X}{\partial \theta_{0_R}} = -\frac{\partial D_R}{\partial \theta_{0_R}} \cos(\alpha - \epsilon_R) + \left(\frac{\partial L_R}{\partial \theta_{0_R}} \cos A_{1_R} + \frac{\partial Y_R}{\partial \theta_{0_R}} \sin A_{1_R}\right) \sin(\alpha - \epsilon_R)$$

7.2.4.2 Y_{θ_C}

$$Y_{\theta_C} = \frac{1}{d_4}\left(\frac{\partial Y}{\partial \theta_{0_F}} + \frac{\partial Y}{\partial \theta_{0_R}}\right)$$

where

$$\frac{\partial Y}{\partial \theta_{0_F}} = \frac{\partial L_F}{\partial \theta_{0_F}} \sin A_{1_F} + \frac{\partial Y_F}{\partial \theta_{0_F}} \cos A_{1_F}$$

$$\frac{\partial Y}{\partial \theta_{0_R}} = \frac{\partial L_R}{\partial \theta_{0_R}} \sin A_{1_R} - \frac{\partial Y_R}{\partial \theta_{0_R}} \cos A_{1_R}$$

7.2.4.3 Z_{θ_C}

$$Z_{\theta_C} = \frac{1}{d_4}\left(\frac{\partial Z}{\partial \theta_{0_F}} + \frac{\partial Z}{\partial \theta_{0_R}}\right)$$

where

$$\frac{\partial Z}{\partial \theta_{0_F}} = -\left[\frac{\partial D_F}{\partial \theta_{0_F}} \sin(\alpha-\epsilon_F) + \left(\frac{\partial L_F}{\partial \theta_{0_F}} \cos A_{1_F} - \frac{\partial Y_F}{\partial \theta_{0_F}} \sin A_{1_F}\right) \cos(\alpha-\epsilon_F)\right]$$

$$\frac{\partial Z}{\partial \theta_{0_R}} = -\left[\frac{\partial D_R}{\partial \theta_{0_R}} \sin(\alpha-\epsilon_R) + \left(\frac{\partial L_R}{\partial \theta_{0_R}} \cos A_{1_R} + \frac{\partial Y_R}{\partial \theta_{0_R}} \sin A_{1_R}\right) \cos(\alpha-\epsilon_R)\right]$$

7.2.4.4 $\mathcal{L}_{\theta_C}$

$$\mathcal{L}_{\theta_C} = \frac{1}{d_4}\left(\frac{\partial \mathcal{L}}{\partial \theta_{0_F}} + \frac{\partial \mathcal{L}}{\partial \theta_{0_R}}\right)$$

where

$$\frac{\partial \mathcal{L}}{\partial \theta_{0_F}} = \frac{\partial Z}{\partial \theta_{0_F}} \ell_{Y_F} - \frac{\partial Y}{\partial \theta_{0_F}} \ell_{Z_F} + \frac{\partial \mathcal{L}_{HUB_F}}{\partial \theta_{0_F}}$$

$$\frac{\partial \mathcal{L}}{\partial \theta_{0_R}} = \frac{\partial Z}{\partial \theta_{0_R}} \ell_{Y_R} - \frac{\partial Y}{\partial \theta_{0_R}} \ell_{Z_R} - \frac{\partial \mathcal{L}_{HUB_R}}{\partial \theta_{0_R}}$$

<u>7.2.4.5 M_{θ_c}</u>

$$M_{\theta_c} = \frac{1}{d_4} \left(\frac{\partial M}{\partial \theta_{0_F}} + \frac{\partial M}{\partial \theta_{0_R}} \right)$$

where

$$\frac{\partial M}{\partial \theta_{0_F}} = \frac{\partial X}{\partial \theta_{0_F}} \ell_{Z_F} - \frac{\partial Z}{\partial \theta_{0_F}} \ell_{X_F} + \frac{\partial M_{HUB_F}}{\partial \theta_{0_F}}$$

$$\frac{\partial M}{\partial \theta_{0_R}} = \frac{\partial X}{\partial \theta_{0_R}} \ell_{Z_R} - \frac{\partial Z}{\partial \theta_{0_R}} \ell_{X_R} + \frac{\partial M_{HUB_R}}{\partial \theta_{0_R}}$$

<u>7.2.4.6 N_{θ_c}</u>

$$N_{\theta_c} = \frac{1}{d_4} \left(\frac{\partial N}{\partial \theta_{0_F}} + \frac{\partial N}{\partial \theta_{0_R}} \right)$$

where

$$\frac{\partial N}{\partial \theta_{0_F}} = \frac{\partial Y}{\partial \theta_{0_F}} \ell_{X_F} - \frac{\partial X}{\partial \theta_{0_F}} \ell_{Y_F} + \frac{\partial Q_F}{\partial \theta_{0_F}}$$

$$\frac{\partial N}{\partial \theta_{c_R}} = \frac{\partial Y}{\partial \theta_{0_R}} \ell_{X_R} - \frac{\partial X}{\partial \theta_{0_R}} \ell_{Y_R} - \frac{\partial Q_R}{\partial \theta_{0_R}}$$

7.2.4.7 Stability Augmentation System (θ_s) Derivatives

All required (θ_s) derivatives for stability augmentation system are identical to the vertical control derivatives (θ_c) given above. Thus:

$$X_{\theta_s} = X_{\theta_c} \qquad \mathcal{L}_{\theta_s} = \mathcal{L}_{\theta_c}$$

$$Y_{\theta_s} = Y_{\theta_c} \qquad M_{\theta_s} = M_{\theta_c}$$

$$Z_{\theta_s} = Z_{\theta_c} \qquad N_{\theta_s} = N_{\theta_c}$$

NOTE: In order to obtain vertical control derivatives for a single rotor helicopter, it is necessary to eliminate all the derivatives with respect to (θ_{0_R})

7.2.4.8 The Rate Derivatives $(\dot{\theta}_c$ and $\dot{\theta}_s)$

The rate derivatives $(\dot{\theta}_c)$ and $(\dot{\theta}_s)$ are considered to be small and are herein neglected.

7.3 LOCAL DERIVATIVES

The local stability derivatives contained in this section are presented as partial differentials of local forces and moments of aerodynamic components with respect to local wind conditions. These derivatives are expressed in a suitable nondimensional form and are obtained for the following aircraft components:

(a) Main Rotor (or Rotors)
(b) Fuselage
(c) Wing
(d) Horizontal and Vertical Tailplanes
(e) Tail Rotor
(f) Propulsion System

7.3.1 Single Rotor (or Front Rotor of a Tandem Rotor Helicopter)

The rotor local stability derivatives must be evaluated at the required rotor solidity (σ). These derivatives are obtained by utilizing the values corresponding to rotor solidity of $\sigma = 0.1$ and by applying the appropriate solidity correction factors presented in Section 7.4. The rotor derivatives for $\sigma = 0.1$ were obtained from the theoretical results of Reference 1 and are presented in the form of nondimensionalized charts in Section 7.5.

Some of the rotor derivatives, such as Y-force and coning angle (a_0), for which the numerical data was not available were determined analytically using the classical rotor theory. Wherever possible, these derivatives were expressed in terms of rotor parameters for which the performance data of Reference 1 could be utilized.

7.3.1.1 The Longitudinal Speed (u_F) Derivatives

$$\frac{\partial L_F}{\partial u_F} = \left[\frac{(T.F.)\sigma}{\Omega R}\right]_F \left[\frac{\partial(\frac{C_L'}{\sigma})}{\partial\mu}\right]_F$$

$$\frac{\partial D_F}{\partial u_F} = \left[\frac{(T.F.)\sigma}{\Omega R}\right]_F \left[\frac{\partial(\frac{C_D'}{\sigma})}{\partial\mu}\right]_F$$

$$\frac{\partial Y_F}{\partial u_F} = \left[\frac{(T.F.)\sigma}{\Omega R}\right]_F \left[\frac{\partial(\frac{C_Y}{\sigma})}{\partial\mu}\right]_F$$

$$\frac{\partial Q_F}{\partial u_F} = \left[\frac{(T.F.)\sigma}{\Omega}\right]_F \left[\frac{\partial(\frac{C_Q}{\sigma})}{\partial\mu}\right]_F$$

$$\frac{\partial a_{1_F}}{\partial u_F} = (\frac{1}{\Omega R})_F \, (\frac{\partial a_1}{\partial\mu})_F$$

$$\frac{\partial b_{1_F}}{\partial u_F} = (\frac{1}{\Omega R})_F \, (\frac{\partial b_1}{\partial\mu})_F$$

$$\frac{\partial \mathcal{L}_{HUB_F}}{\partial u_F} = (\frac{eb\Omega^2 M_S}{2})_F \, (\frac{\partial b_{1_F}}{\partial u_F})$$

$$\frac{\partial M_{HUB_F}}{\partial u_F} = (\frac{eb\Omega^2 M_S}{2})_F \, (\frac{\partial a_{1_F}}{\partial u_F})$$

7.3.1.2 The Angle of Attack (α_F) Derivatives

$$\frac{\partial L_F}{\partial \alpha_F} = \left[(T.F.)\sigma\right]_F \left[\frac{\partial(\frac{C_L'}{\sigma})}{\partial\alpha_C}\right]_F$$

$$\frac{\partial D_F}{\partial \alpha_F} = \left[(T.F.)\sigma\right]_F \left[\frac{\partial(\frac{C_D'}{\sigma})}{\partial\alpha_C}\right]_F$$

$$\frac{\partial Y_F}{\partial \alpha_F} = \left[(T.F.)\sigma\right]_F \left[\frac{\partial(\frac{C_Y}{\sigma})}{\partial\alpha_C}\right]_F$$

$$\frac{\partial Q_F}{\partial \alpha_F} = \left[(T.F.)\,\sigma R\right]_F \left[\frac{\partial(\frac{C_Q}{\sigma})}{\partial \alpha_C}\right]_F$$

$$\frac{\partial a_{1_F}}{\partial \alpha_F} = \left(\frac{\partial a_1}{\partial \alpha_C}\right)_F$$

$$\frac{\partial b_{1_F}}{\partial \alpha_F} = \left(\frac{\partial b_1}{\partial \alpha_C}\right)_F$$

$$\frac{\partial \mathcal{L}_{HUB_F}}{\partial \alpha_F} = \left(\frac{eb\Omega^2 M_S}{2}\right)_F \left(\frac{\partial b_{1_F}}{\partial \alpha_F}\right)$$

$$\frac{\partial M_{HUB_F}}{\partial \alpha_F} = \left(\frac{eb\Omega^2 M_S}{2}\right)_F \left(\frac{\partial a_{1_F}}{\partial \alpha_F}\right)$$

7.3.1.3 The Side Slip (β_S) Derivatives

$$\frac{\partial a_{1_F}}{\partial \beta_S} = b_{1_F}$$

$$\frac{\partial b_{1_F}}{\partial \beta_S} = -a_{1_F}$$

$$\frac{\partial Q_F}{\partial \beta_S} = \left(\frac{\partial Q_F}{\partial a_{1_F}}\right)\left(\frac{\partial a_{1_F}}{\partial \beta_S}\right) + \left(\frac{\partial Q_F}{\partial b_{1_F}}\right)\left(\frac{\partial b_{1_F}}{\partial \beta_S}\right)$$

$$\frac{\partial \mathcal{L}_{HUB_F}}{\partial \beta_S} = -M_{HUB_F}$$

$$\frac{\partial M_{HUB_F}}{\partial \beta_S} = \mathcal{L}_{HUB_F}$$

7.3.1.4 The Angular Pitching Velocity (q) Derivatives

$$\frac{\partial a_{1_F}}{\partial q} = \left(\frac{\partial a_1}{\partial q}\right)_F = -\left[\frac{34}{\gamma\Omega(1.883-\mu^2)}\right]_F$$

$$\frac{\partial b_{1_F}}{\partial q} = \left(\frac{\partial b_1}{\partial q}\right)_F = -\left[\frac{1.883}{\Omega\,(1.883+\mu^2)}\right]_F$$

7.3.1.5 The Angular Rolling Velocity (p) Derivatives

$$\frac{\partial a_{1_F}}{\partial p} = \left(\frac{\partial a_1}{\partial p}\right)_F = \left[\frac{1.883}{\Omega\,(1.883-\mu^2)}\right]_F$$

$$\frac{\partial b_{1_F}}{\partial p} = \left(\frac{\partial b_1}{\partial p}\right)_F = -\left[\frac{34}{\gamma\Omega(1.883+\mu^2)}\right]_F$$

7.3.1.6 The Angular Yawing Velocity (r) Derivatives

$$\frac{\partial a_{1_F}}{\partial r} = \left(\frac{\partial a_1}{\partial r}\right)_F = -\frac{\partial a_{1_F}}{\partial \Omega} = \left(\frac{\partial a_1}{\partial \mu}\right)_F \left(\frac{\mu}{\Omega}\right)_F$$

$$\frac{\partial b_{1_F}}{\partial r} = \left(\frac{\partial b_1}{\partial \mu}\right)_F \left(\frac{\mu}{\Omega}\right)_F$$

7.3.1.7 The Longitudinal Flapping Angle (a_{1_F}) Derivatives

$$\frac{\partial L_F}{\partial a_{1_F}} = -D_F$$

$$\frac{\partial D_F}{\partial a_{1_F}} = L_F$$

$$\frac{\partial Y_F}{\partial a_{1_F}} = 0$$

$$\left[\frac{\partial (\frac{C_Q}{\sigma})}{\partial a_1}\right]_F = -\frac{a}{2}\left[a_1\left(\frac{1}{4}+\frac{3}{8}\mu^2\right)+\frac{1}{2}\mu\lambda\right]_F$$

$$\frac{\partial Q_F}{\partial a_{1_F}} = \left[(T.F.)\sigma R\right]_F\left[\frac{\partial (\frac{C_Q}{\sigma})}{\partial a_1}\right]_F$$

7.3.1.8 The Lateral Flapping Angle (b_{1_F}) Derivatives

$$\frac{\partial L_F}{\partial b_{1_F}} = -Y_F$$

$$\frac{\partial D_F}{\partial b_{1_F}} = 0$$

$$\frac{\partial Y_F}{\partial b_{1_F}} = L_F$$

$$\left[\frac{\partial (\frac{C_Q}{\sigma})}{\partial b_1}\right]_F = -\frac{a}{2}\left[b_1\left(\frac{1}{4}+\frac{1}{8}\mu^2\right)-\frac{1}{3}\mu a_0\right]_F$$

$$\frac{\partial Q_F}{\partial b_{1_F}} = \left[(T.F.)\sigma R\right]_F\left[\frac{\partial (\frac{C_Q}{\sigma})}{\partial b_1}\right]_F$$

7.3.1.9 Rotor Collective Pitch (θ_{0F}) Derivatives

$$\frac{\partial L_F}{\partial \theta_{0F}} = \left[(T.F.)\sigma\right]_F\left[\frac{\partial (\frac{C_L'}{\sigma})}{\partial \theta_{.75}}\right]_F$$

$$\frac{\partial D_F}{\partial \theta_{0F}} = \left[(T.F.)\sigma\right]_F \left[\frac{\partial(\frac{C_D'}{\sigma})}{\partial \theta_{.75}}\right]_F$$

$$\frac{\partial Y_F}{\partial \theta_{0F}} = \left[(T.F.)\sigma\right]_F \left[\frac{\partial(\frac{C_Y}{\sigma})}{\partial \theta_{.75}}\right]_F$$

$$\frac{\partial Q_F}{\partial \theta_{0F}} = \left[(T.F.)\sigma R\right]_F \left[\frac{\partial(\frac{C_Q}{\sigma})}{\partial \theta_{.75}}\right]_F$$

$$\frac{\partial \mathcal{L}_{HUB_F}}{\partial \theta_{0F}} = \left(\frac{eb\Omega^2 M_S}{2}\right)_F \left(\frac{\partial b_1}{\partial \theta_{.75}}\right)_F$$

$$\frac{\partial M_{HUB_F}}{\partial \theta_{0F}} = \left(\frac{eb\Omega^2 M_S}{2}\right)_F \left(\frac{\partial a_1}{\partial \theta_{.75}}\right)_F$$

7.3.2 Rear Rotor of a Tandem Rotor Configuration

The local derivatives for the rear rotor of a tandem rotor helicopter can be obtained in exactly the same manner as those for the single rotor presented in Subsection 7.3.1. However, to avoid duplication, the majority of the rear rotor derivatives can be formulated by changing suffix (F) to suffix (R) in the equations of Subsection 7.3.1 with the exception of the following:

$$\frac{\partial b_{1_R}}{\partial q} = \left[\frac{1.883}{\Omega(1.883+\mu^2)}\right]_R$$

$$\frac{\partial b_{1_R}}{\partial p} = \left[\frac{34}{\gamma\Omega(1.883+\mu^2)}\right]_R$$

$$\frac{\partial a_{1_R}}{\partial r} = -\left(\frac{\partial a_1}{\partial \mu}\right)_R \left(\frac{\mu}{\Omega}\right)_R$$

$$\frac{\partial b_{1_R}}{\partial r} = -\left(\frac{\partial b_1}{\partial \mu}\right)_R \left(\frac{\mu}{\Omega}\right)_R$$

$$\frac{\partial a_{1_R}}{\partial \beta_s} = -b_{1_R}$$

$$\frac{\partial b_{1_R}}{\partial \beta_s} = a_{1_R}$$

$$\frac{\partial \mathcal{L}_{HUB_R}}{\partial \beta_s} = M_{HUB_R}$$

$$\frac{\partial M_{HUB_R}}{\partial \beta_s} = -\mathcal{L}_{HUB_R}$$

7.3.3 Fuselage Derivatives

The required local fuselage derivatives are obtained by taking slopes of the appropriate fuselage data. It is recommended that the actual test data such as presented in References 2 through 4 for various fuselage shapes be utilized for this purpose.

7.3.3.1 The Forward Speed (u_{FUS}) Derivatives

$$\frac{\partial L_{FUS}}{\partial u_{FUS}} = \frac{2}{V_0} L_{FUS}$$

$$\frac{\partial D_{FUS}}{\partial u_{FUS}} = \frac{2}{V_0} D_{FUS}$$

$$\frac{\partial Y_{FUS}}{\partial u_{FUS}} = \frac{2}{V_0} Y_{FUS}$$

$$\frac{\partial \mathcal{L}_{FUS}}{\partial u_{FUS}} = \frac{2}{V_0} \mathcal{L}_{FUS}$$

$$\frac{\partial M_{FUS}}{\partial u_{FUS}} = \frac{2}{V_0} M_{FUS}$$

$$\frac{\partial N_{FUS}}{\partial u_{FUS}} = \frac{2}{V_0} N_{FUS}$$

7.3.3.2 The Angle of Attack (α_{FUS}) Derivatives

$$\frac{\partial L_{FUS}}{\partial \alpha_{FUS}} = q\, A_{Z_{FUS}} \left(\frac{\partial C_{L_{FUS}}}{\partial \alpha_{FUS}}\right)$$

$$\frac{\partial D_{FUS}}{\partial \alpha_{FUS}} = q\, A_{X_{FUS}} \left(\frac{\partial C_{D_{FUS}}}{\partial \alpha_{FUS}}\right)$$

$$\frac{\partial Y_{FUS}}{\partial \alpha_{FUS}} = q\, A_{Y_{FUS}} \left(\frac{\partial C_{Y_{FUS}}}{\partial \alpha_{FUS}}\right)$$

$$\frac{\partial \mathcal{L}_{FUS}}{\partial \alpha_{FUS}} = q\, A_{X_{FUS}} \ell_{FUS} \left(\frac{\partial C_{\mathcal{L}_{FUS}}}{\partial \alpha_{FUS}}\right)$$

$$\frac{\partial M_{FUS}}{\partial \alpha_{FUS}} = q\, A_{X_{FUS}} \ell_{FUS} \left(\frac{\partial C_{M_{FUS}}}{\partial \alpha_{FUS}}\right)$$

$$\frac{\partial N_{FUS}}{\partial \alpha_{FUS}} = q\, A_{X_{FUS}} \ell_{FUS} \left(\frac{\partial C_{N_{FUS}}}{\partial \alpha_{FUS}}\right)$$

7.3.3.3 The Side Slip Angle (β_s) Derivatives

$$\frac{\partial L_{FUS}}{\partial \beta_s} = q\, A_{Z_{FUS}} \left(\frac{\partial C_{L_{FUS}}}{\partial \beta_s}\right)$$

$$\frac{\partial D_{FUS}}{\partial \beta_s} = q\, A_{X_{FUS}} \left(\frac{\partial C_{D_{FUS}}}{\partial \beta_s}\right)$$

$$\frac{\partial Y_{FUS}}{\partial \beta_s} = q\, A_{Y_{FUS}} \left(\frac{\partial C_{Y_{FUS}}}{\partial \beta_s}\right)$$

$$\frac{\partial \mathcal{L}_{FUS}}{\partial \beta_s} = q\, A_{X_{FUS}} \ell_{FUS} \left(\frac{\partial C_{\mathcal{L}_{FUS}}}{\partial \beta_s}\right)$$

$$\frac{\partial M_{FUS}}{\partial \beta_s} = q\, A_{X_{FUS}} \ell_{FUS} \left(\frac{\partial C_{M_{FUS}}}{\partial \beta_s}\right)$$

$$\frac{\partial N_{FUS}}{\partial \beta_s} = q\, A_{X_{FUS}}\, \ell_{FUS} \left(\frac{\partial C_{N_{FUS}}}{\partial \beta_s}\right)$$

NOTE: The remaining fuselage derivatives can be neglected

7.3.4 Wing Derivatives

7.3.4.1 The Forward Speed (u_W) Derivatives

$$\frac{\partial L_W}{\partial u_W} = \frac{2}{V_a} L_W$$

$$\frac{\partial D_W}{\partial u_W} = \frac{2}{V_a} D_W$$

7.3.4.2 The Angle of Attack (α_W) Derivatives

$$\frac{\partial L_W}{\partial \alpha_W} = q\, a_W S_W$$

$$\frac{\partial D_W}{\partial \alpha_W} = \frac{2 L_W}{\pi (AR)_W} a_W$$

The remaining wing derivatives may be neglected. However, if required, the additional wing derivatives can be obtained from Reference 5.

7.3.5 Horizontal Tail Derivatives

The horizontal tailplane derivatives can be obtained in exactly the same way as for the wing by changing suffix (W) to suffix (T).

7.3.6 Vertical Tail (Fin) Derivatives

7.3.6.1 The Forward Speed (u_{VT}) Derivatives

$$\frac{\partial L_{VT}}{\partial u_{VT}} = \frac{2}{V_0} L_{VT}$$

$$\frac{\partial D_{VT}}{\partial u_{VT}} = \frac{2}{V_0} D_{VT}$$

7.3.6.2 The Angle of Attack (α_{VT}) Derivatives

$$\frac{\partial L_{VT}}{\partial \alpha_{VT}} = \frac{\partial D_{VT}}{\partial \alpha_{VT}} = 0$$

7.3.6.3 The Side Slip Angle (β_S) Derivatives

$$\frac{\partial L_{VT}}{\partial \beta_S} = q\, o_{VT} S_{VT}$$

$$\frac{\partial D_{VT}}{\partial \beta_S} = \frac{2L_{VT}}{\pi(AR)_{VT}} a_{VT}$$

7.3.7 Tail Rotor Derivatives

7.3.7.1 The Forward Speed (u_{TR}) Derivatives

$$\frac{\partial T_{TR}}{\partial u_{TR}} = \left[\frac{(T.F.)\sigma}{\Omega R}\right]_{TR} \left[\frac{\partial(\frac{C_L'}{\sigma})}{\partial \mu}\right]_{TR}$$

$$\frac{\partial D_{TR}}{\partial u_{TR}} = \left[\frac{(T.F.)\sigma}{\Omega R}\right]_{TR} \left[\frac{\partial(\frac{C_D'}{\sigma})}{\partial \mu}\right]_{TR}$$

$$\frac{\partial Y_{TR}}{\partial u_{TR}} = \left[\frac{(T.F.)\sigma}{\Omega R}\right]_{TR} \left[\frac{\partial(\frac{C_Y}{\sigma})}{\partial \mu}\right]_{TR}$$

$$\frac{\partial Q_{TR}}{\partial u_{TR}} = \left[\frac{(T.F.)\sigma}{\Omega}\right]_{TR} \left[\frac{\partial(\frac{C_Q}{\sigma})}{\partial \mu}\right]_{TR}$$

7.3.7.2 The Angle of Attack (α_{TR}) Derivatives

$$\frac{\partial T_{TR}}{\partial \alpha_{TR}} = \frac{\partial D_{TR}}{\partial \alpha_{TR}} = \frac{\partial Y_{TR}}{\partial \alpha_{TR}} = \frac{\partial Q_{TR}}{\partial \alpha_{TR}} = 0$$

7.3.7.3 The Side Slip Angle (β_S) Derivatives

$$\frac{\partial T_{TR}}{\partial \beta_S} = -\left[(T.F.)\sigma\right]_{TR} \left[\frac{\partial(\frac{C_L'}{\sigma})}{\partial \alpha_C}\right]_{TR}$$

$$\frac{\partial D_{TR}}{\partial \beta_S} = -\left[(T.F.)\sigma\right]_{TR}\left[\frac{\partial(\frac{C_D'}{\sigma})}{\partial \alpha_C}\right]_{TR}$$

$$\frac{\partial Y_{TR}}{\partial \beta_S} = -\left[(T.F.)\sigma\right]_{TR}\left[\frac{\partial(\frac{C_Y'}{\sigma})}{\partial \alpha_C}\right]_{TR}$$

$$\frac{\partial Q_{TR}}{\partial \beta_S} = -\left[(T.F.)\sigma R\right]_{TR}\left[\frac{\partial(\frac{C_Q}{\sigma})}{\partial \alpha_C}\right]_{TR}$$

7.3.7.4 The Tail Rotor Collective (θ_{oTR}) Derivatives

$$\frac{\partial T_{TR}}{\partial \theta_{oTR}} = \left[(T.F.)\sigma\right]_{TR}\left[\frac{\partial(\frac{C_L'}{\sigma})}{\partial \theta_{.75}}\right]_{TR}$$

$$\frac{\partial D_{TR}}{\partial \theta_{oTR}} = \left[(T.F.)\sigma\right]_{TR}\left[\frac{\partial(\frac{C_D'}{\sigma})}{\partial \theta_{.75}}\right]_{TR}$$

$$\frac{\partial Y_{TR}}{\partial \theta_{oTR}} = \left[(T.F.)\sigma\right]_{TR}\left[\frac{\partial(\frac{C_Y}{\sigma})}{\partial \theta_{.75}}\right]_{TR}$$

$$\frac{\partial Q_{TR}}{\partial \theta_{oTR}} = \left[(T.F.)\sigma R\right]_{TR}\left[\frac{\partial(\frac{C_Q}{\sigma})}{\partial \theta_{.75}}\right]_{TR}$$

The remaining tail rotor derivatives, if any, can be neglected.

7.3.8 Propeller Derivatives

The propeller local derivatives can be obtained from appropriate propeller charts. A good compilation of propeller data is contained in References 6 and 7.

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7. Olcott, J. W., Tests of a Hamilton Standard Four-Way 21-Inch-Diameter Model Propeller Employing the U.S. Navy Airborne Model Test Facility, Princeton Report No. 6[illegible], April 1964.

7.4 CORRECTIONS OF ISOLATED ROTOR DERIVATIVES FOR VARIATION OF ROTOR SOLIDITY (σ)

The isolated rotor stability derivatives presented as charts in Section 7.5 apply only for a rotor solidity of $\sigma = 0.1$. In order to evaluate the required stability derivatives for rotor having solidity $\sigma \neq 0.1$, the following solidity corrections must be applied:

7.4.1 Solidity Corrections for (μ) Derivatives

$$\frac{\partial(\frac{C_L'}{\sigma})}{\partial\mu} = K_1 \left\{ \left[\frac{\partial(\frac{C_L'}{\sigma})}{\partial\mu}\right]_{0.1} + \frac{\Delta\sigma}{\mu^3}\left(\frac{C_L'}{\sigma}\right)\left[\frac{\partial(\frac{C_L'}{\sigma})}{\partial\alpha_c}\right]_{0.1} \right\}$$

where

$$\Delta\sigma = \sigma - 0.1$$

$$K_1 = \frac{1}{1 + \frac{\Delta\sigma}{2\mu^2}\left[\frac{\partial(\frac{C_L'}{\sigma})}{\partial\alpha_c}\right]_{0.1}}$$

$(\ )_{0.1}$ - denotes stability derivatives for rotor solidity $\sigma = 0.1$. These values can be directly obtained from the charts of Section 7.5.

$$\frac{\partial(\frac{C_D'}{\sigma})}{\partial\mu} = \left[\frac{\partial(\frac{C_D'}{\sigma})}{\partial\mu}\right]_{0.1} + K_2\left[\frac{\partial(\frac{C_D'}{\sigma})}{\partial\alpha_c}\right]_{0.1} - \left(\frac{C_L'}{\sigma}\right)\left\{K_2 - \frac{\Delta\sigma}{2\mu^2}\left[\frac{\partial(\frac{C_L'}{\sigma})}{\partial\mu}\right]\right\}$$

where

$$K_2 = \frac{\Delta\sigma}{2\mu^2}\left[\frac{2}{\mu}\left(\frac{C_L'}{\sigma}\right) - \frac{\partial(\frac{C_L'}{\sigma})}{\partial\mu}\right]$$

$$\frac{\partial(\frac{C_Q}{\sigma})}{\partial\mu} = \left[\frac{\partial(\frac{C_Q}{\sigma})}{\partial\mu}\right]_{0.1} + K_2\left[\frac{\partial(\frac{C_Q}{\sigma})}{\partial\alpha_c}\right]_{0.1}$$

$$\frac{\partial a_1}{\partial \mu} = \left(\frac{\partial a_1}{\partial \mu}\right)_{0.1} + K_2\left(\frac{\partial a_1}{\partial \alpha_c}\right)_{0.1}$$

$$\frac{\partial b_1}{\partial \mu} = \left(\frac{\partial b_1}{\partial \mu}\right)_{0.1} + K_2\left(\frac{\partial b_1}{\partial \alpha_c}\right)_{0.1}$$

$$\frac{\partial \lambda}{\partial \mu} = \left(\frac{\partial \lambda}{\partial \mu}\right)_{0.1} + K_2\left(\frac{\partial \lambda}{\partial \alpha_c}\right)_{0.1}$$

7.4.2 Solidity Corrections for (α_c) Derivatives

$$\frac{\partial\left(\frac{C_L'}{\sigma}\right)}{\partial \alpha_c} = K_1\left[\frac{\partial\left(\frac{C_L'}{\sigma}\right)}{\partial \alpha_c}\right]_{0.1}$$

$$\frac{\partial\left(\frac{C_D'}{\sigma}\right)}{\partial \alpha_c} = K_1\left\{\left[\frac{\partial\left(\frac{C_D'}{\sigma}\right)}{\partial \alpha_c}\right]_{0.1} - \frac{\Delta\sigma}{\mu^2}\left(\frac{C_L'}{\sigma}\right)\left[\frac{\partial\left(\frac{C_L'}{\sigma}\right)}{\partial \alpha_c}\right]_{0.1}\right\}$$

$$\frac{\partial\left(\frac{C_Q}{\sigma}\right)}{\partial \alpha_c} = K_1\left[\frac{\partial\left(\frac{C_Q}{\sigma}\right)}{\partial \alpha_c}\right]_{0.1}$$

$$\frac{\partial a_1}{\partial \alpha_c} = K_1\left(\frac{\partial a_1}{\partial \alpha_c}\right)_{0.1}$$

$$\frac{\partial b_1}{\partial \alpha_c} = K_1\left(\frac{\partial b_1}{\partial \alpha_c}\right)_{0.1}$$

$$\frac{\partial \lambda}{\partial \alpha_c} = K_1\left(\frac{\partial \lambda}{\partial \alpha_c}\right)_{0.1}$$

7.4.3 Solidity Corrections for $(\theta_{.75})$ Derivatives

$$\frac{\partial\left(\frac{C_L'}{\sigma}\right)}{\partial \theta_{.75}} = K_1\left[\frac{\partial\left(\frac{C_L'}{\sigma}\right)}{\partial \theta_{.75}}\right]_{0.1}$$

$$\frac{\partial(\frac{C_Q'}{\sigma})}{\partial\theta_{.75}} = \left[\frac{\partial(\frac{C_Q'}{\sigma})}{\partial\theta_{.75}}\right]_{0.1} + K_3\left\{2\left(\frac{C_L'}{\sigma}\right) - \left[\frac{\partial(\frac{C_Q'}{\sigma})}{\partial\alpha_c}\right]_{0.1}\right\}$$

where

$$K_3 = \frac{\Delta\sigma}{\partial\mu^2}\left[\frac{\partial(\frac{C_L'}{\sigma})}{\partial\theta_{.75}}\right]$$

$$\frac{\partial(\frac{C_Q}{\sigma})}{\partial\theta_{.75}} = \left[\frac{\partial(\frac{C_Q}{\sigma})}{\partial\theta_{.75}}\right]_{0.1} - K_3\left[\frac{\partial(\frac{C_Q}{\sigma})}{\partial\alpha_c}\right]_{0.1}$$

$$\frac{\partial a_1}{\partial\theta_{.75}} = \left(\frac{\partial a_1}{\partial\theta_{.75}}\right)_{0.1} - K_3\left(\frac{\partial a_1}{\partial\alpha_c}\right)_{0.1}$$

$$\frac{\partial b_1}{\partial\theta_{.75}} = \left(\frac{\partial b_1}{\partial\theta_{.75}}\right)_{0.1} - K_3\left(\frac{\partial b_1}{\partial\alpha_c}\right)_{0.1}$$

$$\frac{\partial\lambda}{\partial\theta_{.75}} = \left(\frac{\partial\lambda}{\partial\theta_{.75}}\right)_{0.1} - K_3\left(\frac{\partial\lambda}{\partial\alpha_c}\right)_{0.1}$$

7.5 ISOLATED ROTOR DERIVATIVES FOR ROTOR SOLIDITY $\sigma = 0.1$

The change of rotor aerodynamic parameters with respect to the basic variables, μ, α_C, and $\theta_{.75}$, are defined here as isolated rotor derivatives. These are functions of the trim values α_C, μ, $\theta_{.75}$, and M_T, as well as of the design variables σ, θ_1, and γ.

The isolated rotor derivatives presented in this section apply to rotor solidity of $\sigma = 0.1$, blade twist of $\theta_1 = 0°$, advancing tip Mach number of $M_T = 0.8$ and a range of Lock inertia number varying between $\gamma = 2.0$ and $\gamma = 25.0$.

One of the prime parameters affecting these derivatives is rotor solidity σ. In order to obtain the required values of the derivatives for rotor solidities other than $\sigma = 0.1$, appropriate solidity corrections must be applied. Such corrections may be obtained by utilizing the equations presented in Section 7.4. The effect of blade twist and advancing tip Mach number may be obtained from the charts presented in Subsections 7.5.4 and 7.5.5, respectively. The Lock inertia number γ, although generally negligible in performance work, primarily affects rotor flapping motion. This effect of γ on rotor flapping derivatives can be easily accounted for, since the parameters such as coning angle a_0, lateral flapping angle b_1, and higher harmonic flapping terms are essentially proportional to γ.

The isolated rotor derivatives have been extracted from the theoretical rotor performance data presented in Reference 1, by utilizing the graphical slope method. The data of Reference 1 include the effects of compressibility, stall, and reverse flow. The assumptions of classical theory, such as uniform induced velocity, rigid blades, no radial flow and two dimensional steady aerodynamic effects are retained. These derivatives cover the range of tip speed ratios between $\mu = 0.3$ and $\mu = 1.0$. The derivatives for the low μ values, $\mu \leq 0.2$, were obtained from Reference 2 and were converted into the form utilized for the derivatives of $\mu \geq 0.3$. The results of Reference 2 are based on classical Bailey theory with all its assumptions and limitations.

7.5.1 Isolated Rotor Derivatives With Respect to Rotor Tip Speed Ratio (μ)

7.5.1.1 $\frac{\partial(\frac{C_L'}{\sigma})}{\partial\mu}$ for $\sigma = 0.1$, $\theta_1 = 0°$, and $M_T = 0.8$

Figures 1(a) through 1(i) present the isolated rotor derivative $\partial(C_L'/\sigma)/\partial\mu$ as a function of C_L'/σ for constant values of $\theta_{.75}$, covering the range of tip speed ratios from $\mu = 0.1$ through $\mu = 1.0$. The values of $\partial(C_L'/\sigma)\partial\mu$ for $\mu = 0.1$ and 0.2 (Figures 1(a) and 1(b)) were obtained from Reference 2 by utilizing the following equation:

$$\frac{\partial(\frac{C_L'}{\sigma})}{\partial\mu} = \frac{\partial(\frac{C_T}{\sigma})}{\partial\mu}\cos\alpha_C - \frac{\partial(\frac{C_H}{\sigma})}{\partial\mu}\sin\alpha_C$$

Values of the $\partial(C_L'/\sigma)\partial\mu$ for $\mu \geq 0.3$ were extracted from the theoretical rotor performance data of Reference 1, as slopes of the C_L'/σ vs. μ relationships for constant values of $\theta_{.75}$ and α_C.

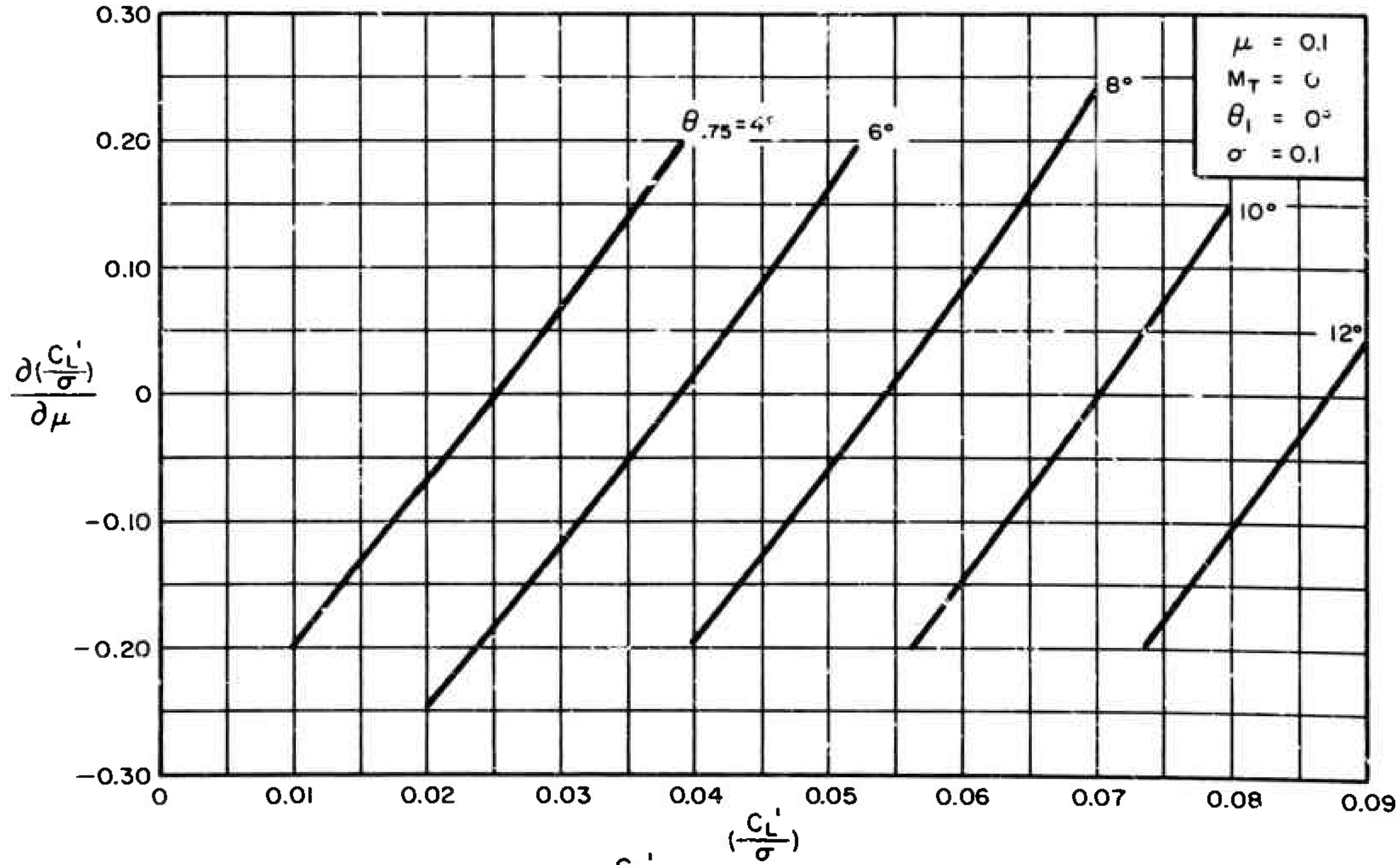

Figure 1. Variation of $\frac{\partial(\frac{C_L'}{\sigma})}{\partial\mu}$ With $\frac{C_L'}{\sigma}$ for Constant Values of $\theta_{.75}$

(a) μ = 0.1

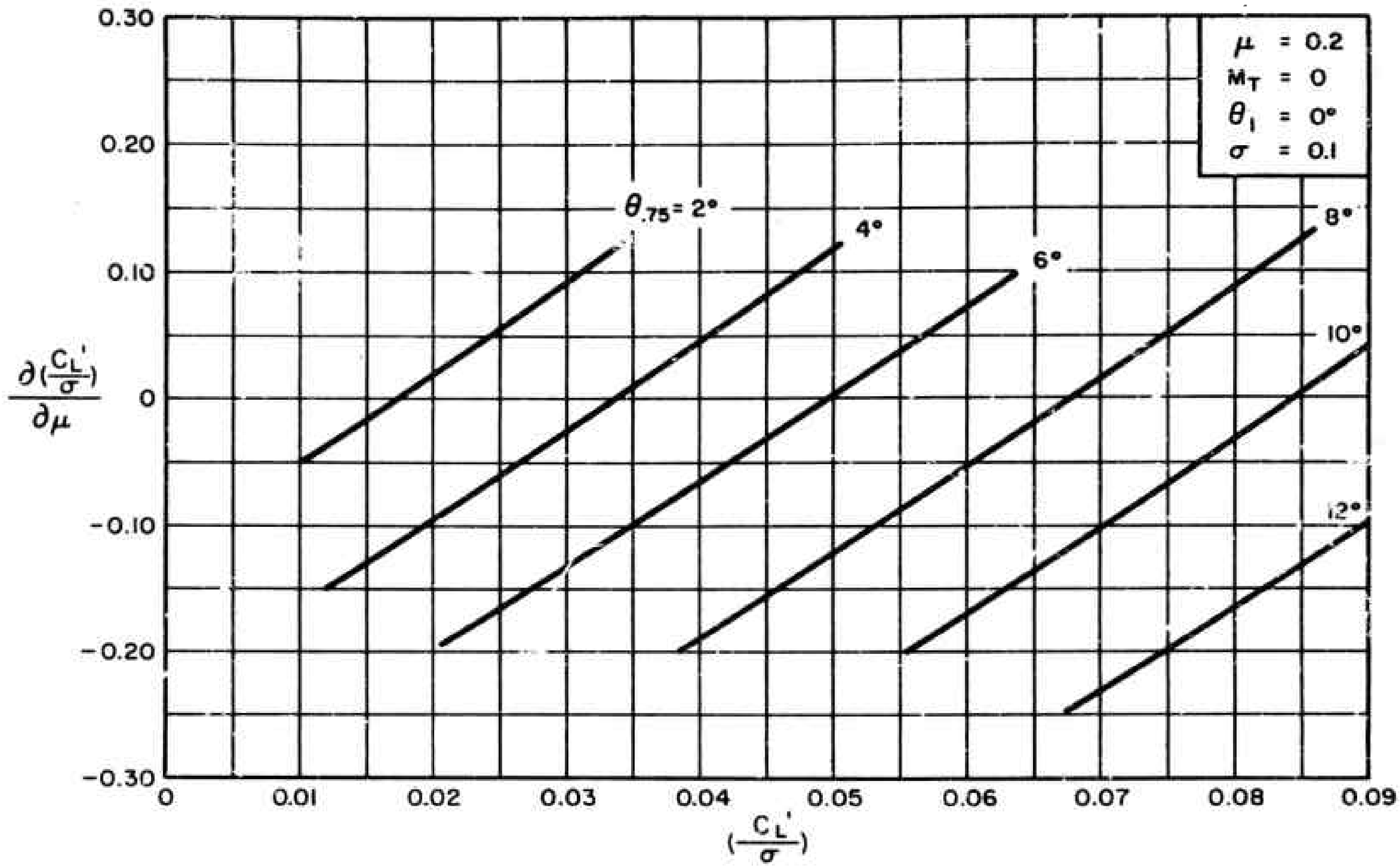

Figure 1. Continued

(b) $\mu = 0.2$

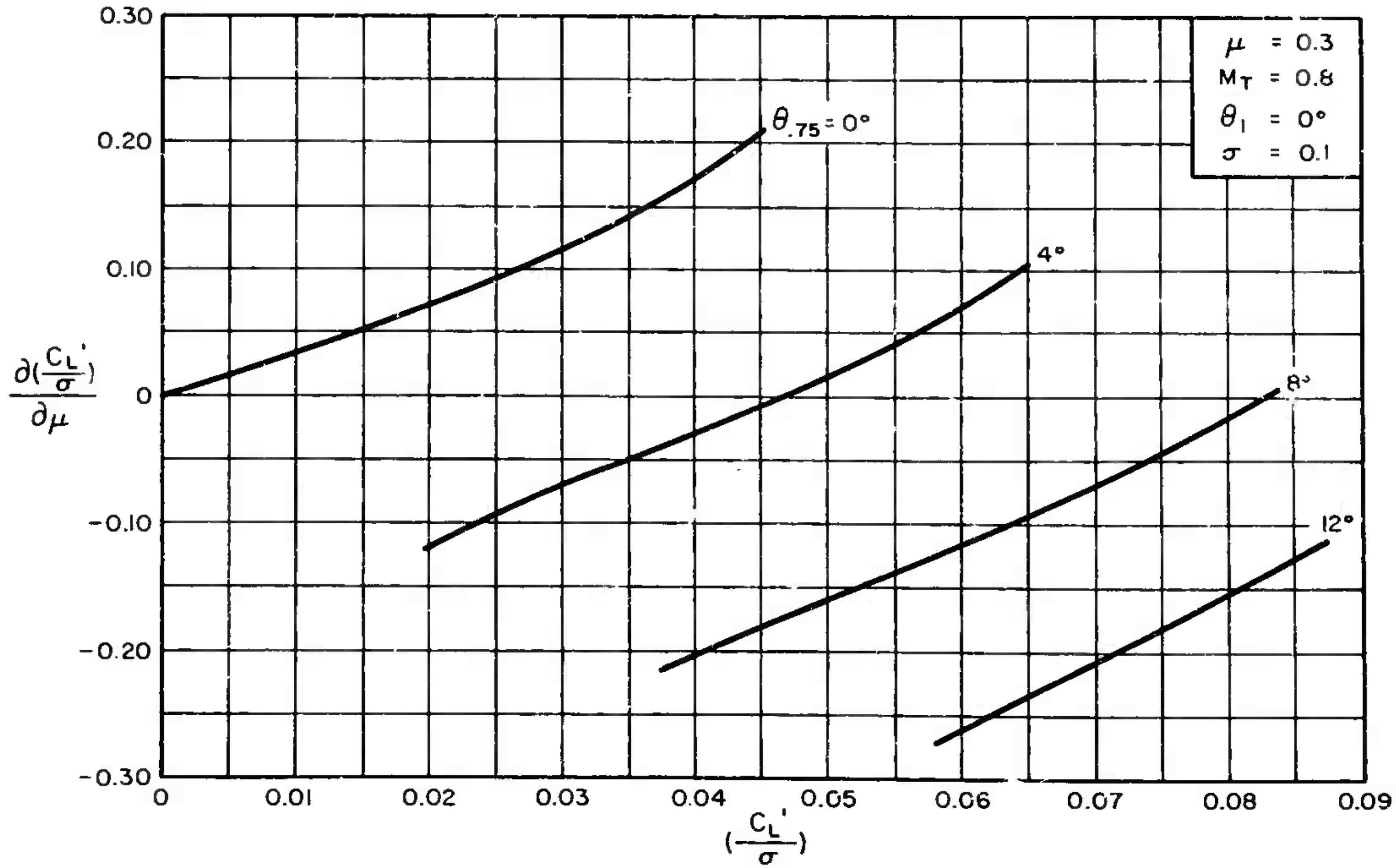

Figure 1. Continued

(c) μ = 0.3

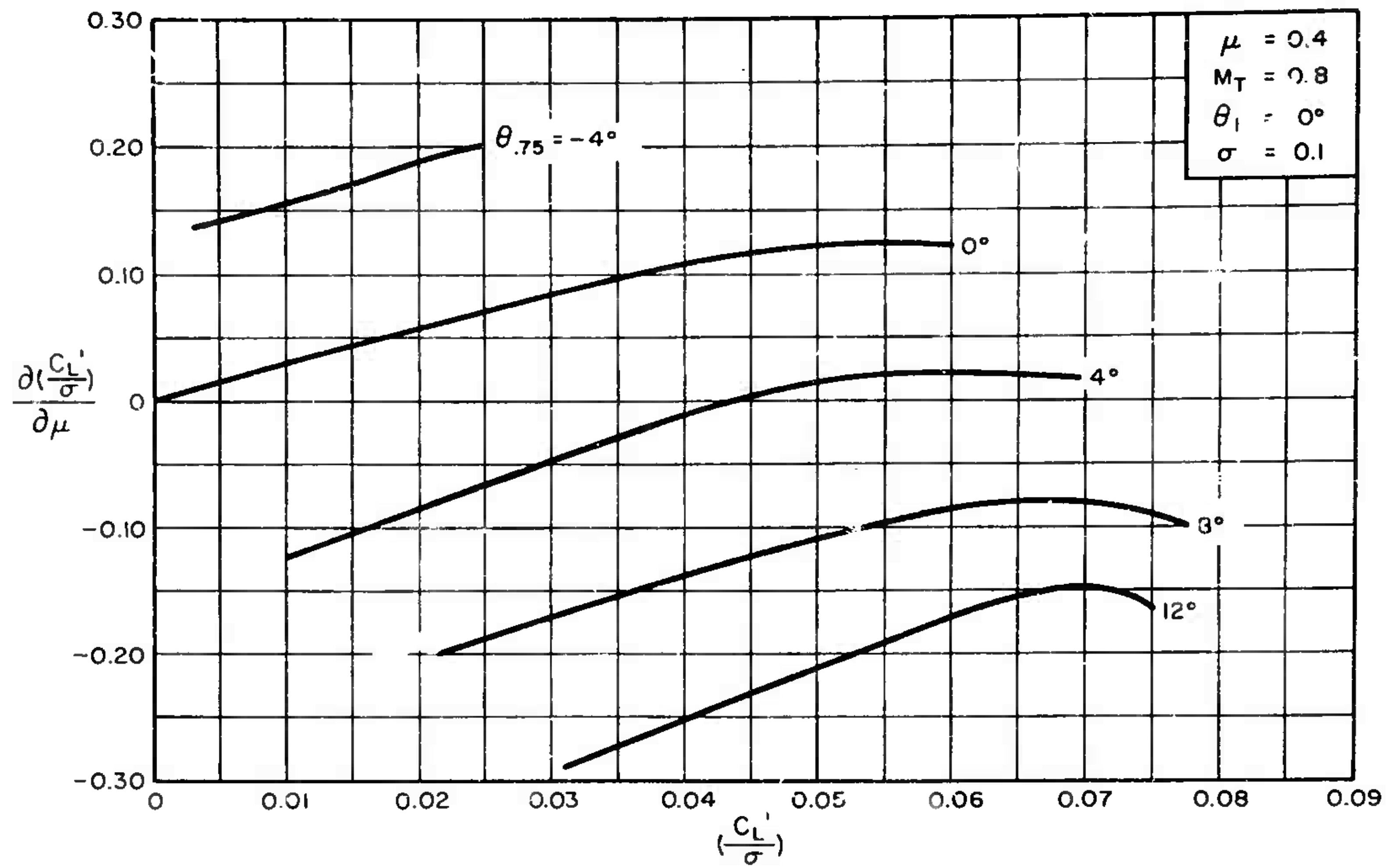

Figure 1. Continued

(d) $\mu = 0.4$

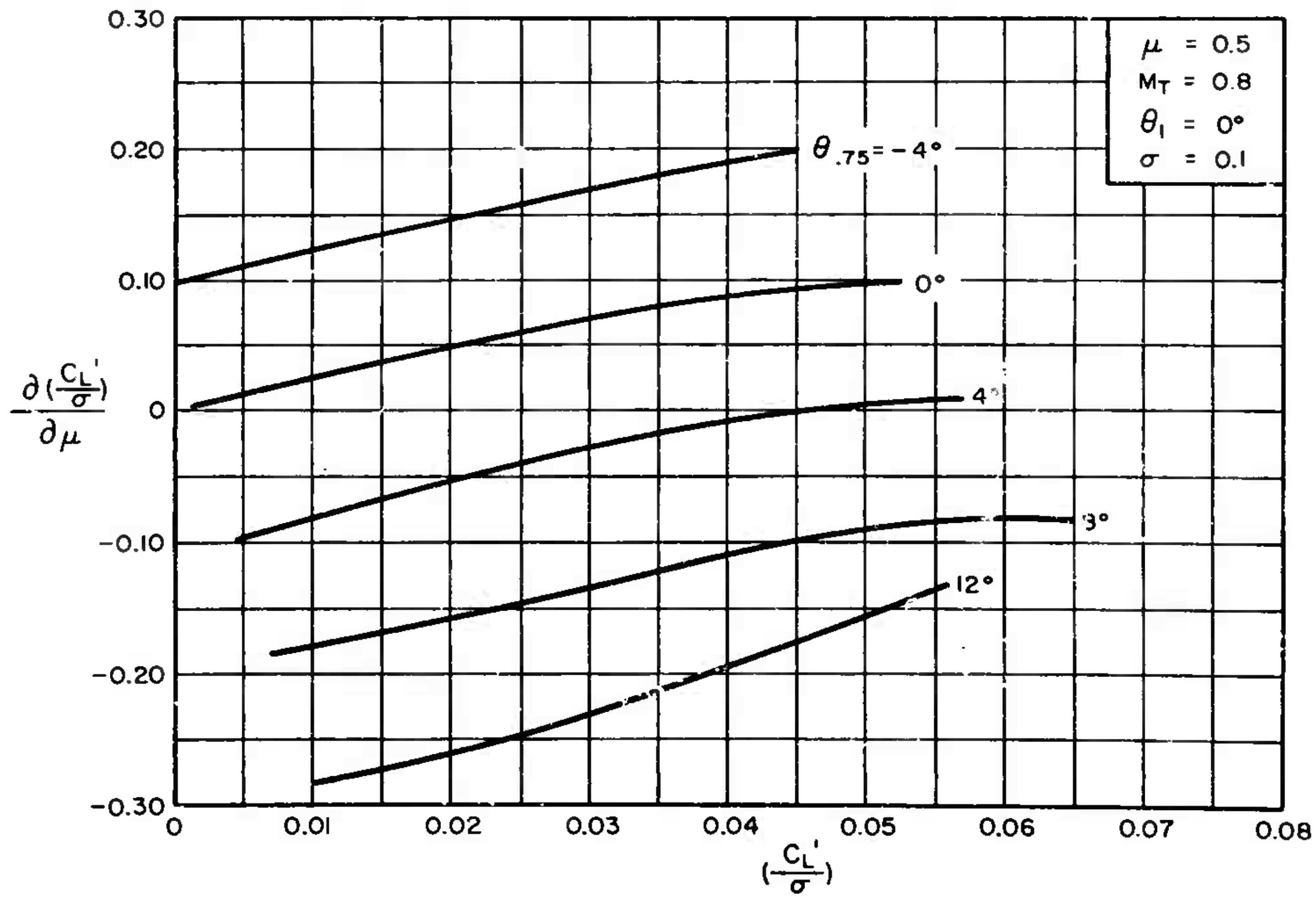

Figure 1. Continued

(e) μ = 0.5

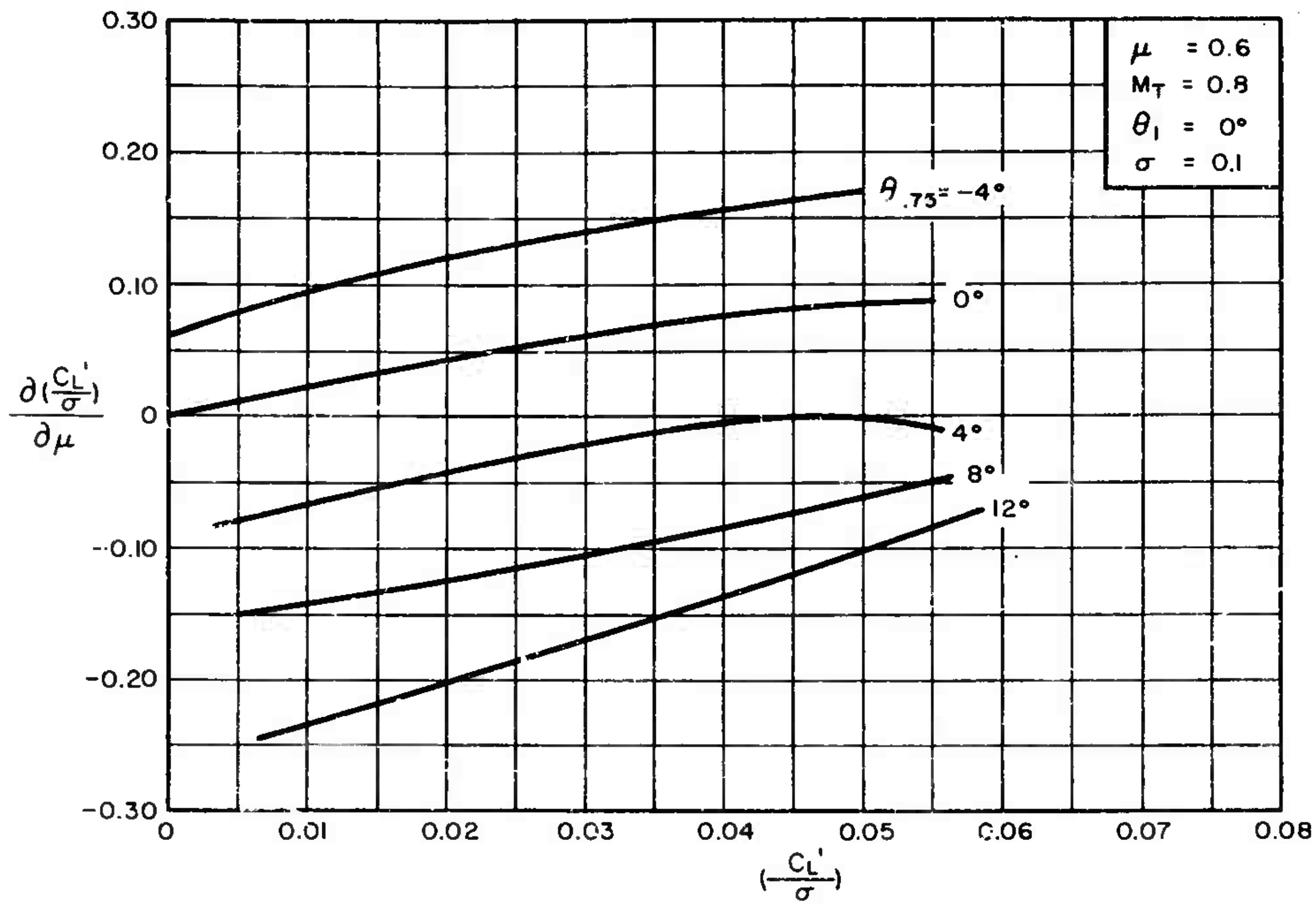

Figure 1. Continued

(f) $\mu = 0.6$

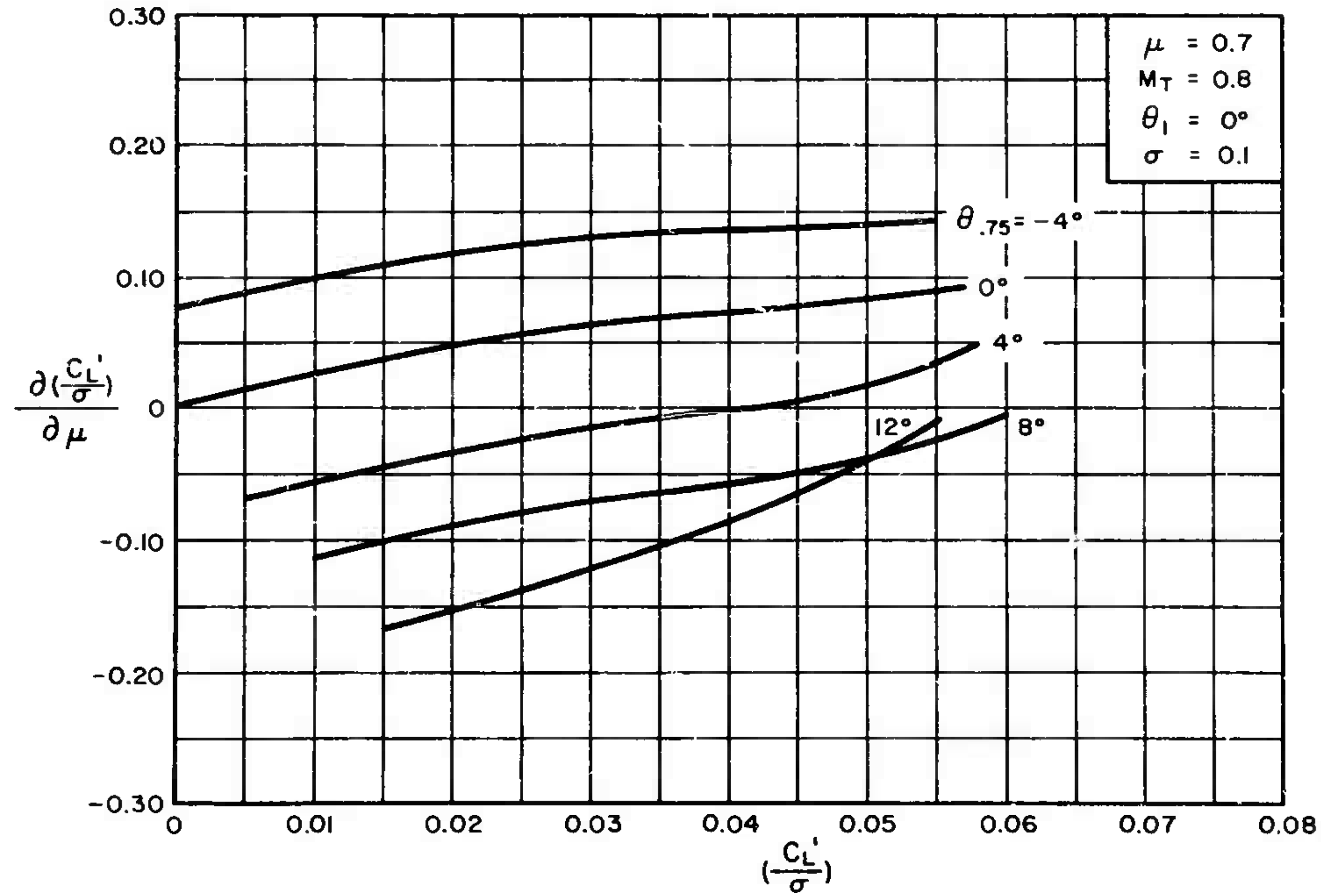

Figure 1. Continued

(g) $\mu = 0.7$

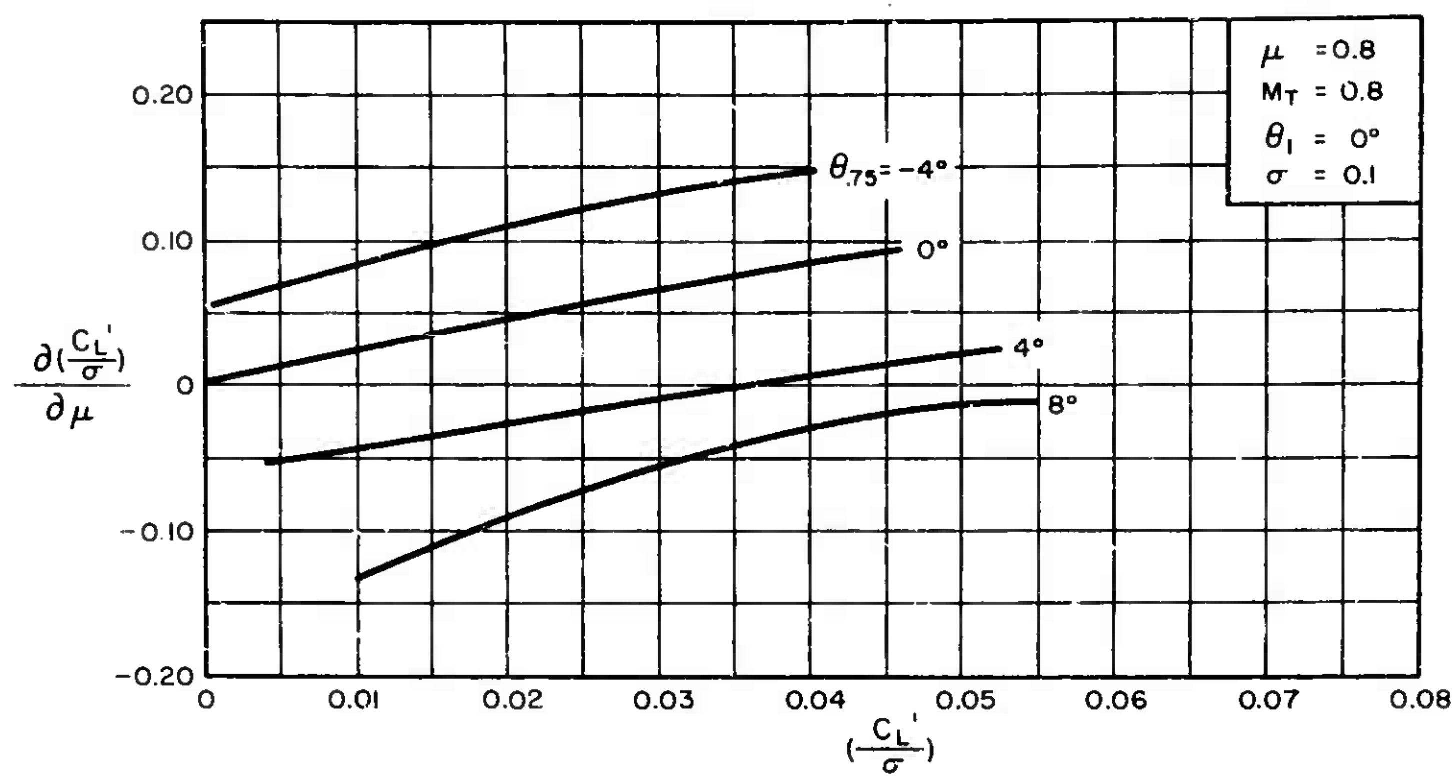

Figure 1. Continued

(h) $\mu = 0.8$

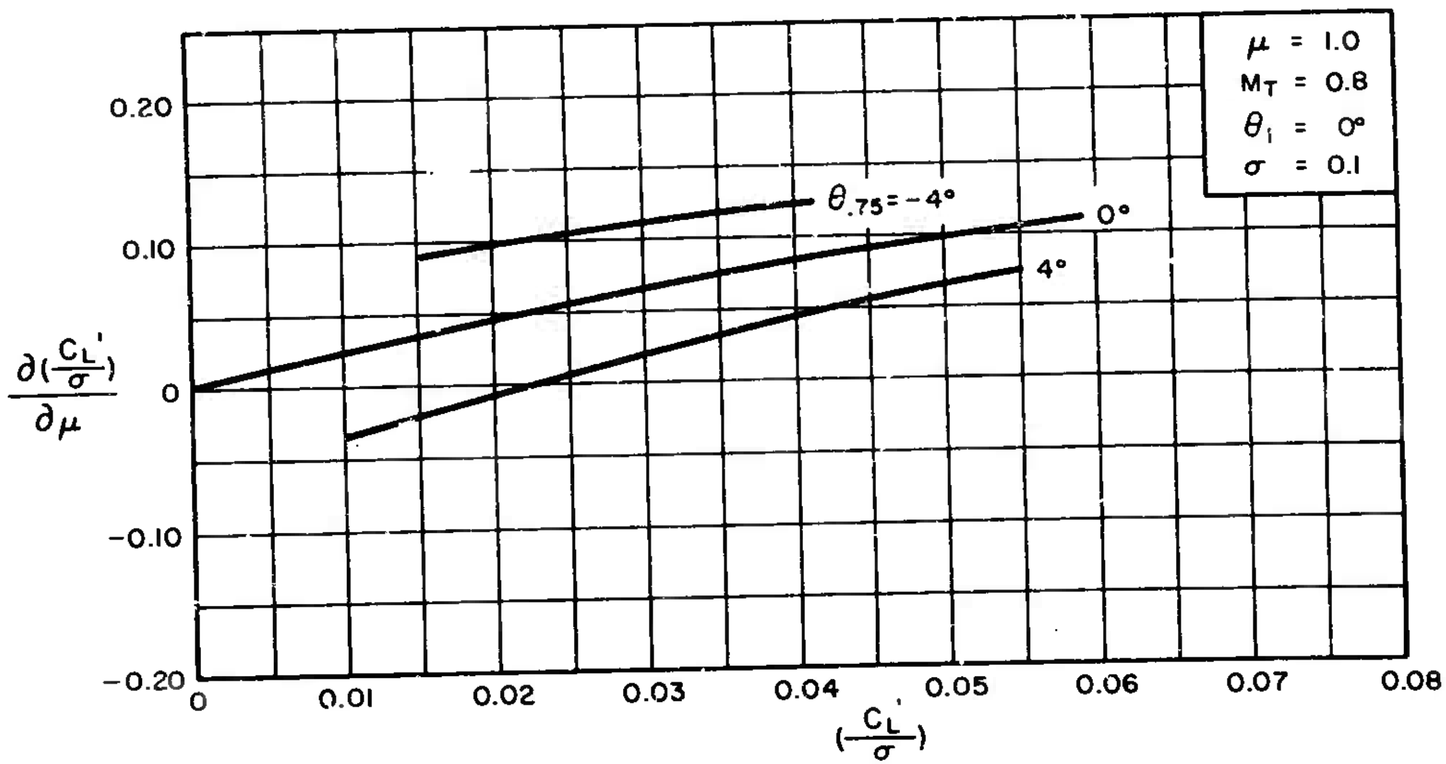

Figure 1. Concluded

(i) $\mu = 1.0$

7.5.1.2 $\frac{\partial(\frac{C_D'}{\sigma})}{\partial\mu}$ for $\sigma = 0.1$, $\theta = 0°$, and $M_T = 0.8$

Figures 2(a) through 2(e) present the isolated rotor derivative $\partial(C_D'/\sigma)/\partial\mu$ as a function of C_L'/σ for constant values of μ covering the collective pitch range ($\theta_{.75}$) between -4° and +12°. The values of the above derivatives for $\mu \geq 0.3$ were extracted from rotor performance data of Reference 1 by graphically obtaining the slopes of the C_D'/σ vs. μ relationships for constant values of $\theta_{.75}$ and α_C . The derivatives for $\mu \leq 0.2$ were obtained from the data of Reference 2 by using the following expression:

$$\frac{\partial(\frac{C_D'}{\sigma})}{\partial\mu} = \frac{\partial(\frac{C_H}{\sigma})}{\partial\mu} \cos \alpha_C + \frac{\partial(\frac{C_T}{\sigma})}{\partial\mu} \sin \alpha_C$$

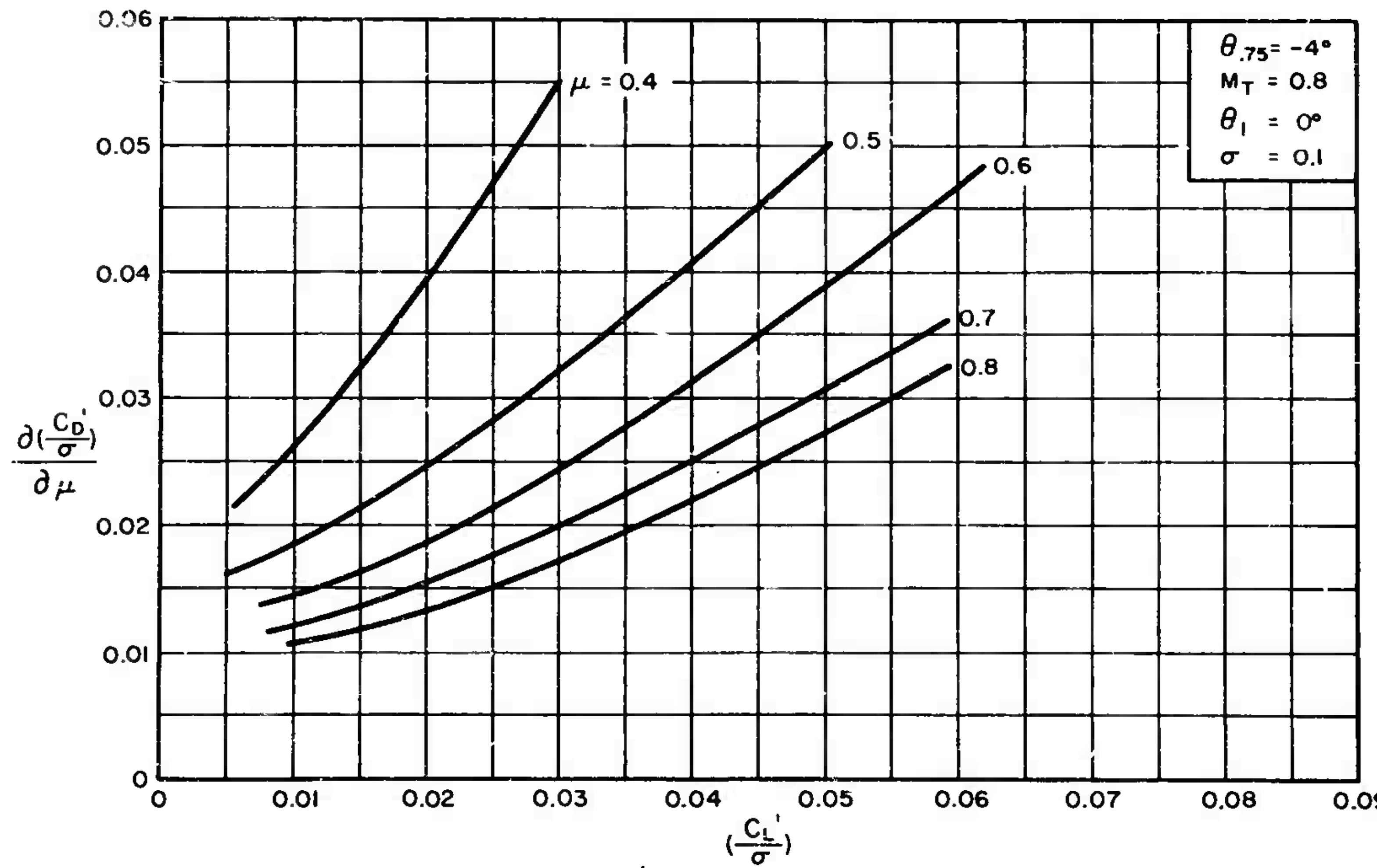

Figure 2. Variation of $\frac{\partial(\frac{C_D'}{\sigma})}{\partial \mu}$ With $\frac{C_L'}{\sigma}$ for Constant Values of μ

(a) $\theta_{.75} = -4°$

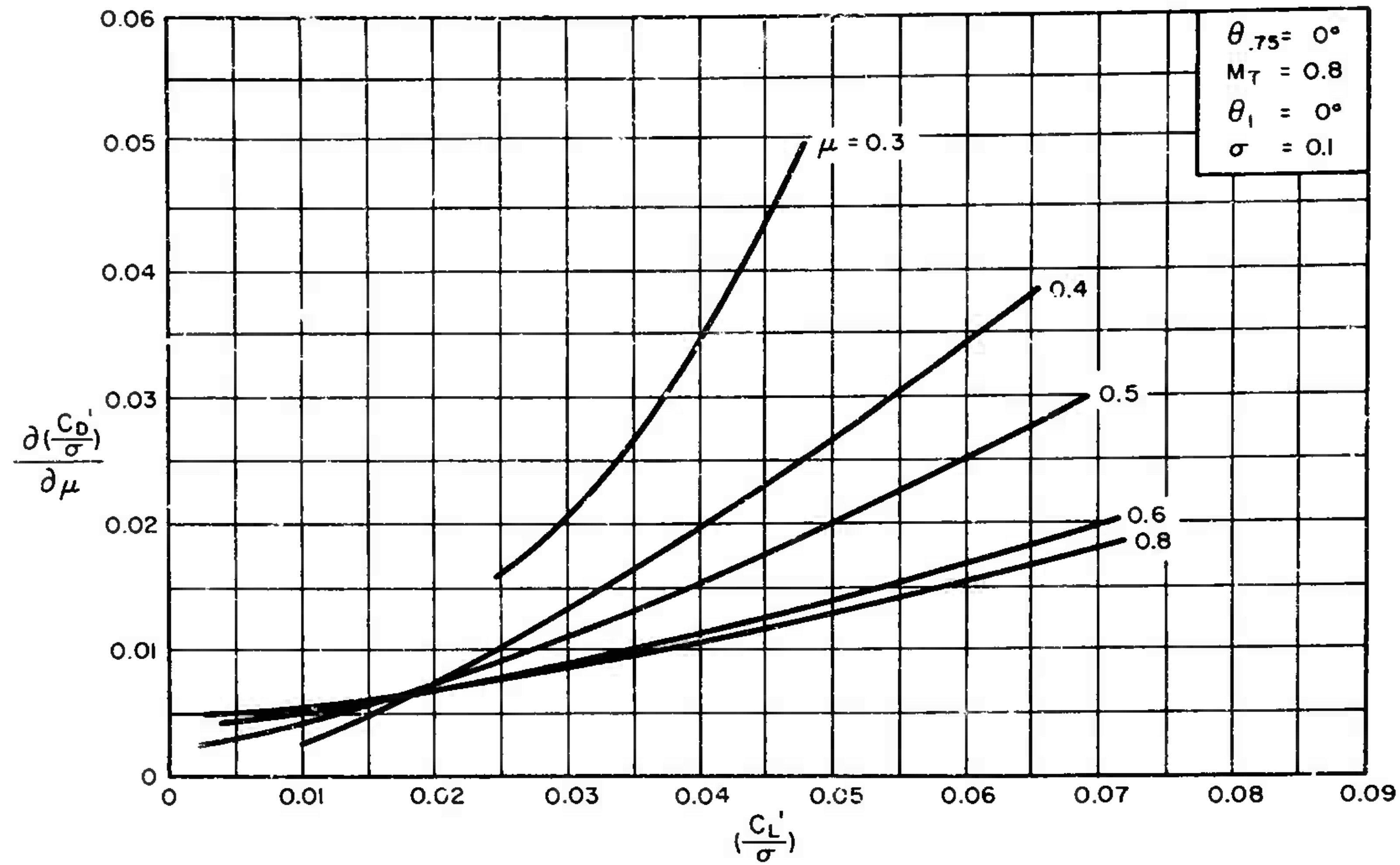

Figure 2. Continued

(b) $\theta_{.75} = 0°$

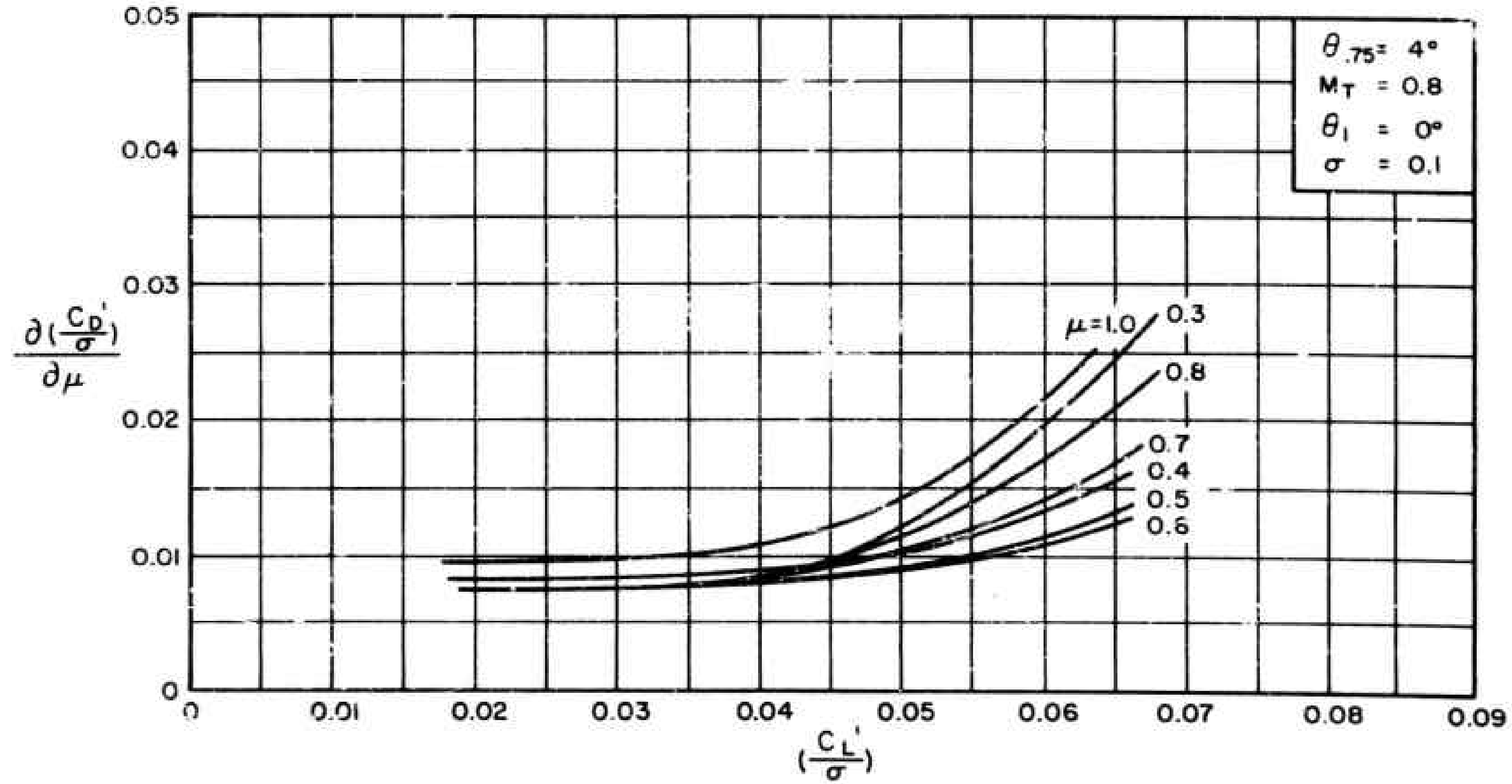

Figure 2. Continued

(c) $\theta_{.75} = 4°$

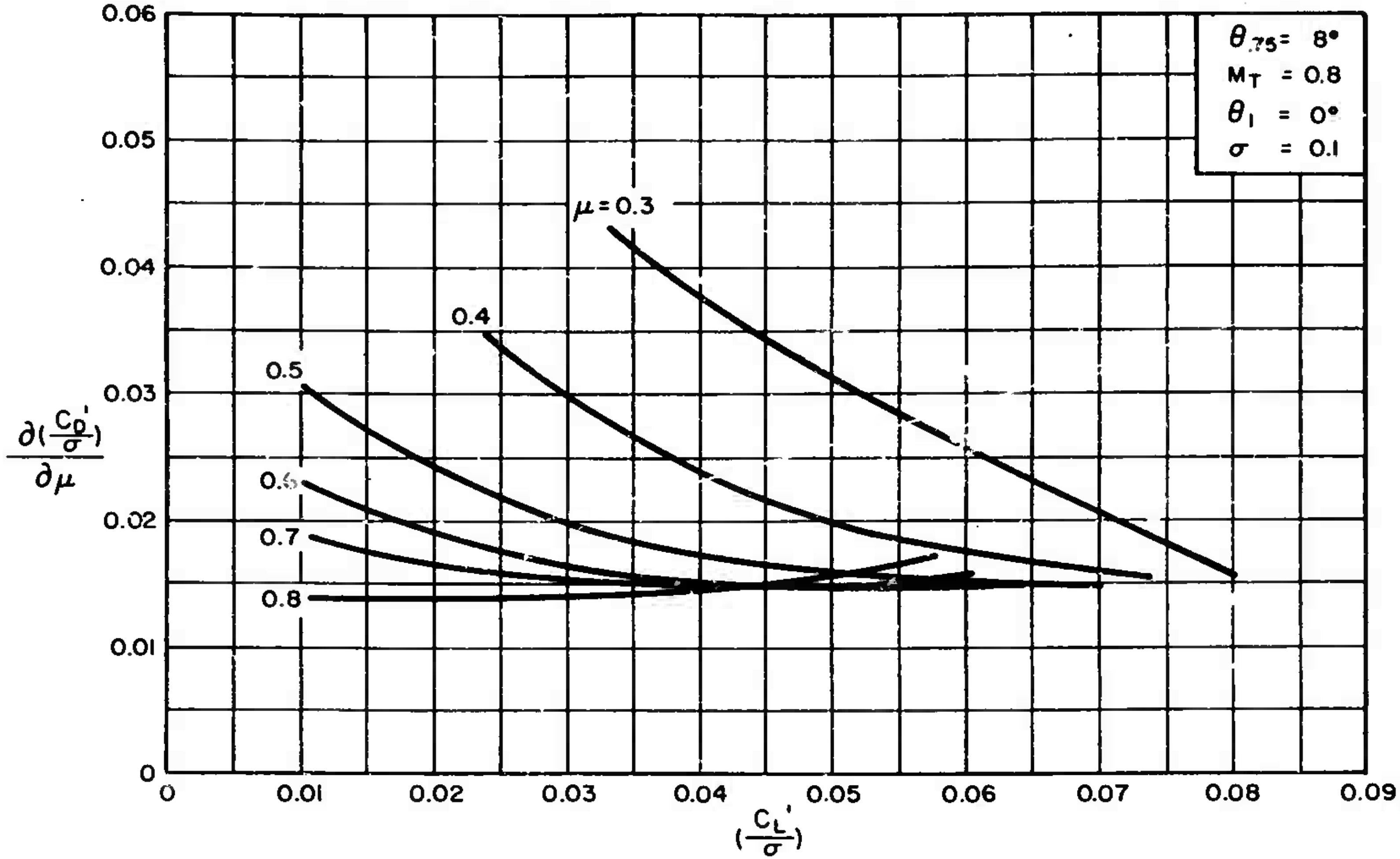

Figure 2. Continued

(d) $\theta_{.75} = 8°$

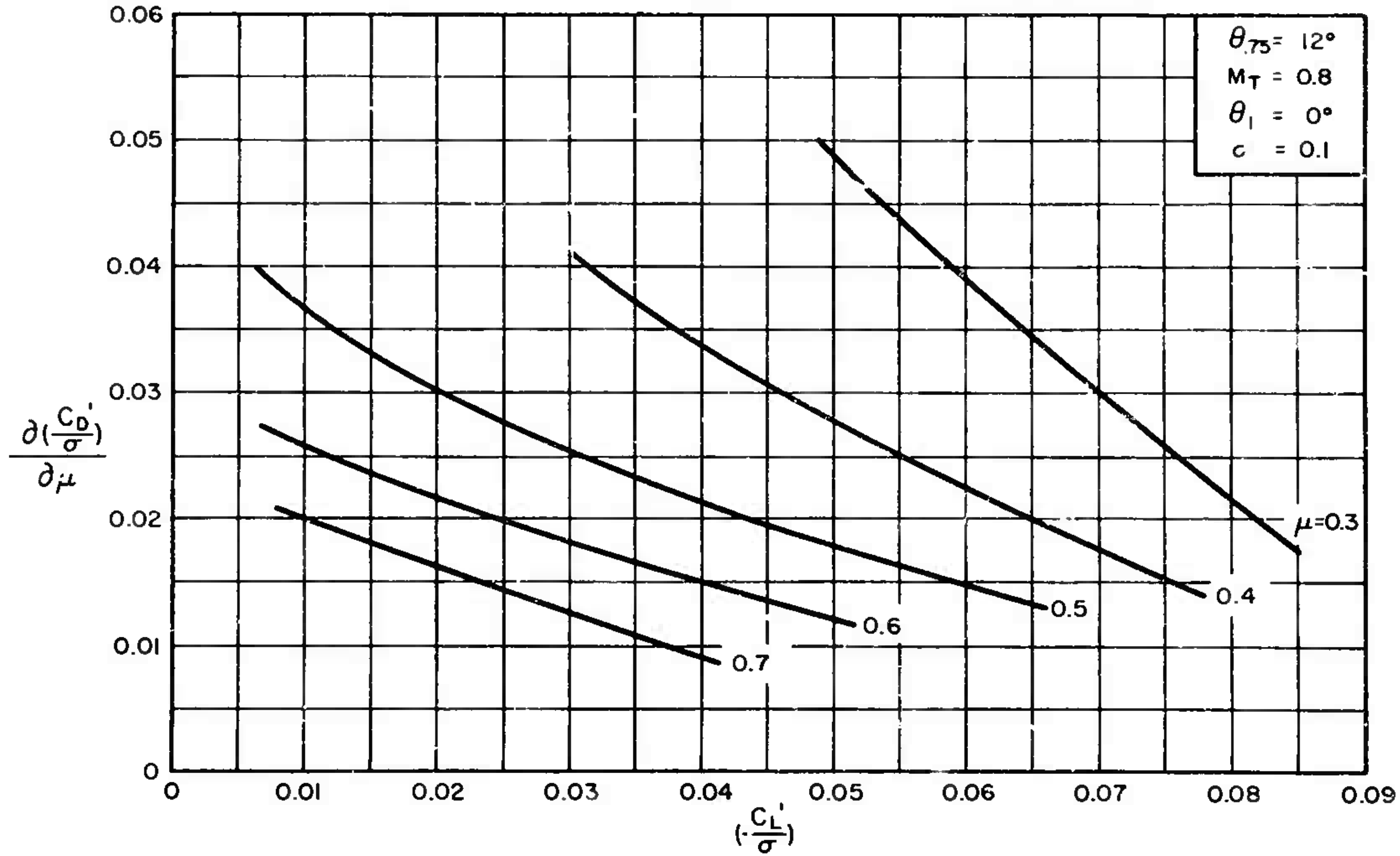

Figure 2. Concluded

(e) $\theta_{.75} = 12°$

7.5.1.3 $\frac{\partial(\frac{C_Q}{\sigma})}{\partial\mu}$ for $\sigma = 0.1$, $\theta_1 = 0°$, and $M_T = 0.8$

Figures 3(a) through 3(e) present the isolated rotor derivative $\partial(C_Q/\sigma)/\partial\mu$ as a function of C_L'/σ for constant values of μ for the collective pitch range from $\theta_{.75} = -4°$ to $\theta_{.75} = 12°$. The values of the derivatives for $\mu \geq 0.3$ were extracted from the rotor performance data of Reference 1 by graphically obtaining the slopes of the C_Q/σ vs. μ relationships for constant values of $\theta_{.75}$ and α_C. For values of $\mu \leq 0.2$, the following expression may be used:

$$\frac{\partial(\frac{C_Q}{\sigma})}{\partial\mu} = \frac{1}{2}\left\{\frac{\delta_0\mu}{2} + \frac{\delta_1\mu\,\theta_{.75}}{2} + \lambda^2\left[\delta_2\frac{\partial t_{55}}{\partial\mu} - a\frac{\partial t_{41}}{\partial\mu}\right]\right.$$

$$+\lambda\,\theta_{.75}\left[\frac{\partial t_{56}}{\partial\mu} - a\frac{\partial t_{42}}{\partial\mu}\right] + \theta_{.75}^2\left[\delta_2\frac{\partial t_{58}}{\partial\mu} - a\frac{\partial t_{44}}{\partial\mu}\right]$$

$$\left.+\frac{\partial\lambda}{\partial\mu}\left[\delta_1 t_{52} + 2\lambda(\delta_2 t_{55} - a_1 t_{41}) + \theta_{.75}(\delta_2 t_{56} - a\,t_{42})\right]\right\}$$

where

$$\frac{\partial t_{55}}{\partial\mu} = 2\mu\left[1.3776 - 0.000648\,\gamma^2\right]$$

$$\frac{\partial t_{41}}{\partial\mu} = 2\mu\left[1.250 + 0.000605\,\gamma^2\right]$$

$$\frac{\partial t_{56}}{\partial\mu} = 2\mu\left[2.835 + 0.000942\,\gamma^2\right] + 0.424\,\mu^2$$

$$\frac{\partial t_{42}}{\partial\mu} = 2\mu\left[0.2587 + 0.00088\,\gamma^2\right] + 0.212\,\mu^2$$

$$\frac{\partial t_{58}}{\partial\mu} = 2\mu\left[1.195 + 0.000343\,\gamma^2\right]$$

$$\frac{\partial t_{44}}{\partial \mu} = 2\mu\left[0.836 + 0.00032\,\gamma^2\right]$$

The value of $\partial\lambda/\partial\mu$ can be obtained from Subsection 7.5.1.6, and the parameters $\delta_0, \delta_1, \delta_2, t_{52}, t_{55}, t_{56}, \ldots\ldots$ can be obtained from Reference 3.

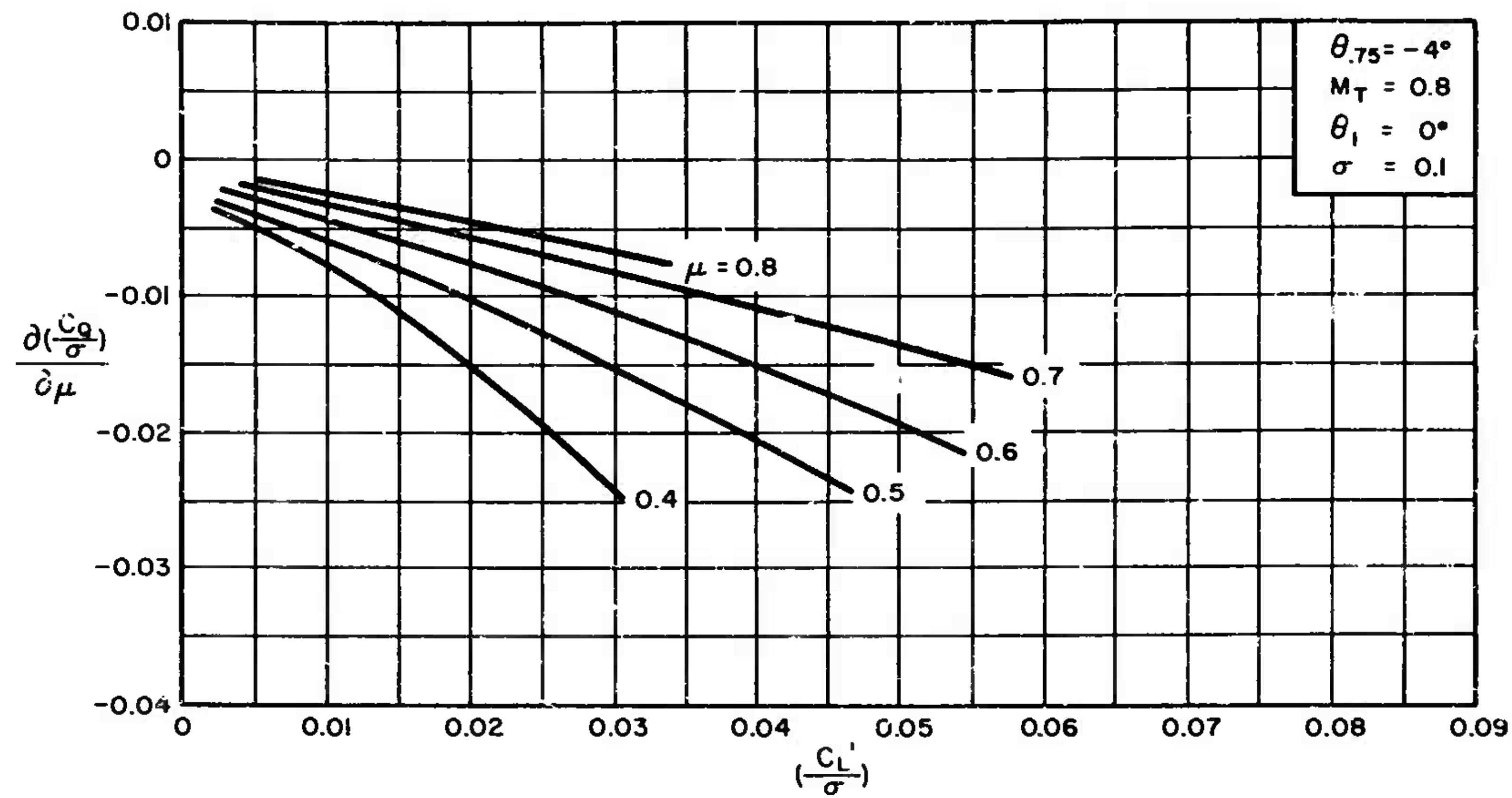

Figure 3. Variation of $\frac{\partial(\frac{C_Q}{\sigma})}{\partial\mu}$ With $\frac{C_L'}{\sigma}$ for Constant Values of μ

(a) $\theta_{.75} = -4°$

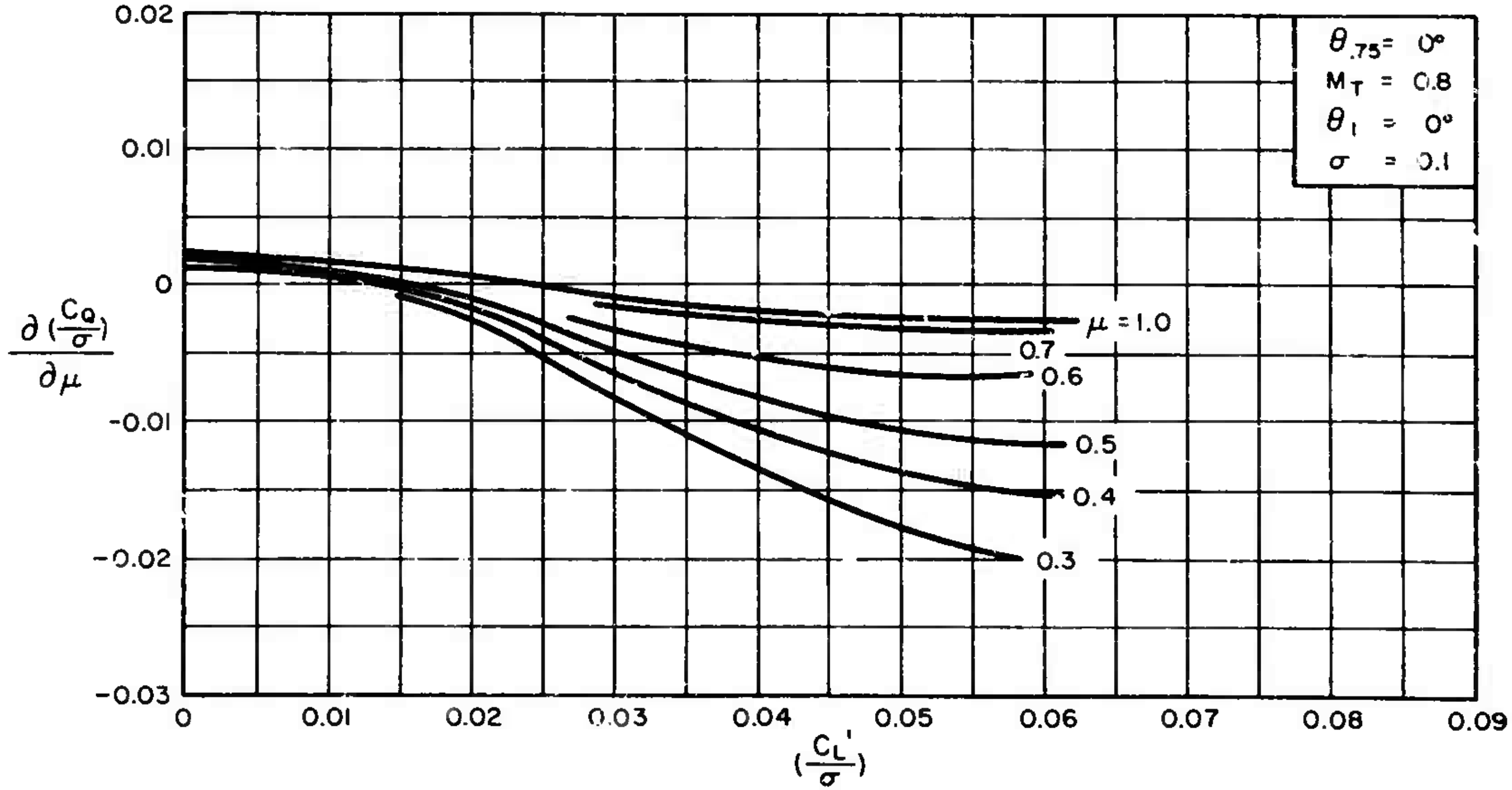

Figure 3. Continued

(b) $\theta_{.75} = 0°$

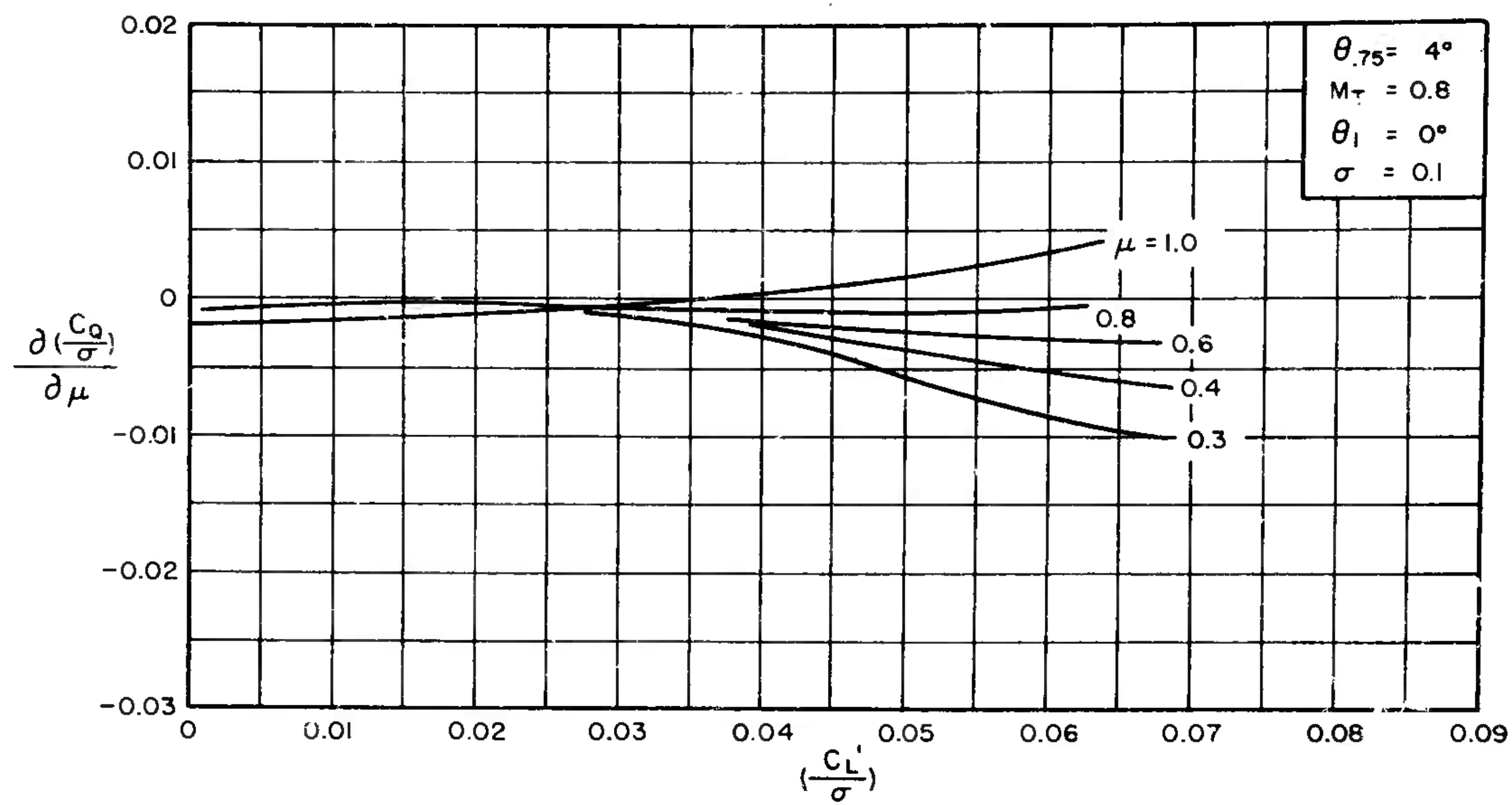

Figure 3. Continued

(c) $\theta_{.75} = 4°$

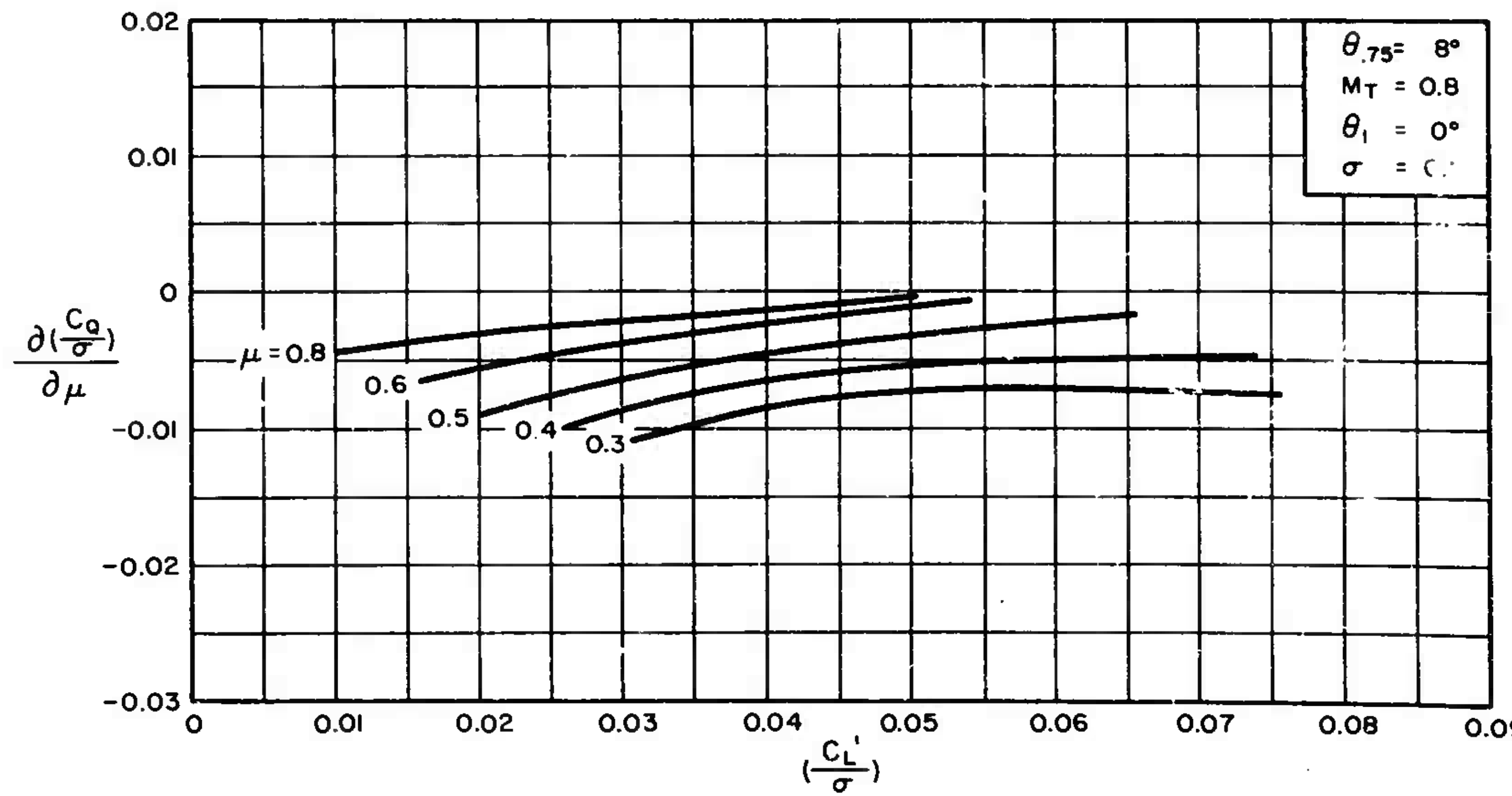

Figure 3. Continued

(d) $\theta_{.75} = 8°$

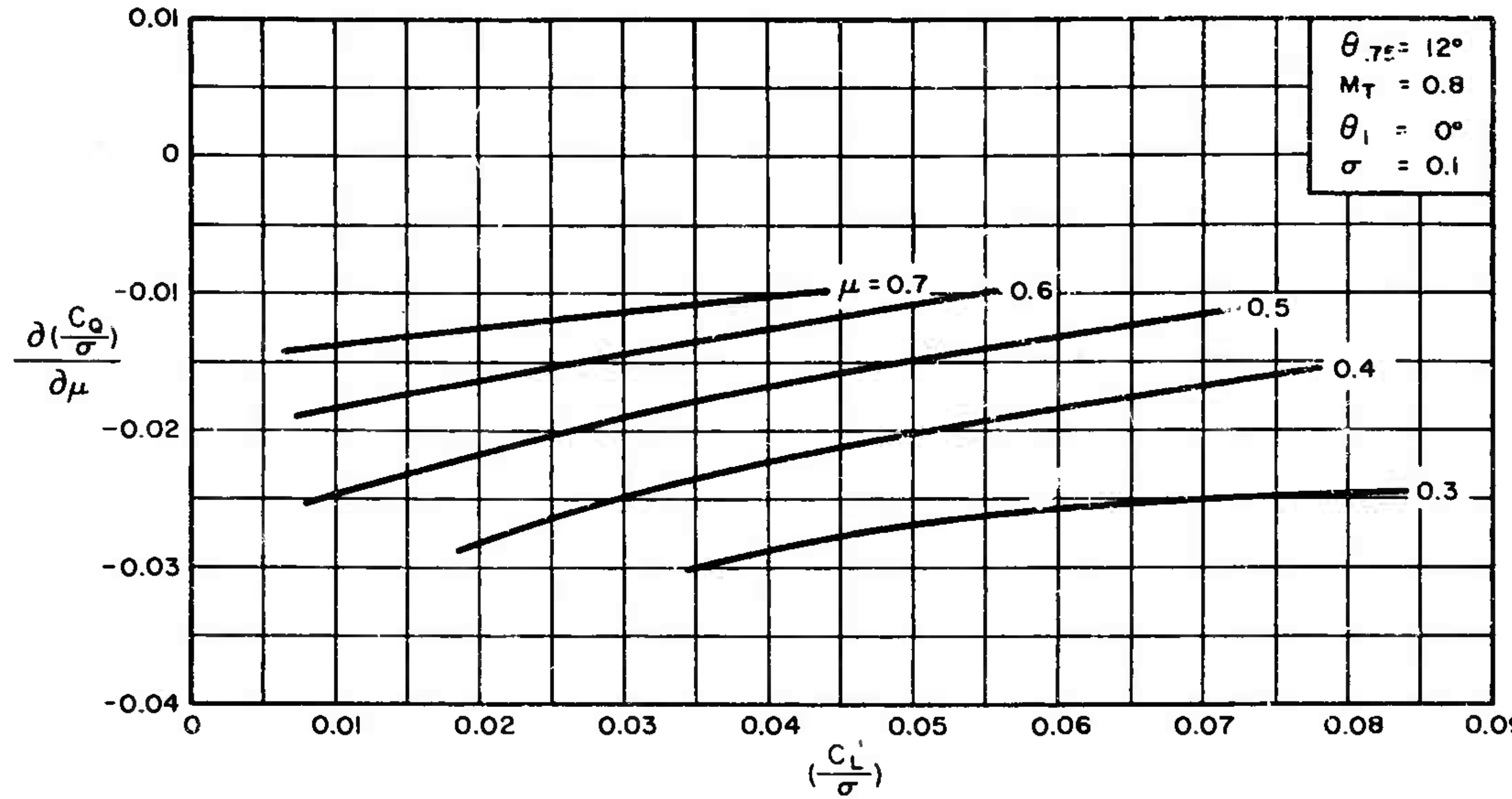

Figure 3. Concluded

(e) $\theta_{.75} = 12°$

7.5.1.4 $\frac{\partial a_1}{\partial \mu}$ for $\sigma = 0.1$, $\theta_1 = 0°$ and $M_T = 0.8$

Figure 4 presents the rotor isolated derivative $\partial a_1/\partial \mu$ as a function of C_L'/σ for all values of $\theta_{.75}$ and $\mu = 0.1$ and 0.2. These derivatives for $\mu \geq 0.3$ are presented in Figures 5(a) through 5(g) as functions of C_L'/σ for constant values of $\theta_{.75}$.

The derivatives $\partial a_1/\partial \mu$ for the values of $\mu \leq 0.2$ were obtained directly from Reference 2. The values for $\mu \geq 0.3$ were extracted from the theoretical rotor performance data of Reference 1 by obtaining the slopes of the a_1 vs. μ relationships for constant values of $\theta_{.75}$ and α_C.

The data of Reference 2, presented herein as Figure 4, show that the derivative of the longitudinal flapping angle a_1 with respect to μ is independent of $\theta_{.75}$ variation and is only a function of μ. However, for high μ values ($\mu \geq 0.3$) the results of Reference 1 indicate a substantial variation of the $(\partial a_1/\partial \mu)$ derivative with $\theta_{.75}$ as well as μ.

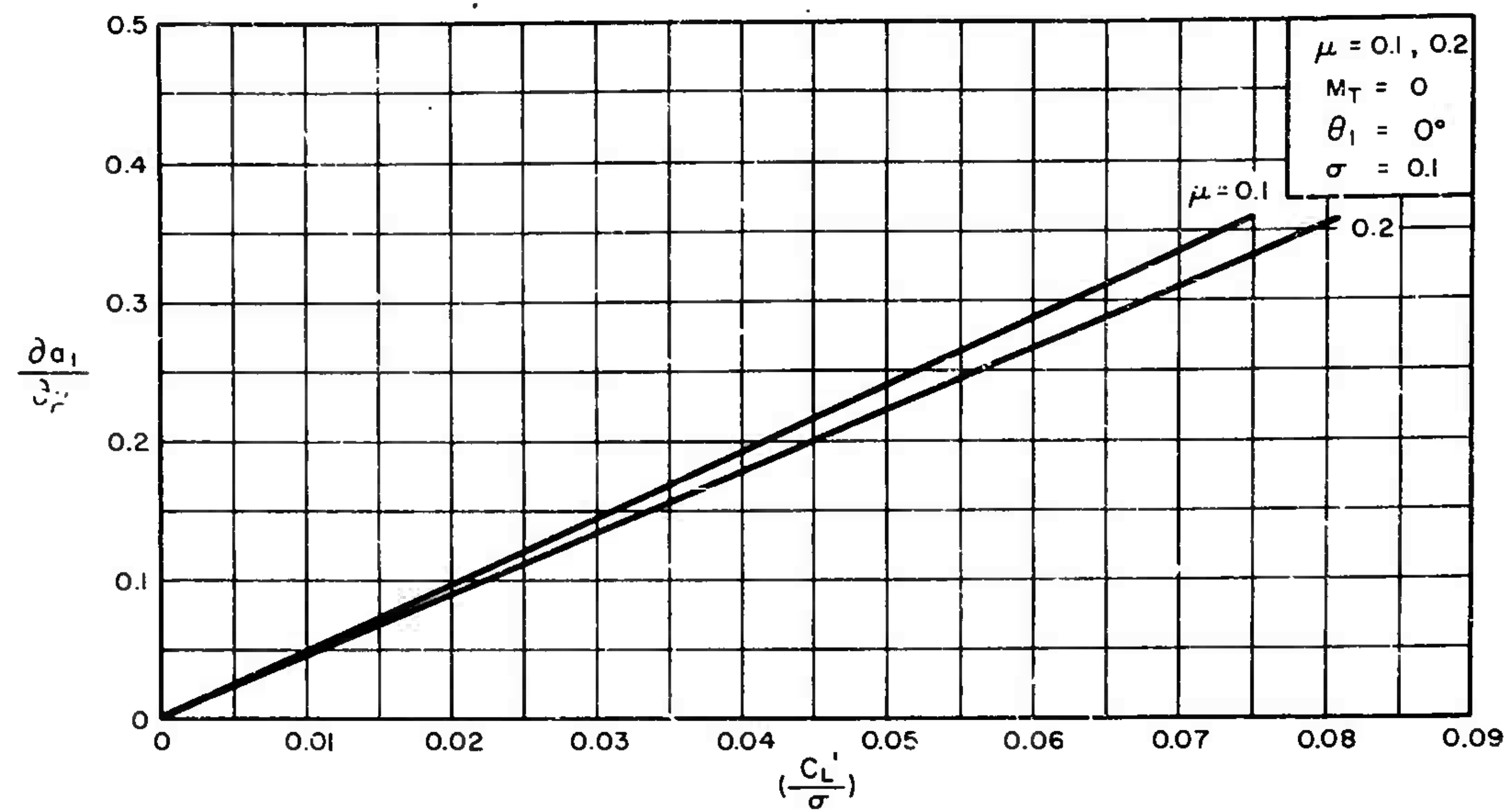

Figure 4. Variation of $\frac{\partial a_1}{\partial \mu}$ With $\frac{C_L'}{\sigma}$ for $\mu = 0.1$ and 0.2

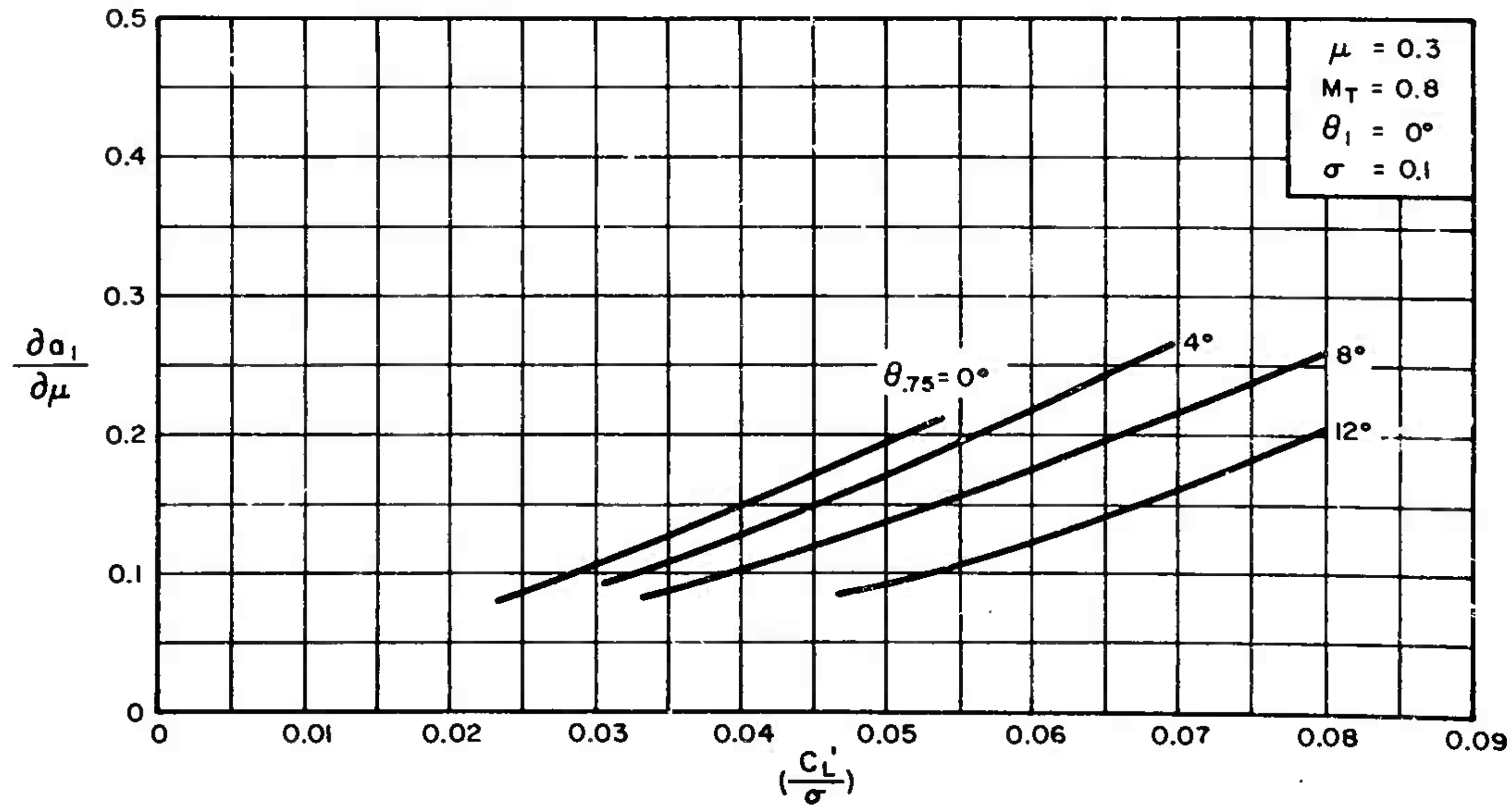

Figure 5. Variation of $\frac{\partial a_1}{\partial \mu}$ With $\frac{C_L'}{\sigma}$ for Constant Values of $\theta_{.75}$

(a) $\mu = 0.3$

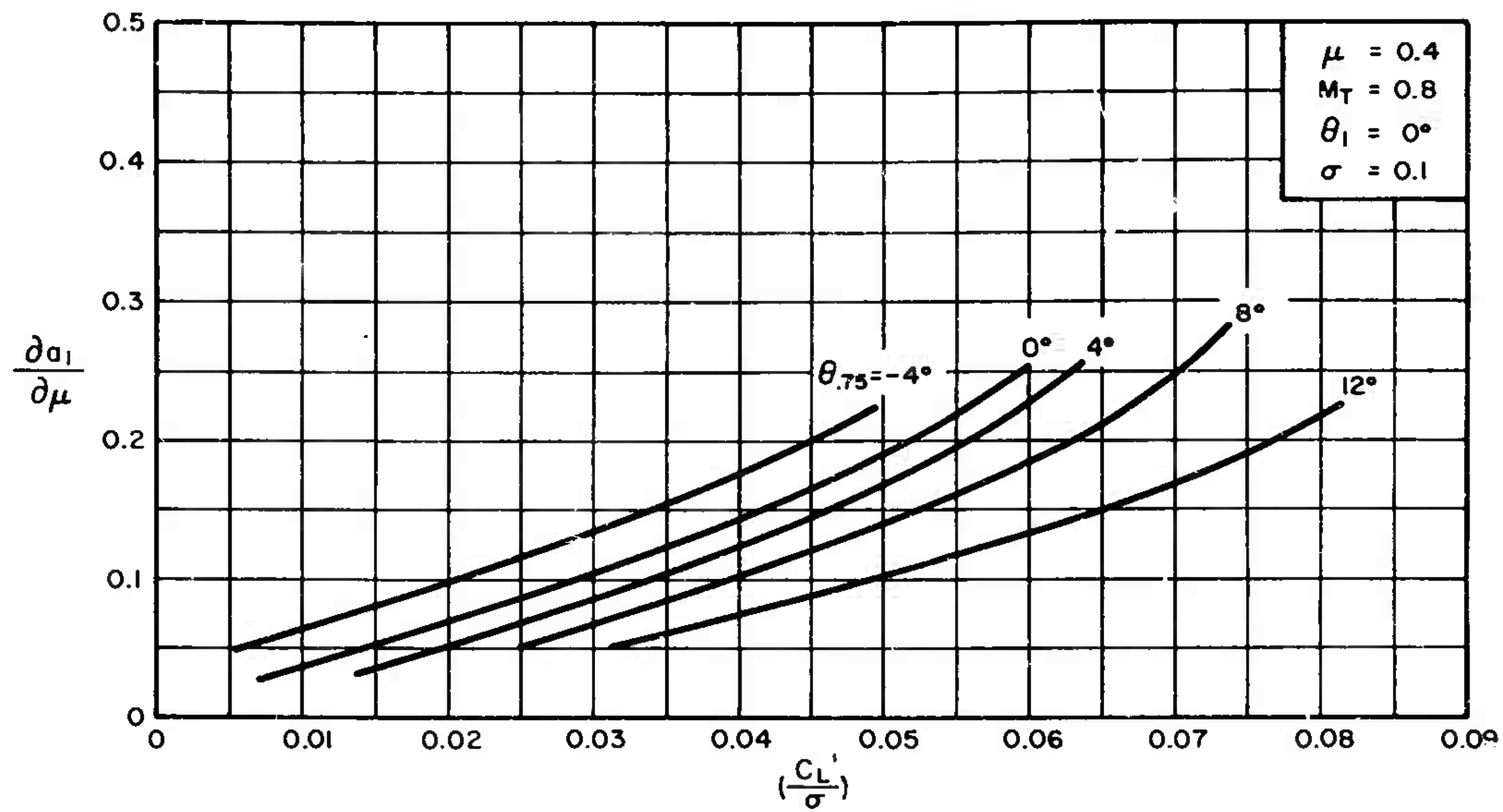

Figure 5. Continued

(b) μ = 0.4

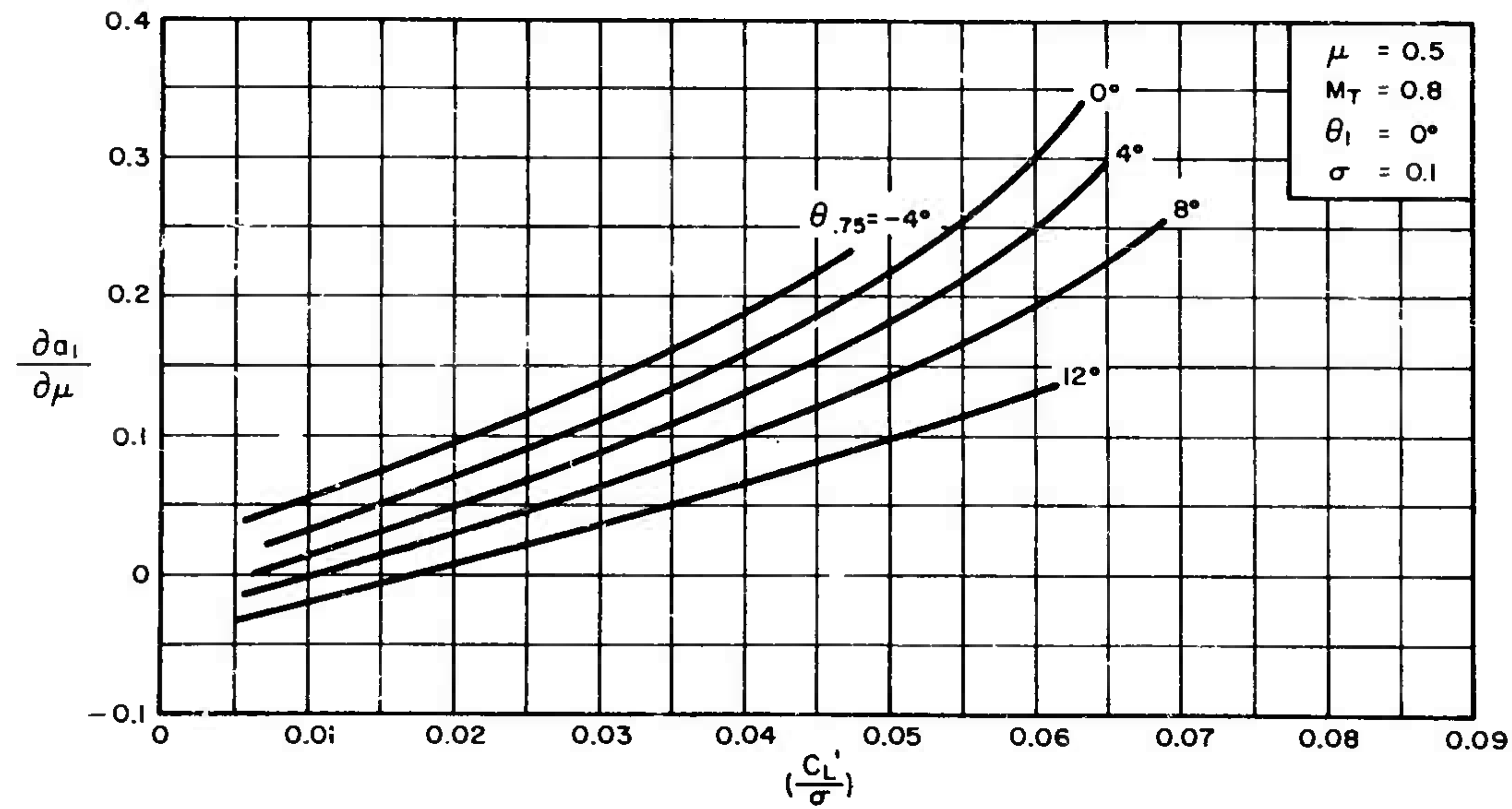

Figure 5. Continued

(c) $\mu = 0.5$

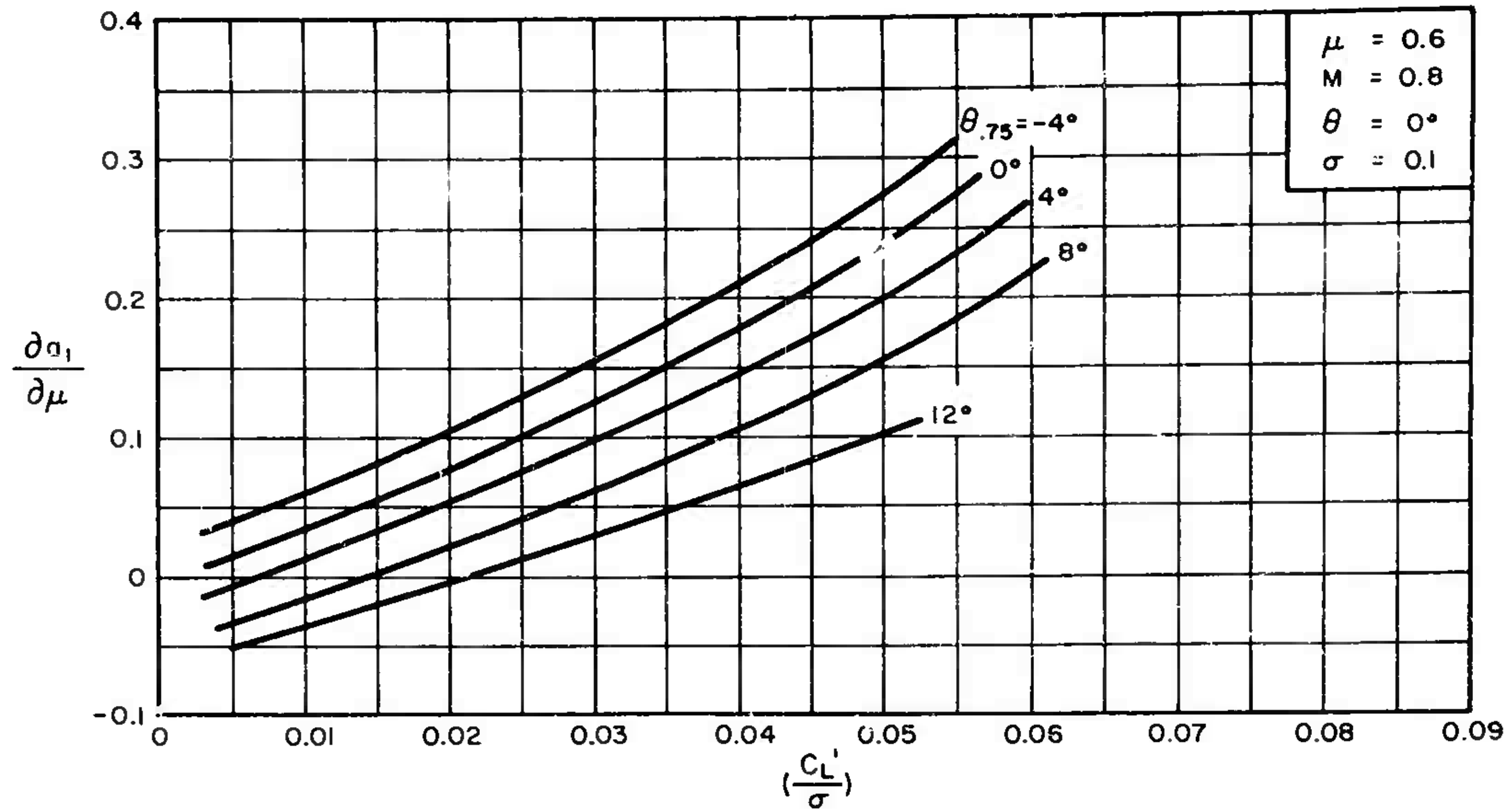

Figure 5. Continued

(d) $\mu = 0.6$

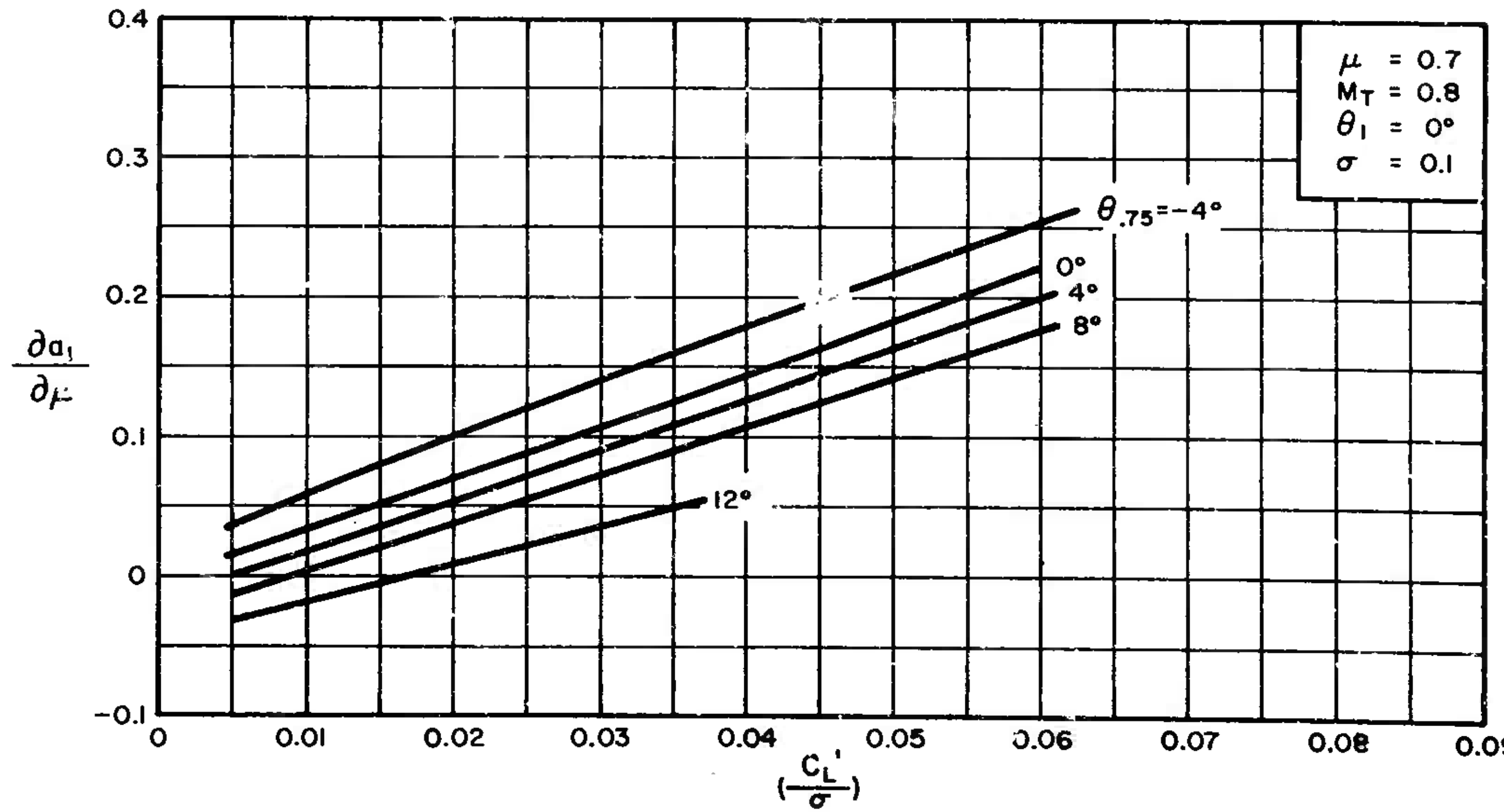

Figure 5. Continued

(e) $\mu = 0.7$

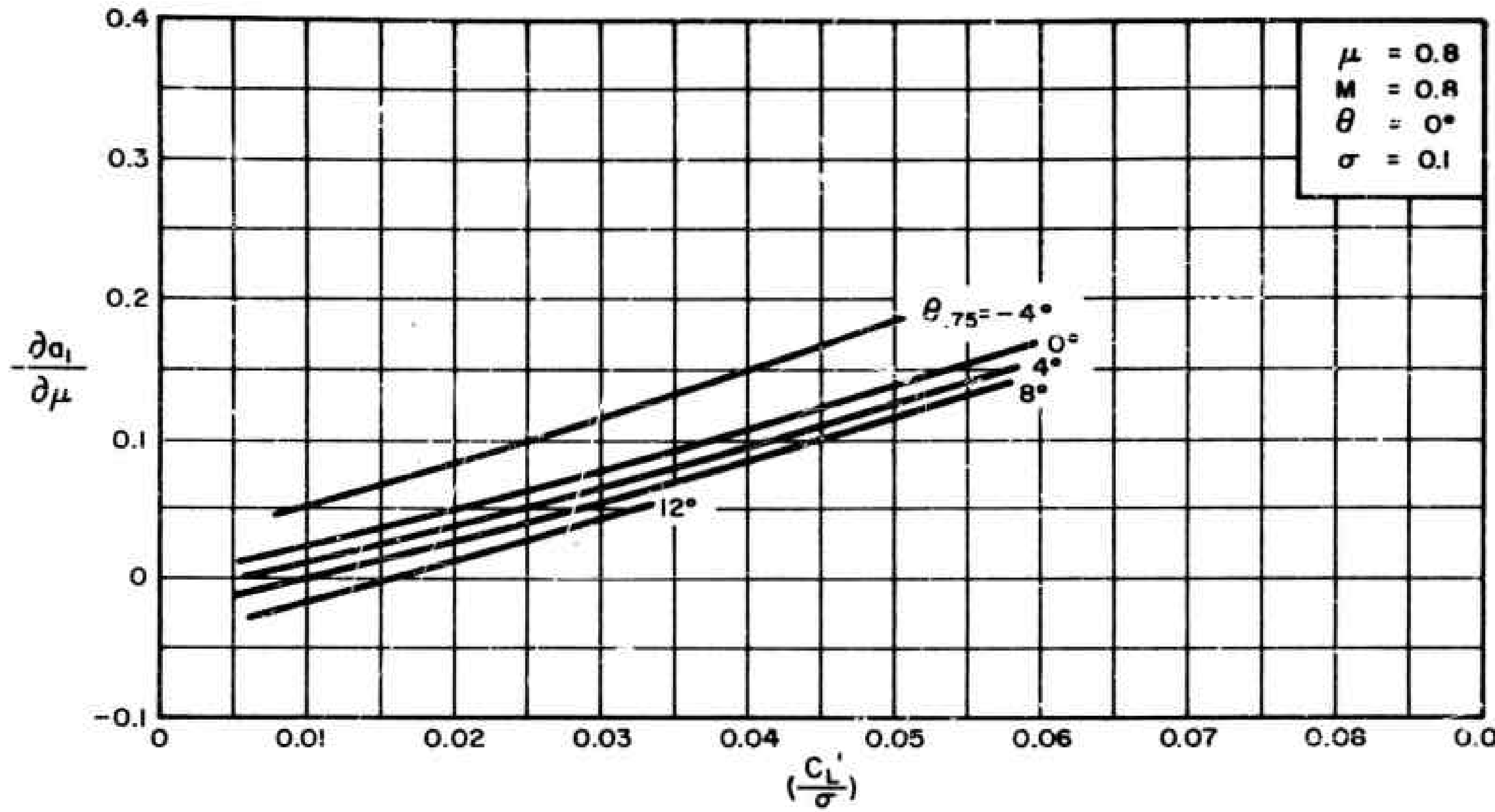

Figure 5. Continued

(f) μ = 0.8

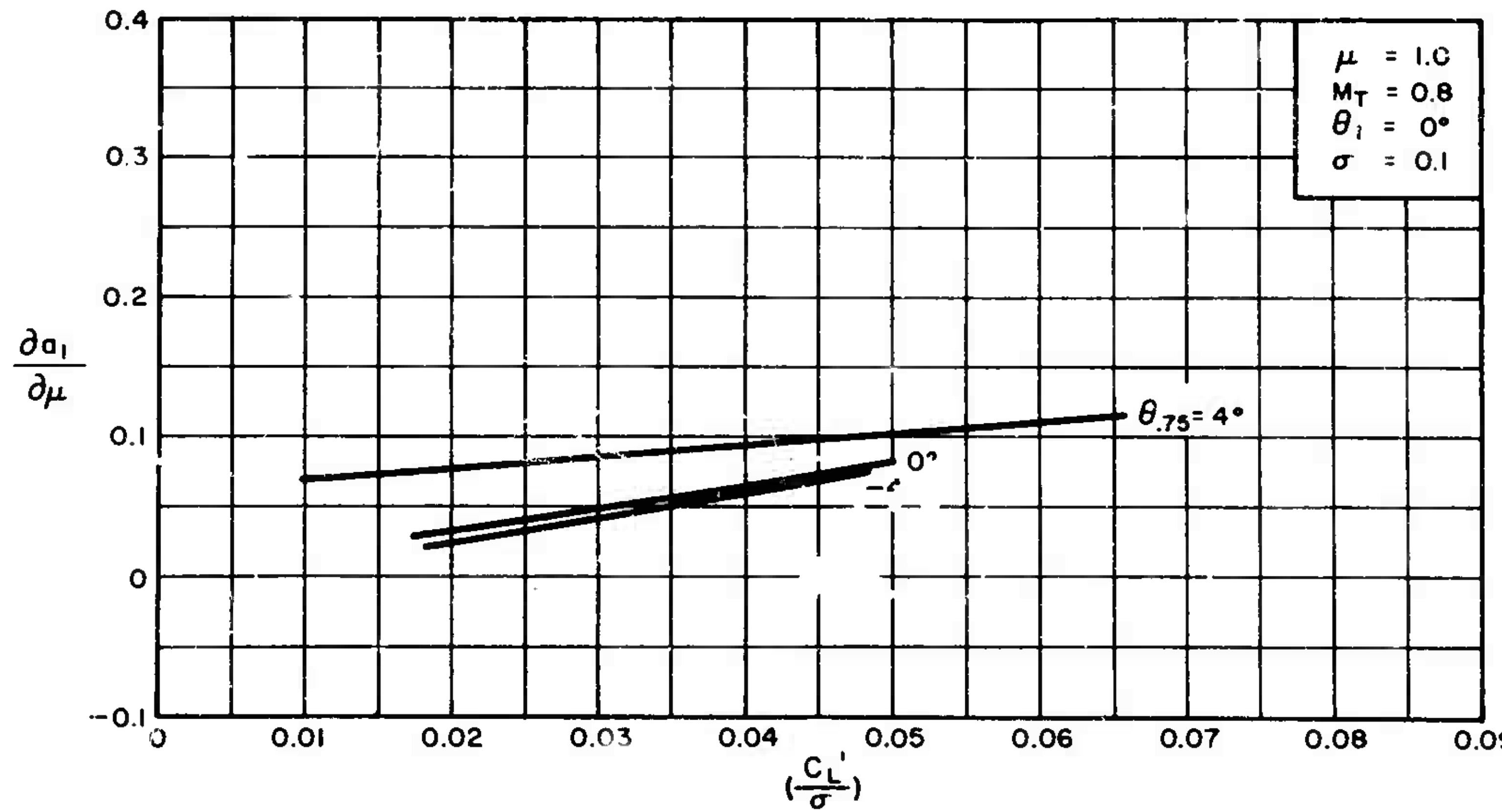

Figure 5. Concluded

(g) μ = 1.0

7.5.1.5 $\frac{\partial b_1}{\partial \mu}$ for $\sigma = 0.1$, $\theta_1 = 0°$, and $M_T = 0.8$

Figures 6(a) through 6(g) present the isolated rotor derivative $\partial b_1/\partial \mu$ as a function of C_L'/σ for constant values of $\theta_{.75}$ and a range of tip speed ratios from $\mu = 0.3$ through $\mu = 1.0$. The values of the above derivatives were extracted from the theoretical rotor performance data of Reference 1 by graphically obtaining the slopes of the b_1 vs. μ relationships for constant values of $\theta_{.75}$ and α_C. These derivatives are specifically applicable to rotors having Lock inertia number $\gamma = 8.0$. However, since the lateral flapping angle b_1 is essentially proportional to γ, a correction factor of $\gamma/8.0$ may be utilized to compute $\partial b_1/\partial \mu$ derivatives applicable to rotors having γ values other than 8.0. Thus:

$$\left(\frac{\partial b_1}{\partial \mu}\right)_\gamma = \frac{\gamma}{8.0}\left(\frac{\partial b_1}{\partial \mu}\right)_{\gamma = 8.0}$$

The $\frac{\partial b_1}{\partial \mu}$ derivatives for $\mu \leq 0.2$ can be computed by using the following equation:

$$\frac{\partial b_1}{\partial \mu} = \gamma\left[\lambda\left(0.2091 - \frac{\mu^2}{3}\right) + \frac{\partial \lambda}{\partial \mu}(t_{17}) + \theta_{.75}(0.1388 + 0.2425\mu^2)\right]$$

where $\frac{\partial \lambda}{\partial \mu}$ can be obtained from Subsection 7.5.1.6 and where values of t_{17} can be obtained from Reference 3.

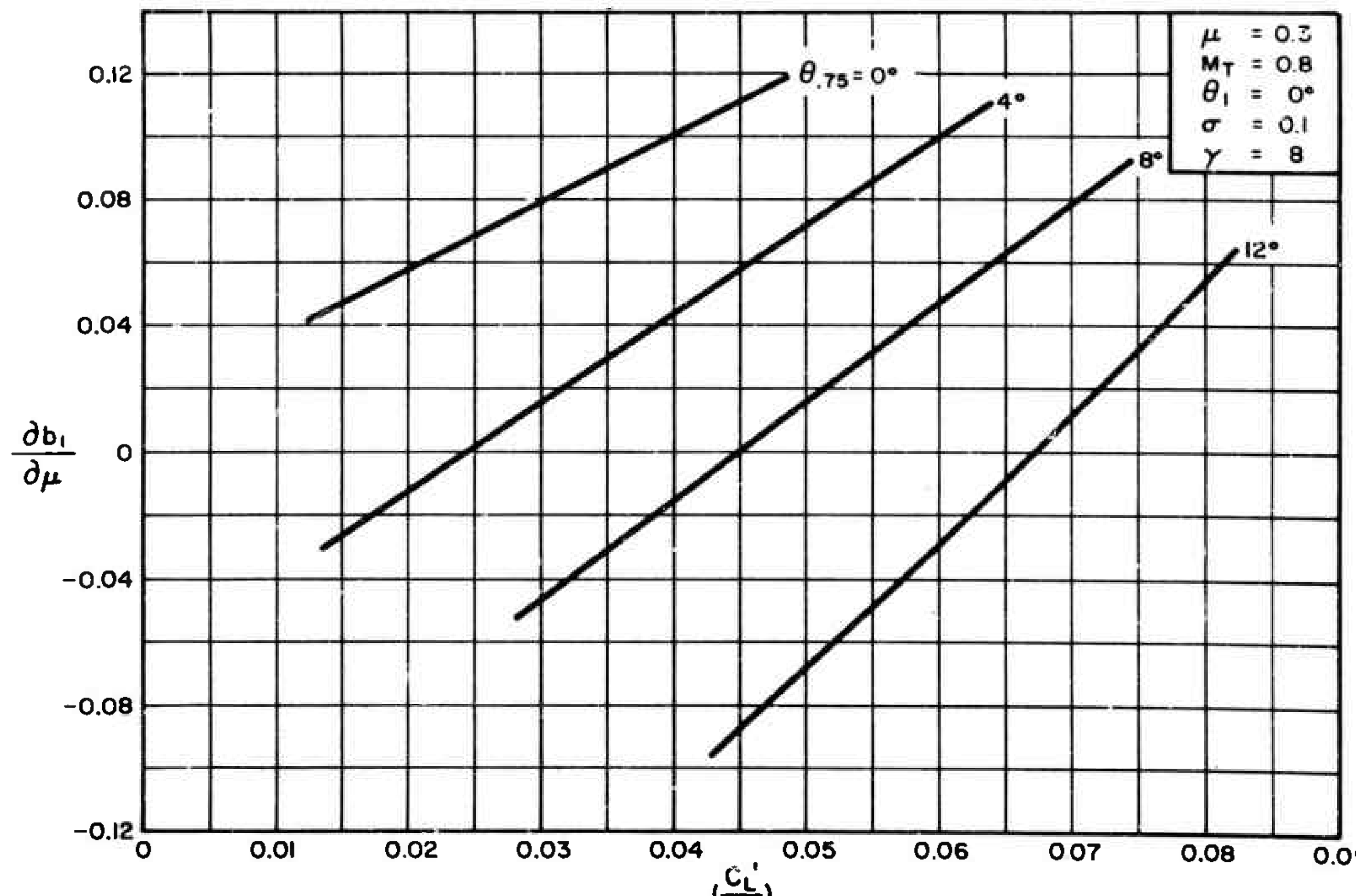

Figure 6. Variation of $\frac{\partial b_1}{\partial \mu}$ With $\frac{C_L}{\sigma}$ for Constant Values of $\theta_{.75}$

(a) $\mu = 0.3$

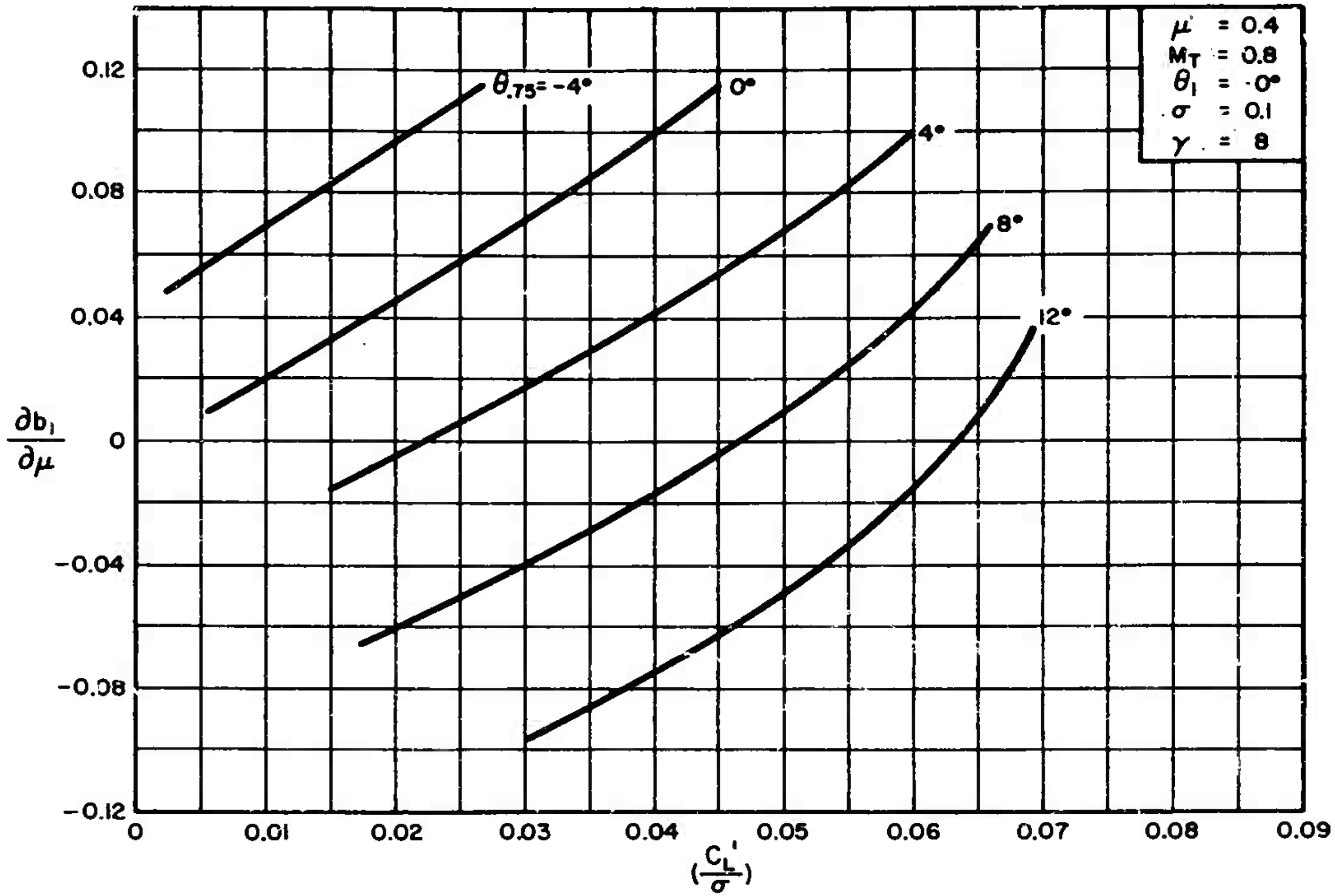

Figure 6. Continued

(b) $\mu = 0.4$

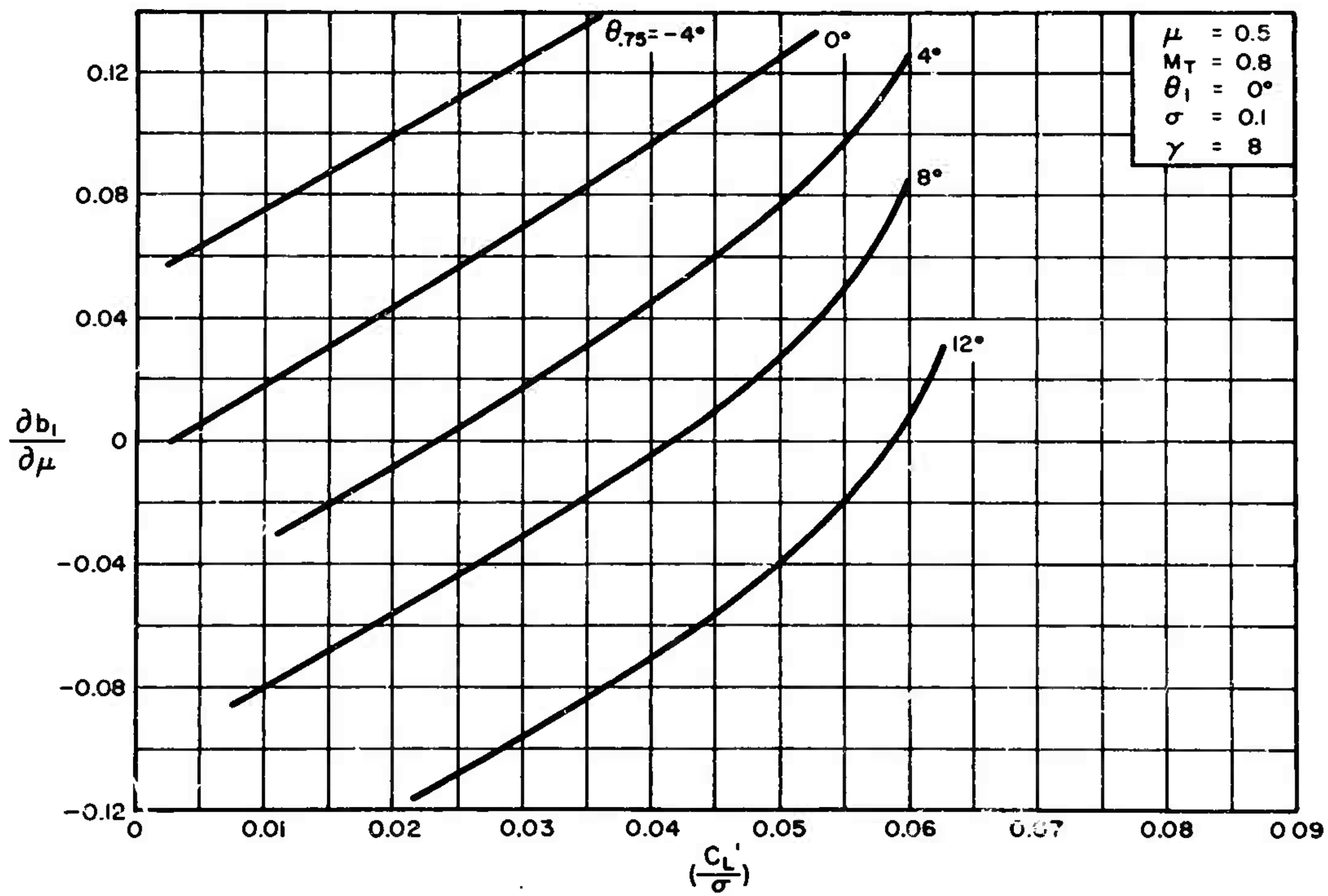

Figure 6. Continued

(c) $\mu = 0.5$

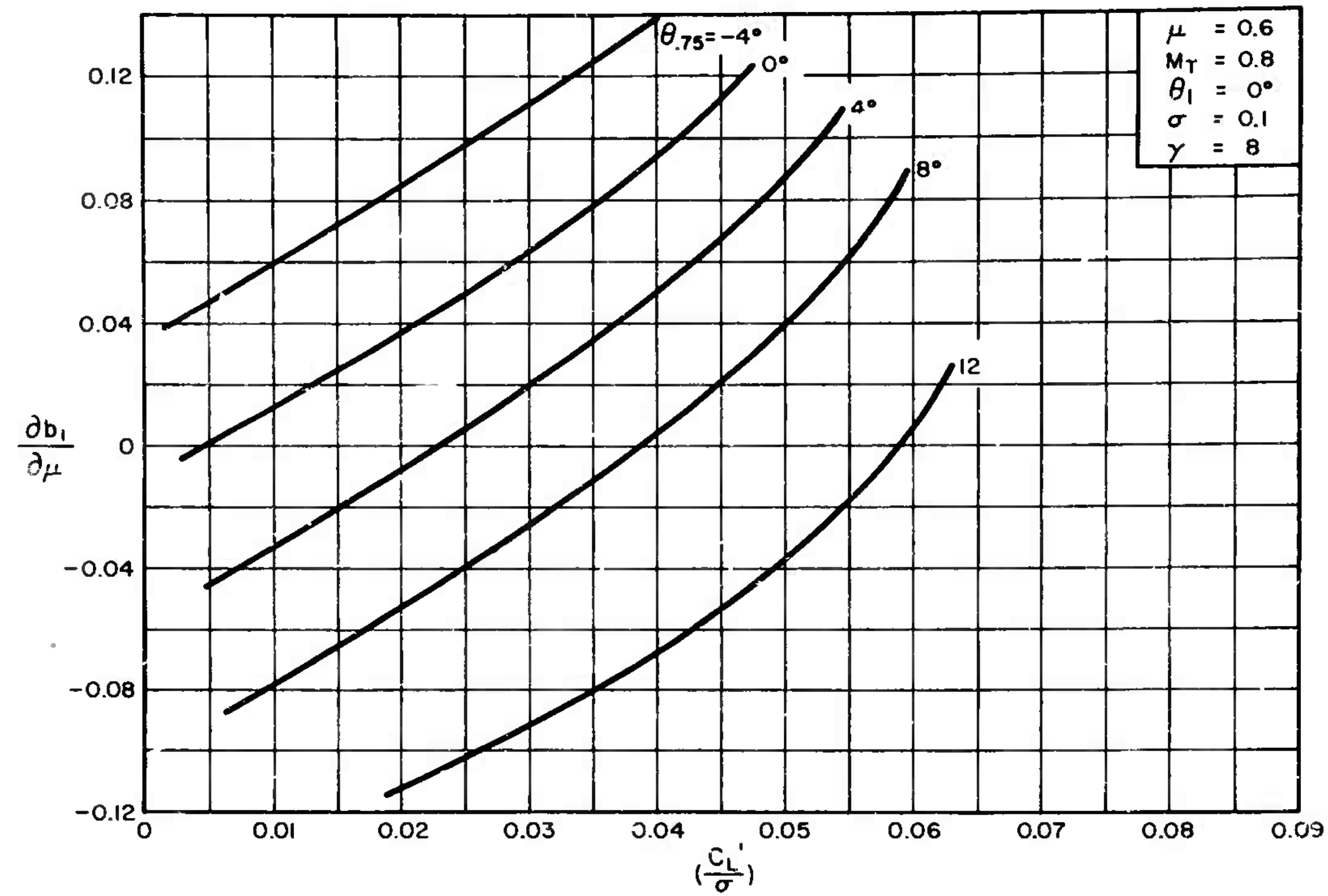

Figure 6. Continued

(d) $\mu = 0.6$

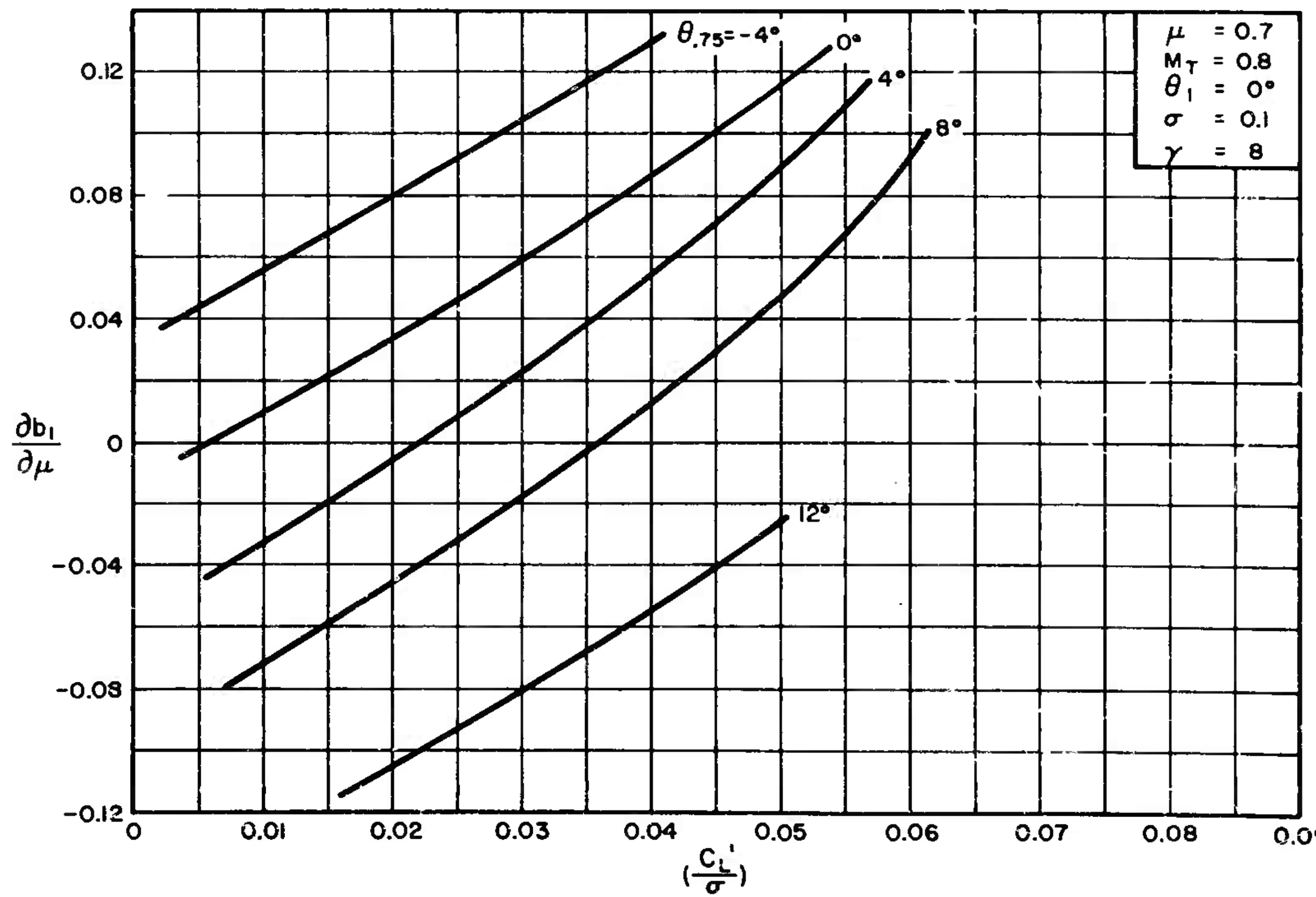

Figure 6. Continued

(e) μ = 0.7

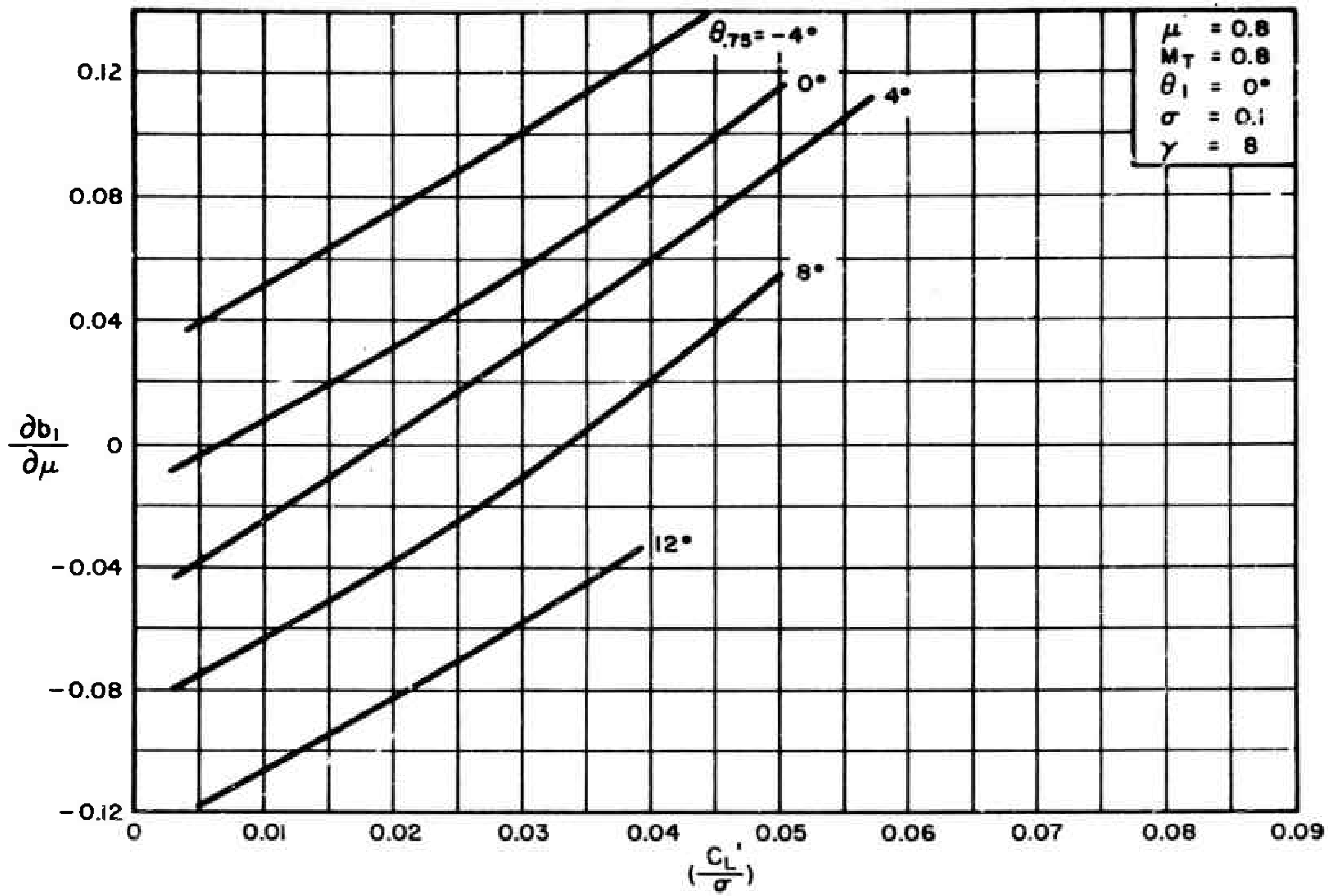

Figure 6. Continued

(f) $\mu = 0.8$

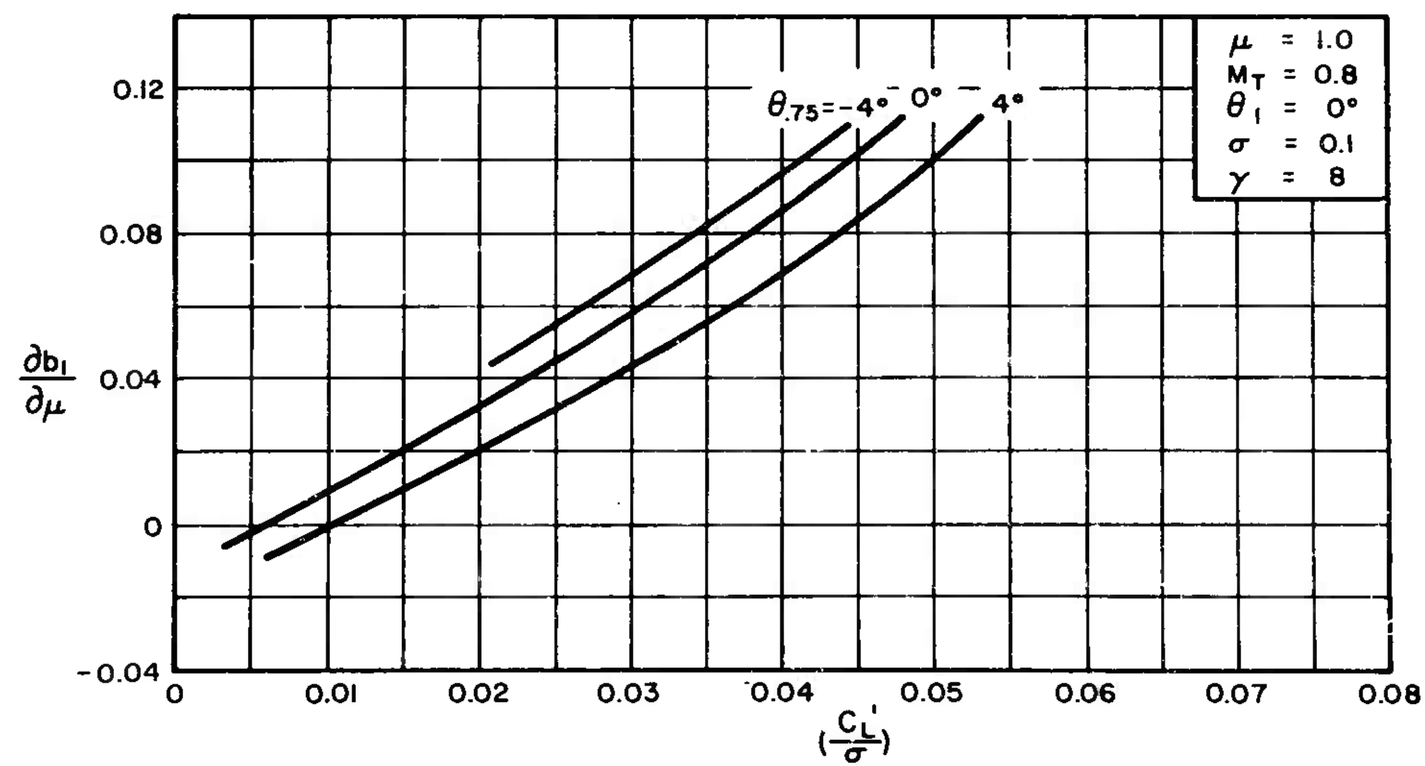

Figure 6. Concluded

(g) μ = 1.0

7.5.1.6 $\frac{\partial\lambda}{\partial\mu}$ for $\sigma = 0.1$, $\theta_1 = 0°$ and $M_T = 0.8$

Figures 7(a) to 7(i) present the isolated rotor derivative $\partial\lambda/\partial\mu$ as a function of C_L'/σ for constant values of $\theta_{.75}$ for a range of μ values of $\mu = 0.1$ through $\mu = 1.0$. The values of $\partial\lambda/\partial\mu$ for $\mu = 0.1$ and 0.2 were obtained directly from Reference 2. The values for $\mu \geq 0.3$ were extracted from the theoretical rotor performance data of Reference 1 by graphically obtaining the slopes of the λ vs. μ relationships for constant values of $\theta_{.75}$ and α_C .

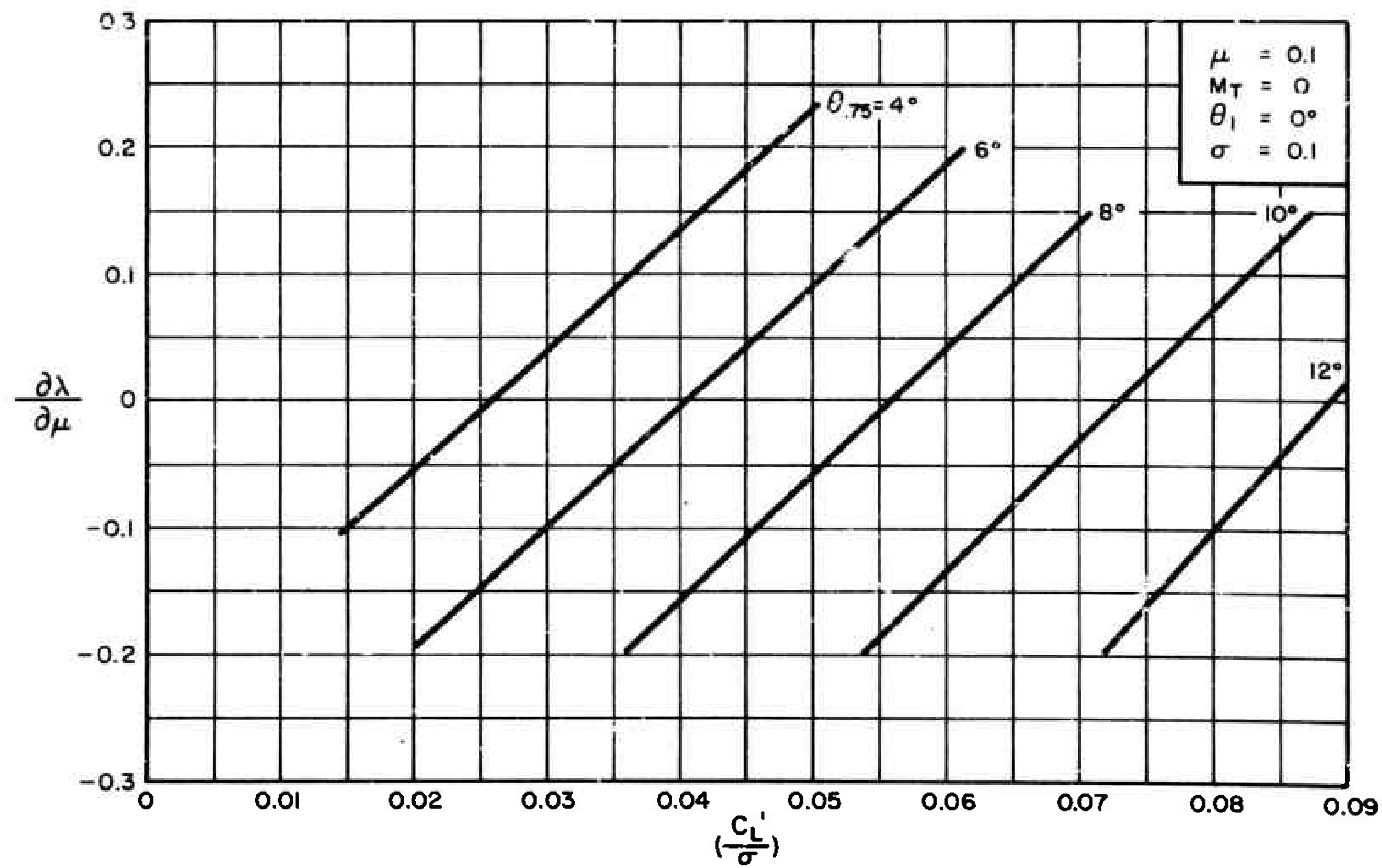

Figure 7. Variation of $\frac{\partial\lambda}{\partial\mu}$ With $\frac{C_L'}{\sigma}$ for Constant Values of $\theta_{.75}$

(a) $\mu = 0.1$

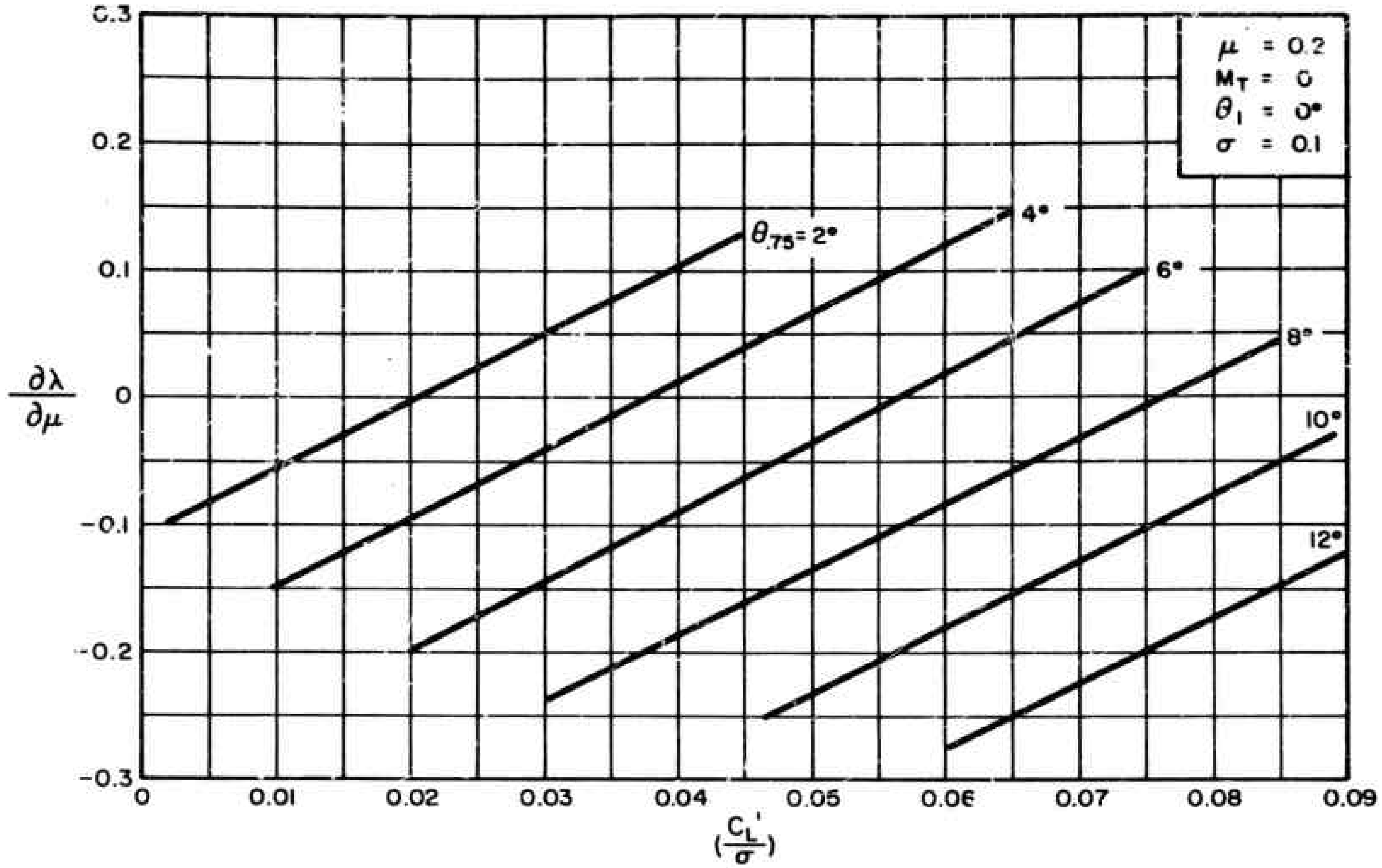

Figure 7. Continued

(b) $\mu = 0.2$

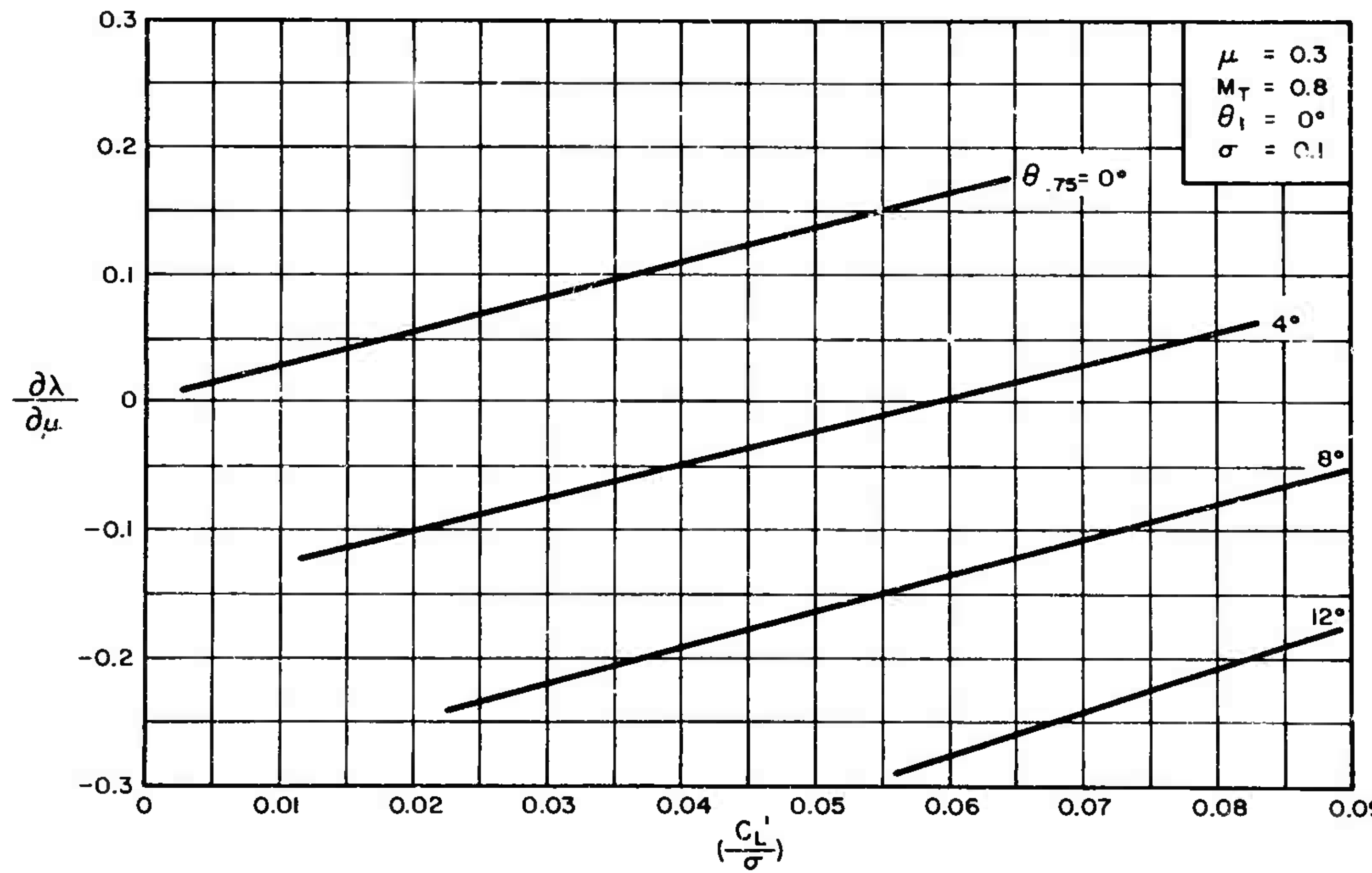

Figure 7. Continued

(c) $\mu = 0.3$

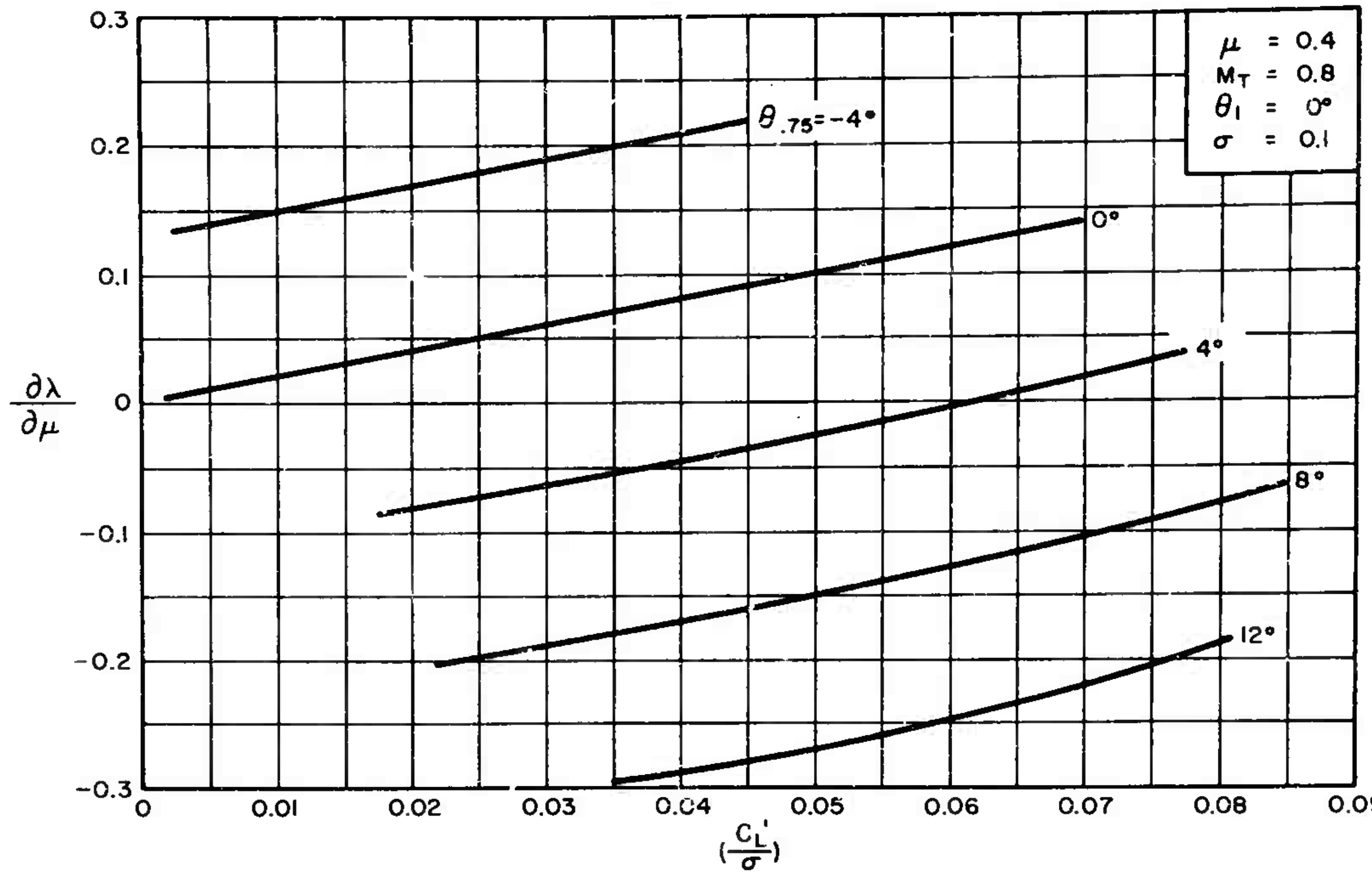

Figure 7. Continued

(d) $\mu = 0.4$

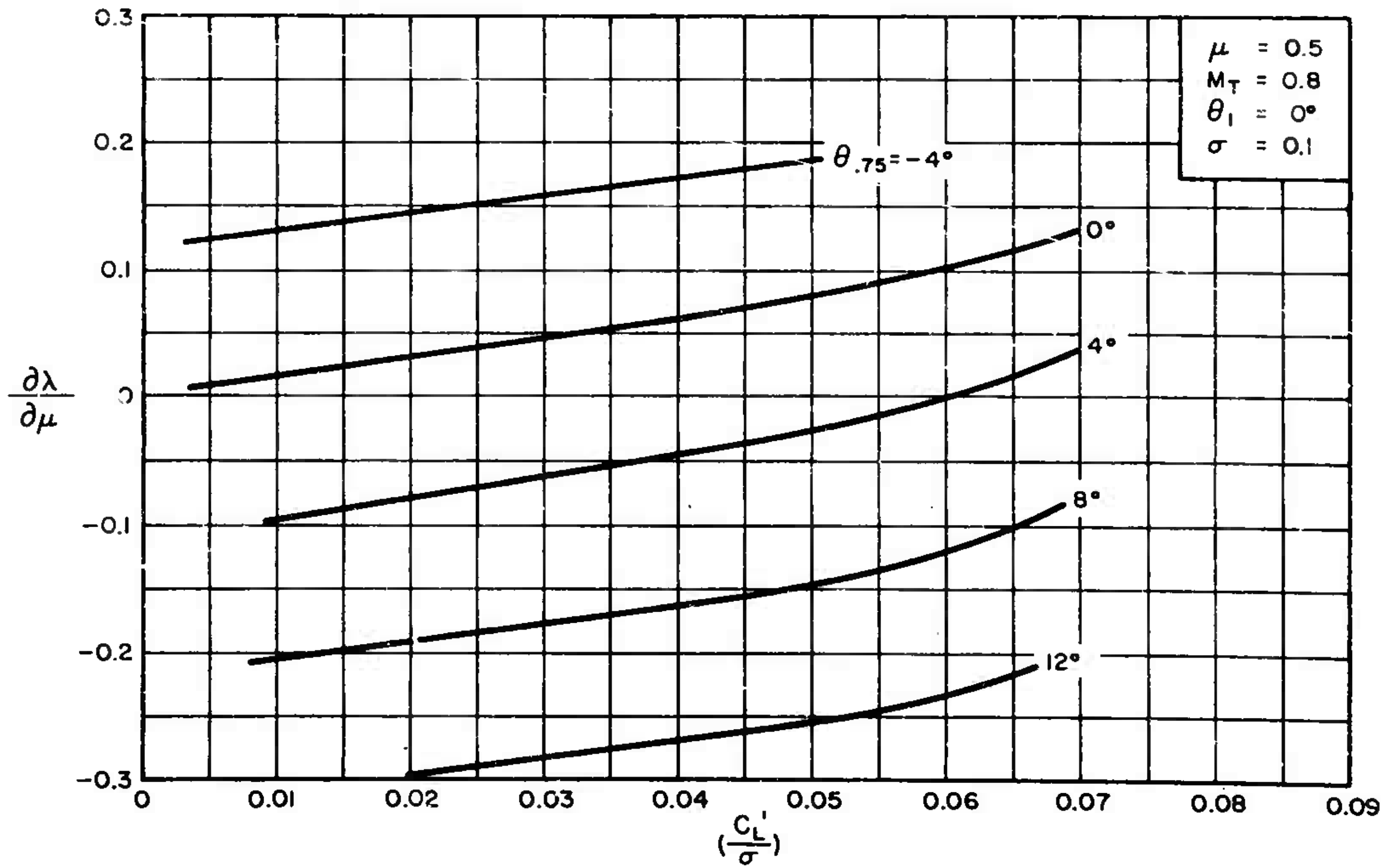

Figure 7. Continued

(e) $\mu = 0.5$

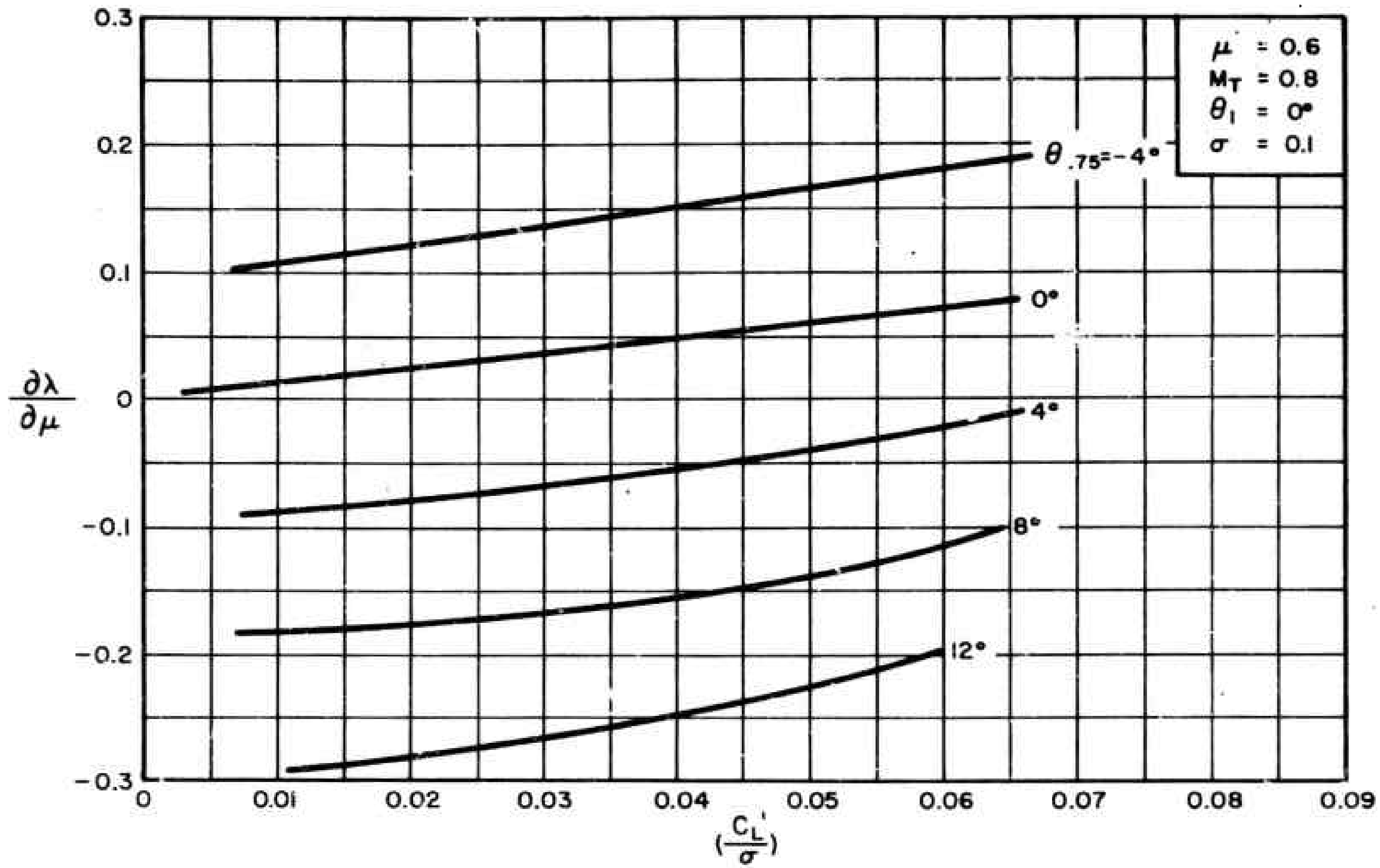

Figure 7. Continued

(f) $\mu = 0.6$

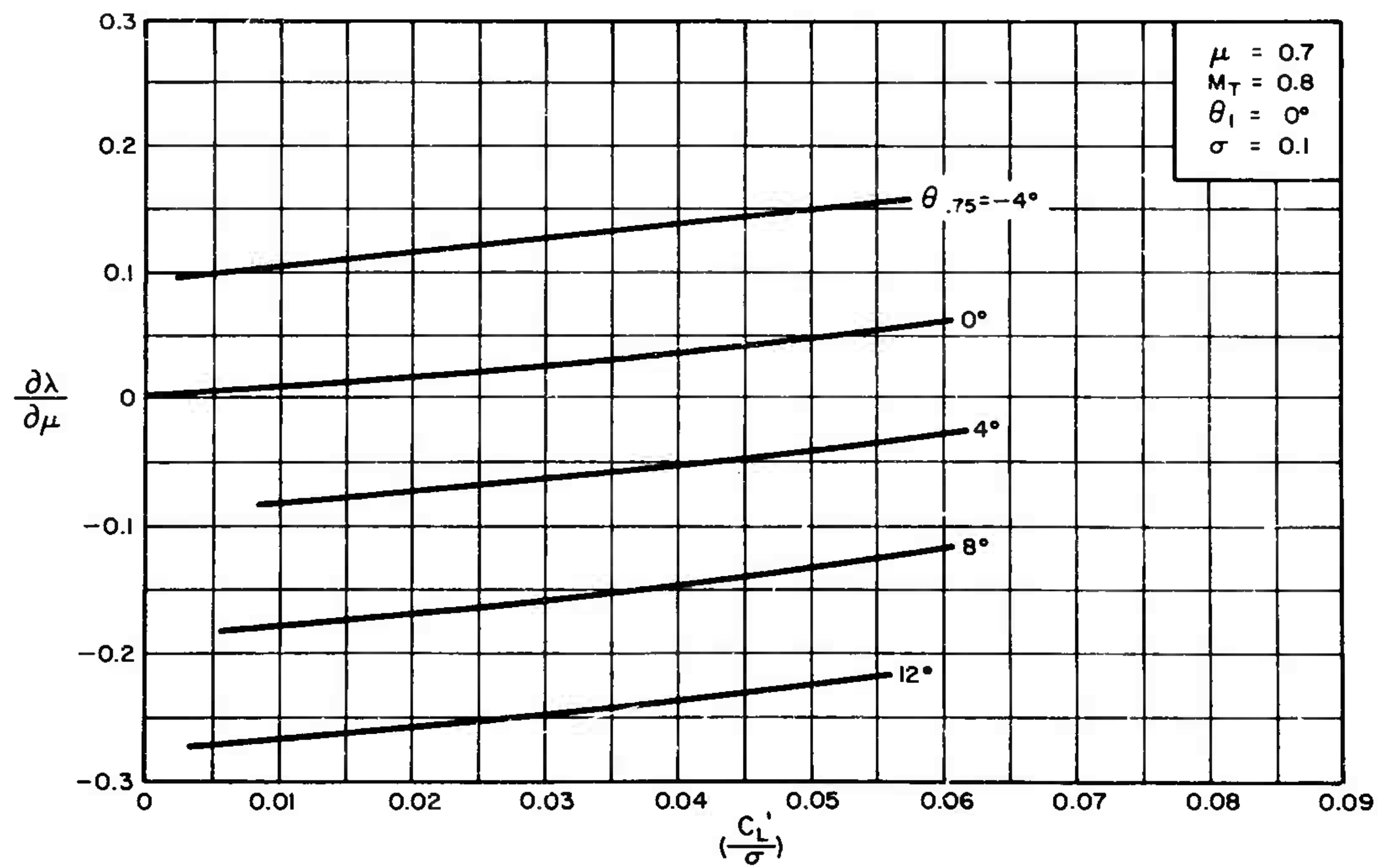

Figure 7. Continued

(g) μ = 0.7

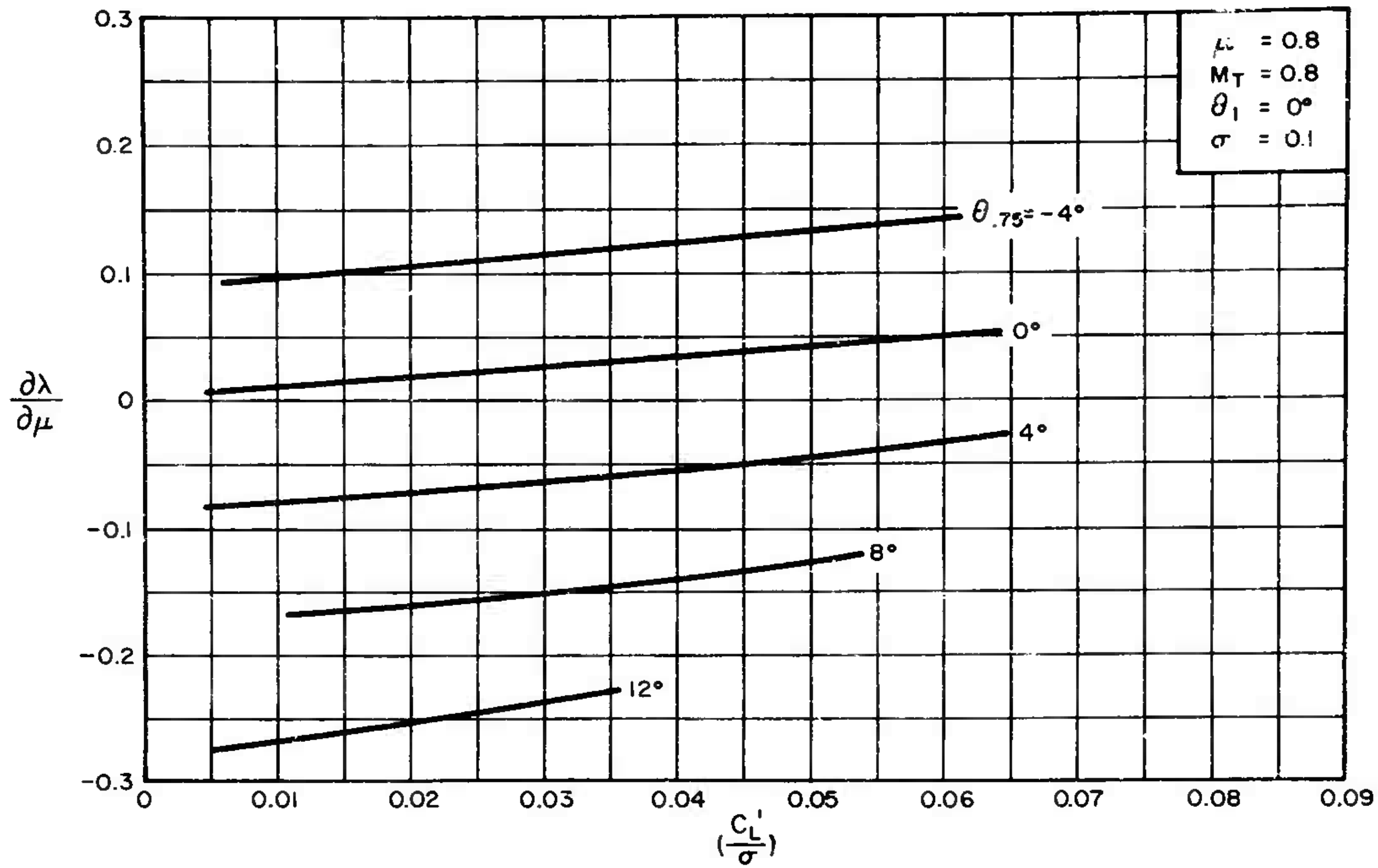

Figure 7. Continued

(h) $\mu = 0.8$

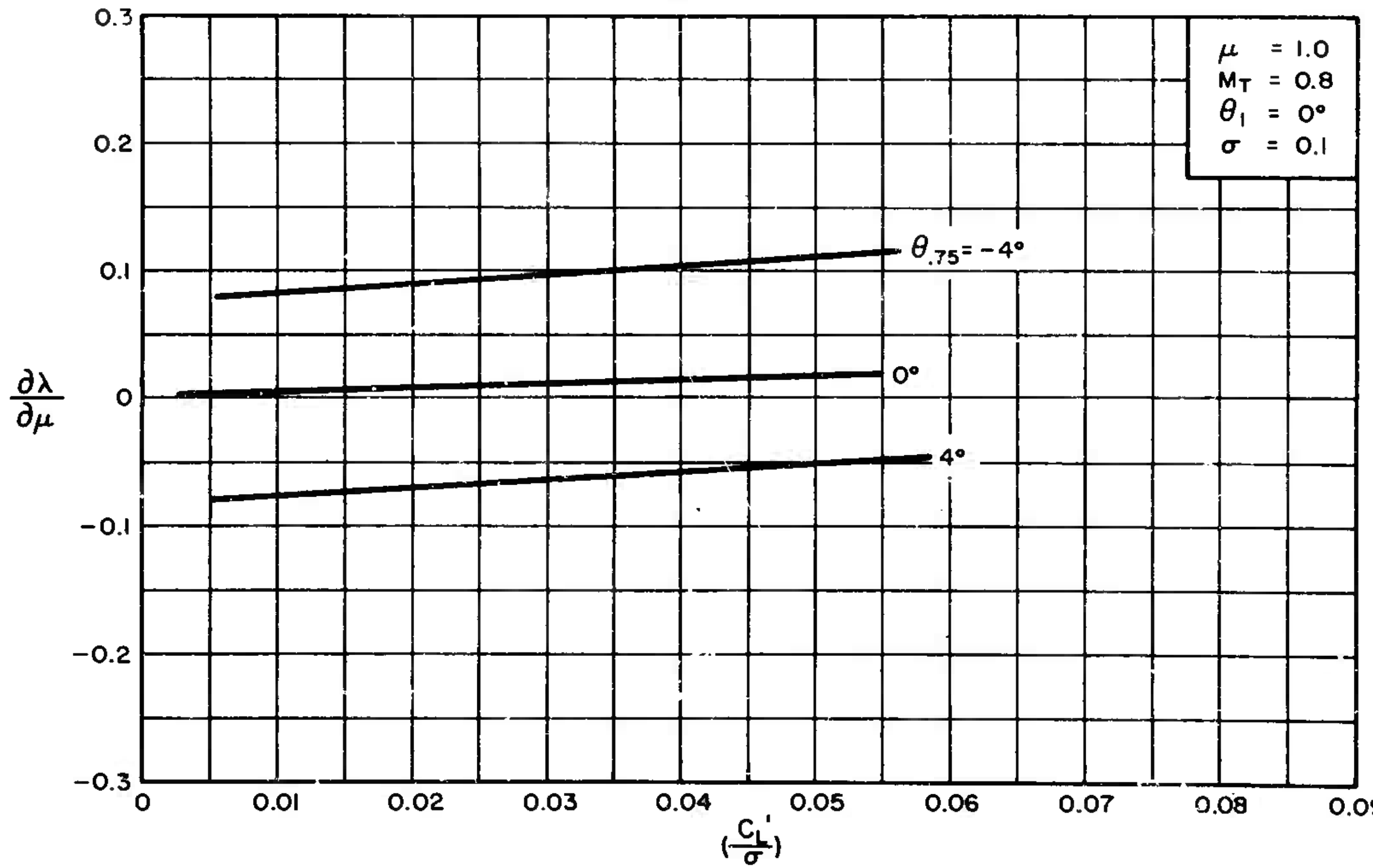

Figure 7. Concluded

(i) $\mu = 1.0$

7.5.1.7 $\frac{\partial(\frac{C_Y'}{\sigma})}{\partial\mu}$ for All Values of σ, θ_1, and M_T

Reference 1 and other reviewed reports do not include the calculated data required to determine the rotor Y-force derivatives. It is therefore suggested that the classical Bailey theory be utilized for this purpose. If the above theory is used, the following expression for $\partial(C_Y'/\sigma)/\partial\mu$ can be derived:

$$\frac{\partial(\frac{C_Y'}{\sigma})}{\partial\mu} = \frac{a}{2}\left\{\frac{\partial a_1}{\partial\mu}\left[a_1(\frac{1}{6}-\mu^2)-\frac{3}{4}\mu(\theta_{.75}+2\lambda)\right]\right.$$

$$+\frac{\partial a_1}{\partial\mu}\left[a_0(\frac{1}{6}-\mu^2)+\frac{1}{4}\mu b_1\right]$$

$$+\frac{\partial b_1}{\partial\mu}\left[\theta_{.75}(\frac{1}{3}+\frac{3}{8}\mu^2)+\lambda(\frac{3}{4}+\frac{1}{8}\mu^2)+\frac{1}{4}\mu a_1\right]$$

$$+\frac{\partial\lambda}{\partial\mu}\left[b_1(\frac{3}{4}+\frac{1}{8}\mu^2)-\frac{3}{2}\mu a_0\right]$$

$$\left.+\frac{1}{4}b_1\left[\mu(3\theta_{.75}+\lambda)+a_1\right]-a_0\left[\frac{3}{4}\theta_{.75}+\frac{2}{3}\lambda-2\mu a_1\right]\right\}$$

where

$$\frac{\partial a_0}{\partial\mu} = \frac{\gamma}{2}\left[\frac{\theta_{.75}}{2}\mu+\frac{1}{3}\frac{\partial\lambda}{\partial\mu}\right]$$

and where $\partial a_1/\partial\mu$, $\partial b_1/\partial\mu$, and $\partial\lambda/\partial\mu$ are given in Subsections 7.5.1.4, 7.5.1.5, and 7.5.1.6, respectively.

The above derivative is applicable to all values of σ, θ_1, and M_T, provided that the pertinent rotor parameters comprising it are evaluated at the required condition.

7.5.2 Isclated Rotor Derivatives With Respect to Rotor Angle of Attack (α_C)

7.5.2.1 $\dfrac{\partial(\frac{C_L'}{\sigma})}{\partial \alpha_C}$ for $\sigma = 0.1$, $\theta_1 = 0°$, and $M_T = 0.8$

Figures 8(a) through 8(d) present the isolated rotor derivative $\partial(C_L'/\sigma)/\partial\alpha_C$ as a function of C_L'/σ for constant values of μ covering a range of collective pitch settings from $\theta_{.75} = 0$ to $\theta_{.75} = 12°$.

The derivatives for $\mu \leq 0.2$ were extracted from the data of Reference 2 by using the following expression:

$$\frac{\partial(\frac{C_L'}{\sigma})}{\partial\alpha_C} = \left[\frac{\partial(\frac{C_T}{\sigma})}{\partial\alpha_C} - \frac{C_H}{\sigma}\right]\cos\alpha_C - \left[\frac{\partial(\frac{C_H}{\sigma})}{\partial\alpha_C} + \frac{C_T}{\sigma}\right]\sin\alpha_C$$

The values of $\partial(C_T/\sigma)/\partial\alpha_C$ and $\partial(C_H/\sigma)/\partial\alpha_C$, obtained from Reference 2, are found to be practically independent of $\theta_{.75}$ and C_L'/σ variations.

The values of $\partial(C_L'/\sigma)/\partial\alpha_C$ for $\mu \geq 0.3$ were extracted from the theoretical data of Reference 1 by graphically obtaining slopes of the C_L'/σ vs. α_C relationships for constant values of μ and $\theta_{.75}$.

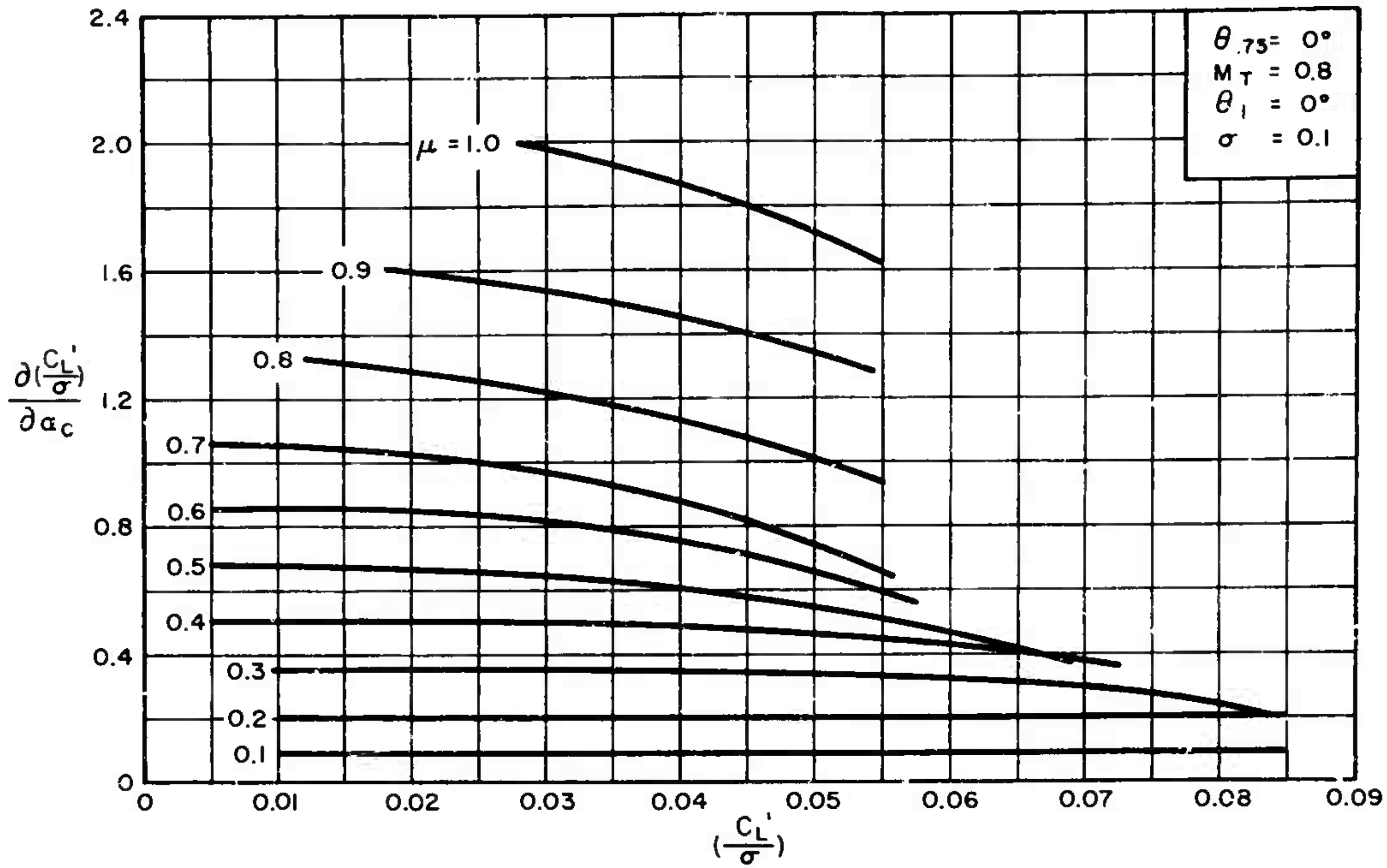

Figure 8. Variation of $\frac{\partial(\frac{C_L'}{\sigma})}{\partial \alpha_c}$ With $\frac{C_L'}{\sigma}$ for Constant Values of μ

(a) $\theta_{.75} = 0°$

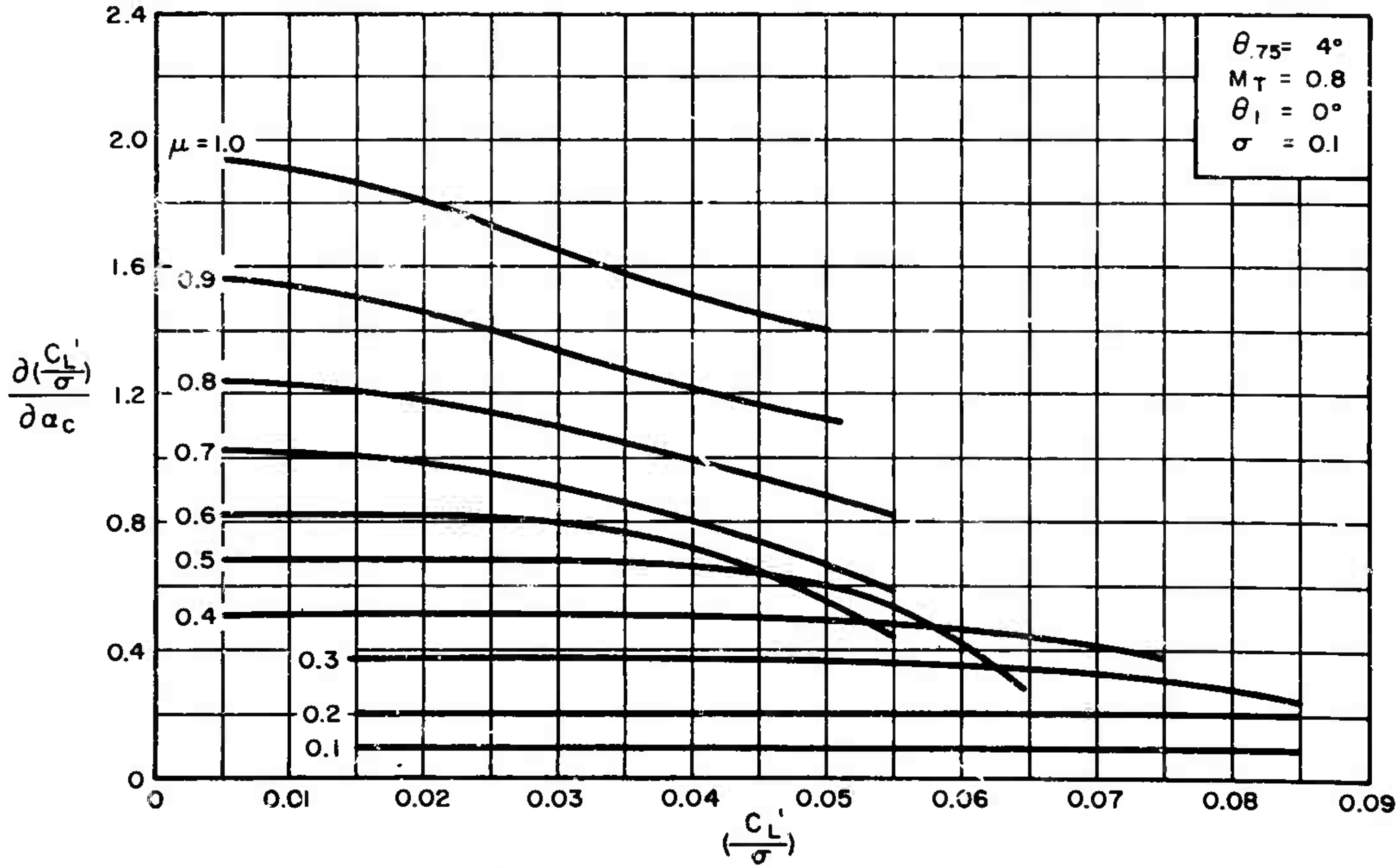

Figure 8. Continued

(b) $\theta_{.75} = 4°$

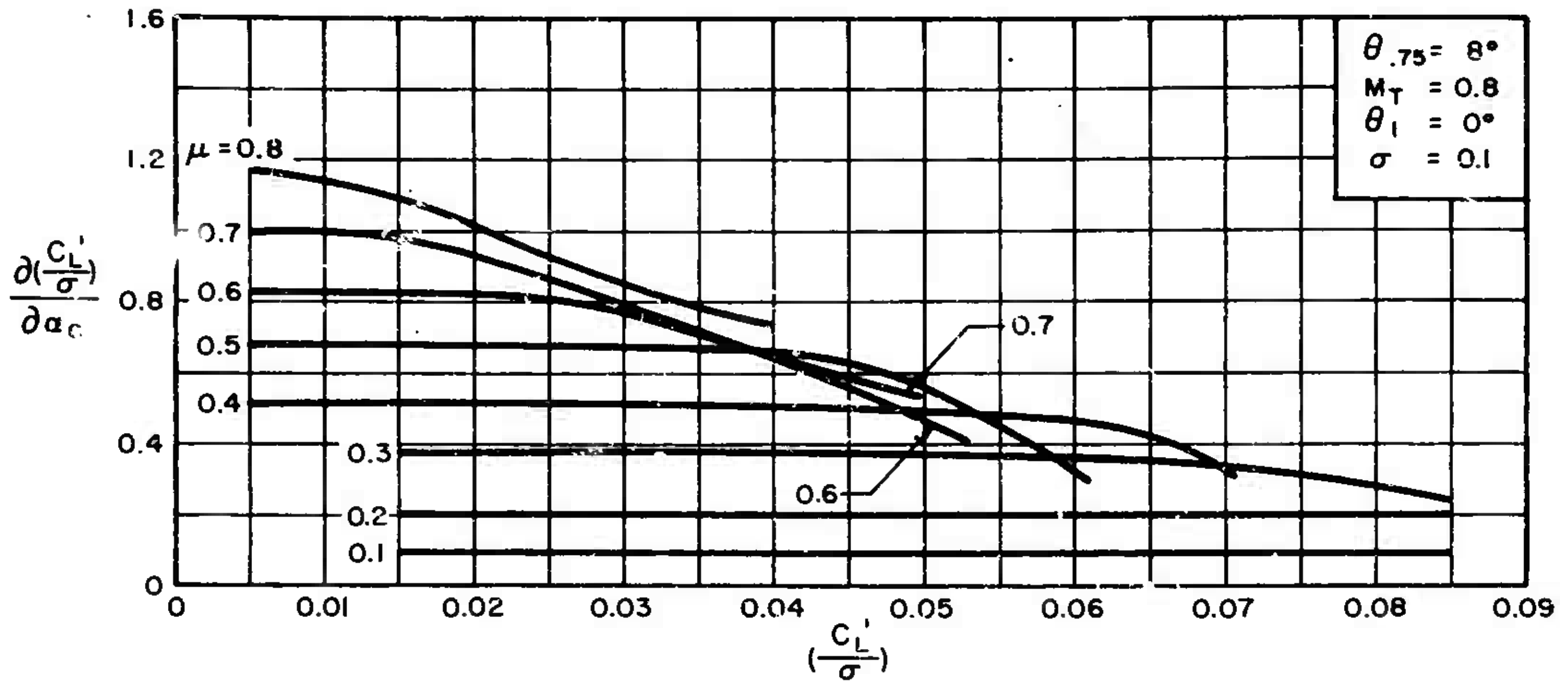

Figure 8. Continued

(c) $\theta_{.75} = 8°$

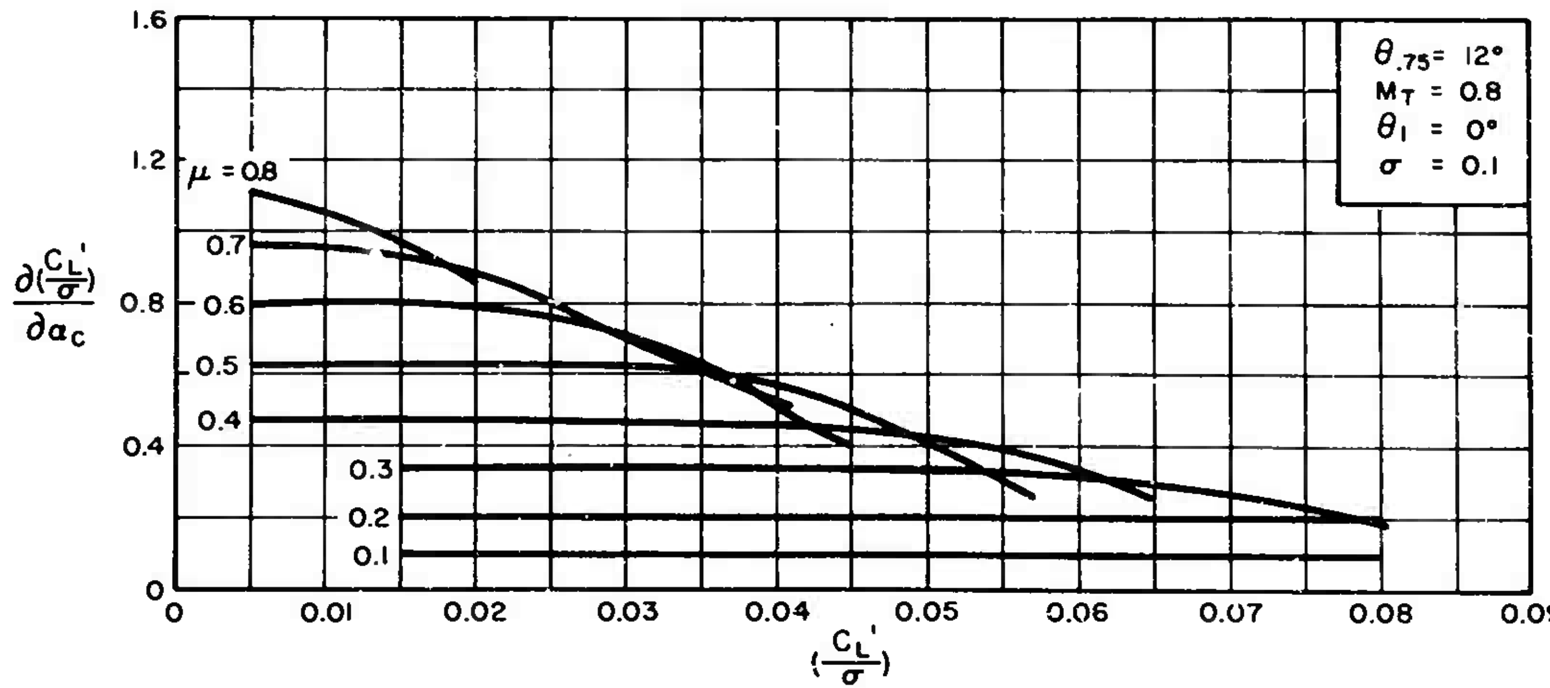

Figure 8. Concluded

(d) $\theta_{.75} = 12°$

7.5.2.2 $\dfrac{\partial(\frac{C_D'}{\sigma})}{\partial\alpha_C}$ for $\sigma = 0.1$, $\theta_1 = 0°$, and $M_T = 0.8$

Figures 9(a) through 9(i) present the isolated rotor derivative $\partial(C_D'/\sigma)/\partial\alpha_C$ as a function of C_L'/σ for constant values of $\theta_{.75}$ and a range of μ from $\mu = 0.1$ through $\mu = 1.0$.

The values of $\partial(C_D'/\sigma)/\partial\alpha_C$ for $\mu = 0.1$ and $\mu = 0.2$ were obtained from Reference 2 by utilizing the following equation:

$$\frac{\partial(\frac{C_D'}{\sigma})}{\partial\alpha_C} = \left[\frac{\partial(\frac{C_H}{\sigma})}{\partial\alpha_C} + \frac{C_T}{\sigma}\right]\cos\alpha_C + \left[\frac{\partial(\frac{C_T}{\sigma})}{\partial\alpha_C} - \frac{C_H}{\sigma}\right]\sin\alpha_C$$

where $\partial(C_T/\sigma)/\partial\alpha_C$ and $\partial(C_H/\sigma)/\partial\alpha_C$ were obtained directly from Reference 2.

The values of $\partial(C_D'/\sigma)/\partial\alpha_C$ for $\mu \geq 0.3$ were extracted from the theoretical rotor performance data of Reference 1 by graphically obtaining slopes of the C_D'/σ vs. α_C relationships for constant values of μ and $\theta_{.75}$.

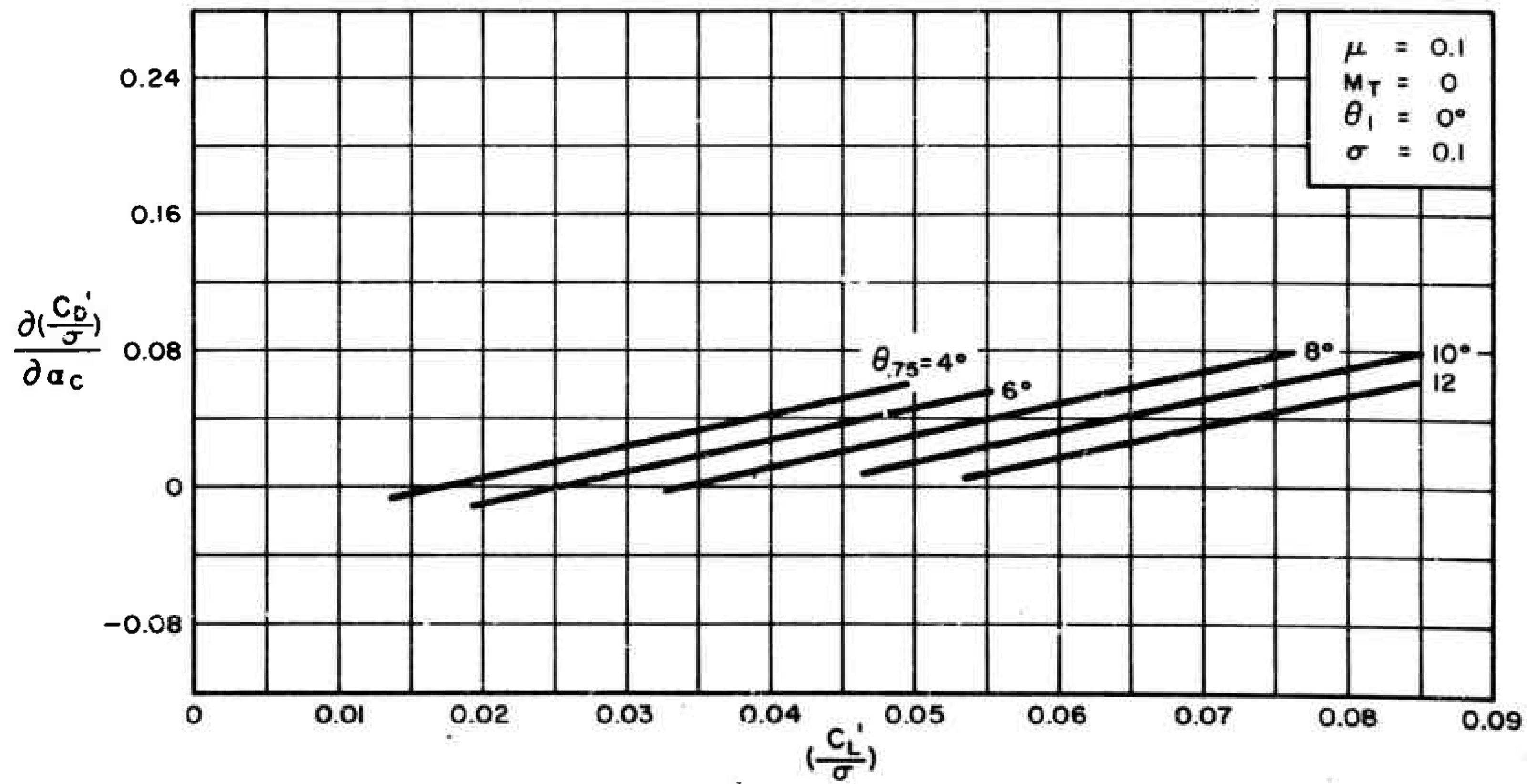

Figure 9. Variation of $\frac{\partial(\frac{C_D'}{\sigma})}{\partial a_c}$ With $\frac{C_L'}{\sigma}$ for Constant Values of $\theta_{.75}$

(a) $\mu = 0.1$

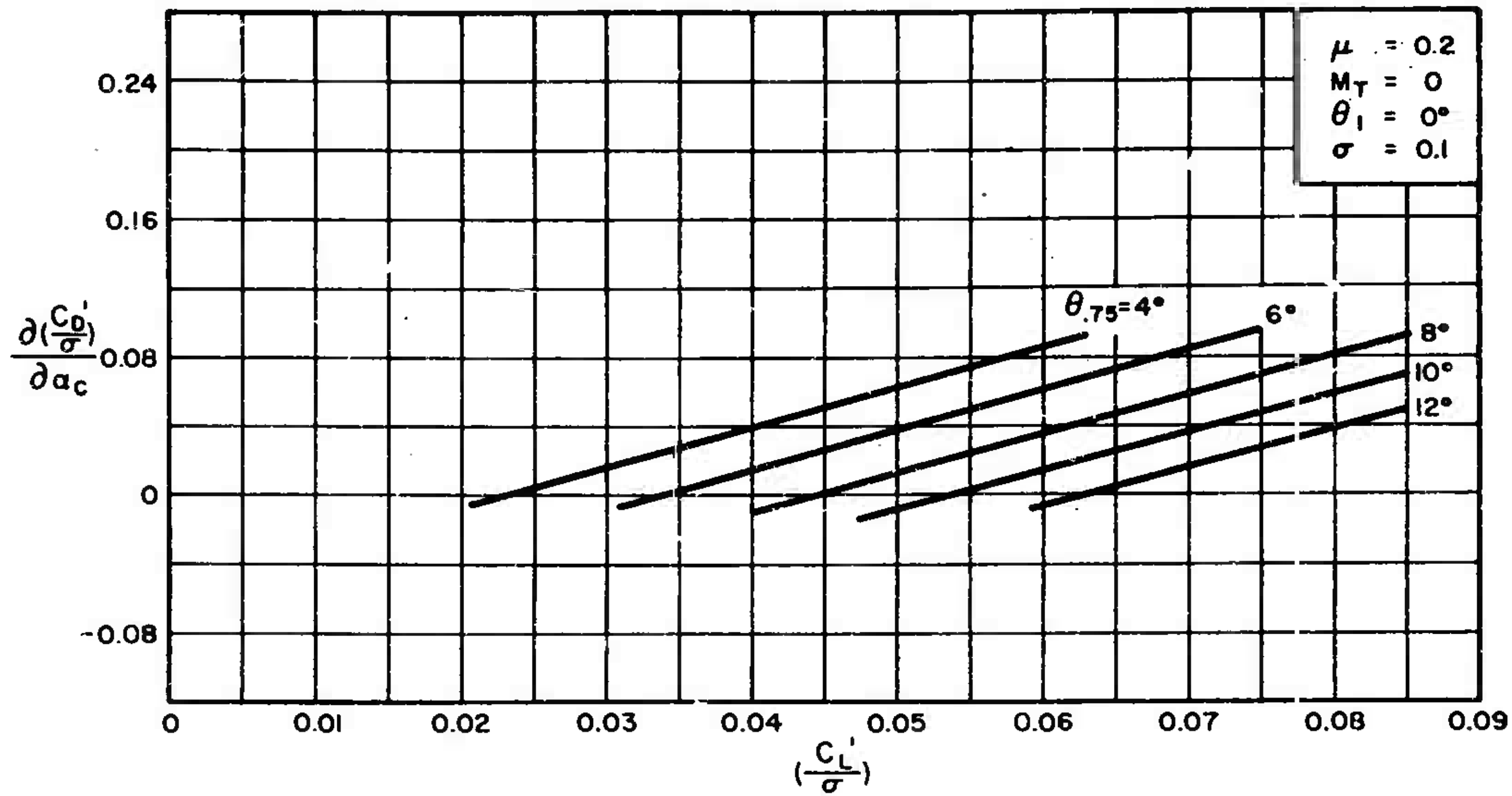

Figure 9. Continued
(b) $\mu = 0.2$

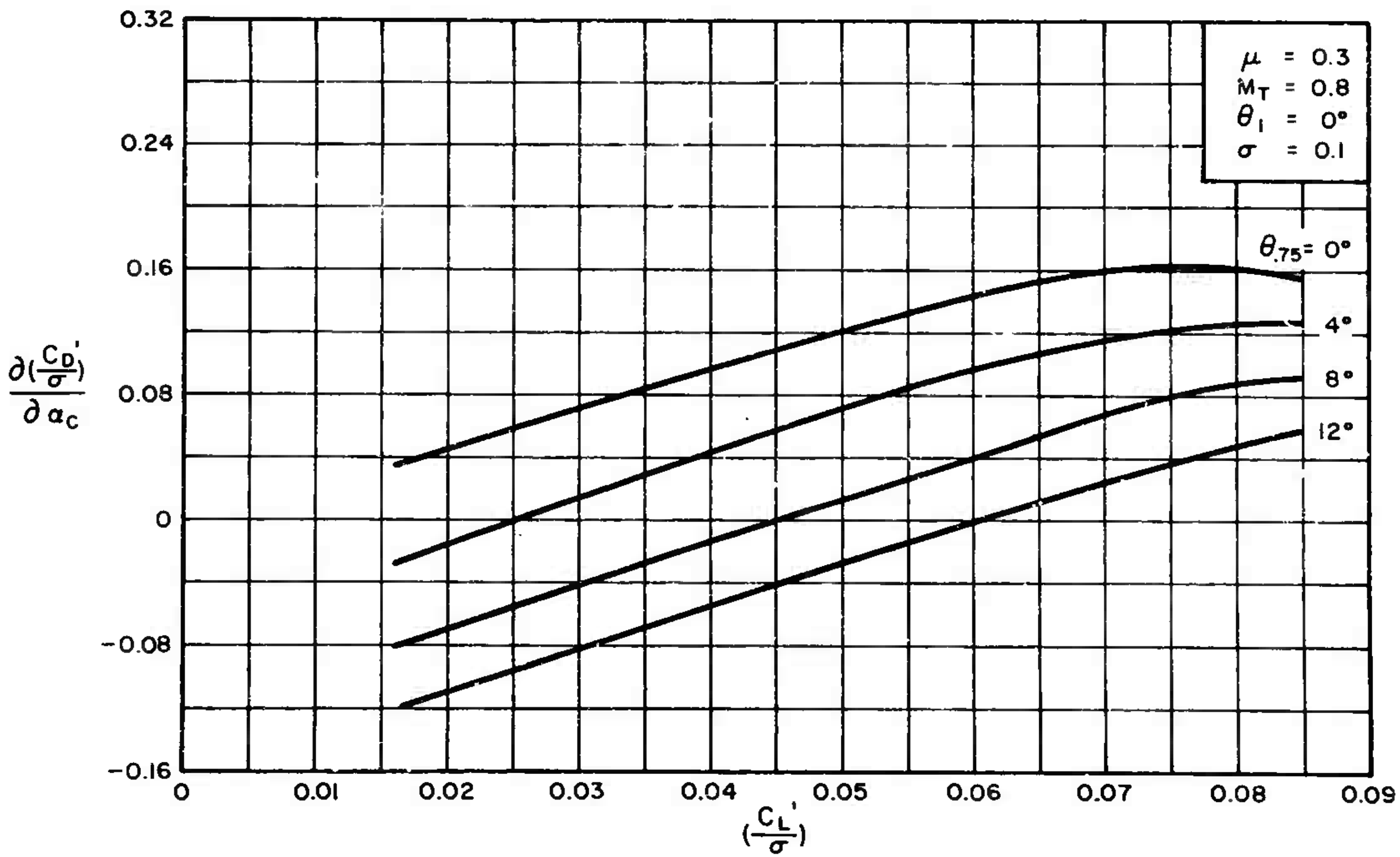

Figure 9. Continued
(c) $\mu = 0.3$

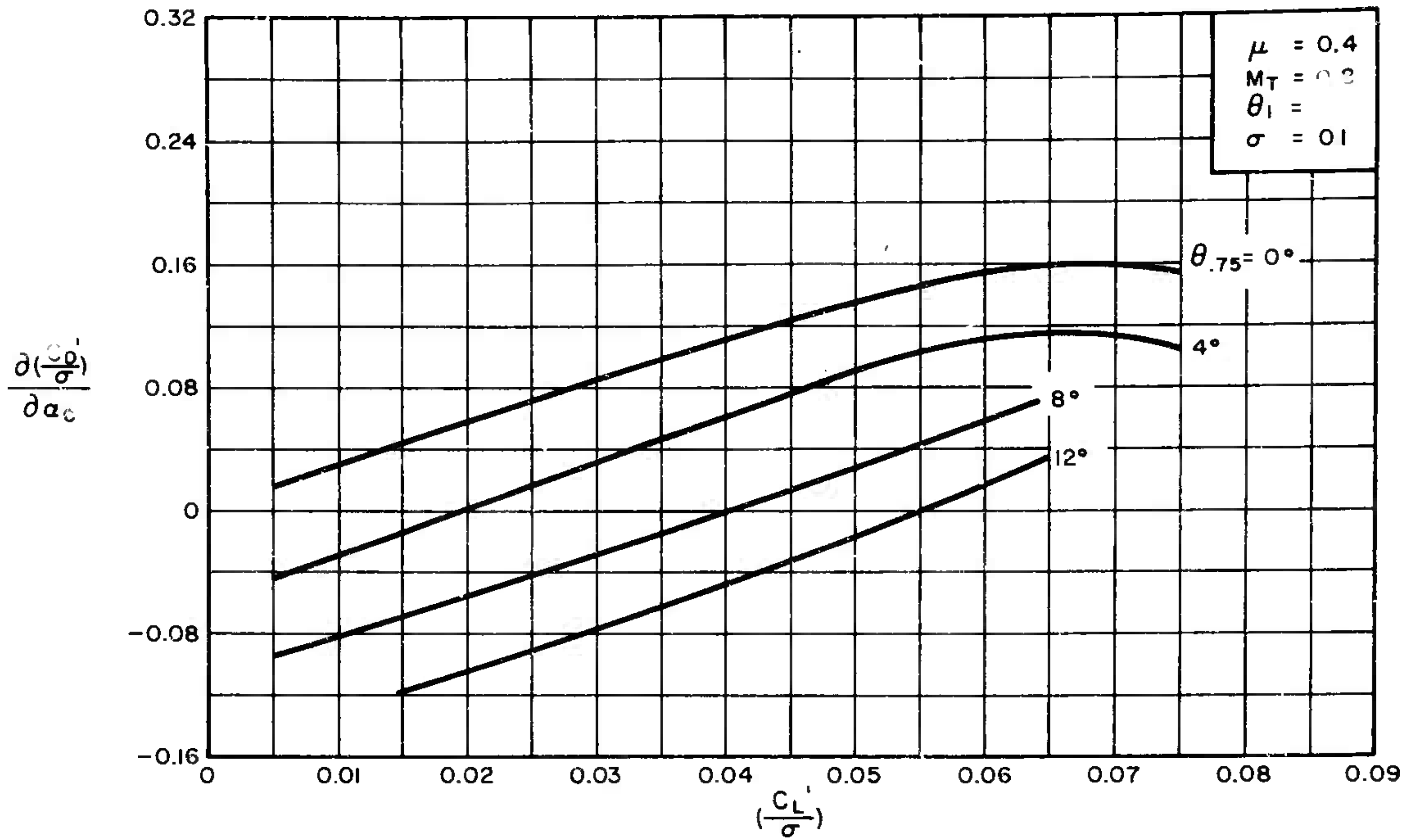

Figure 9 Continued

(d) $\mu = 0.4$

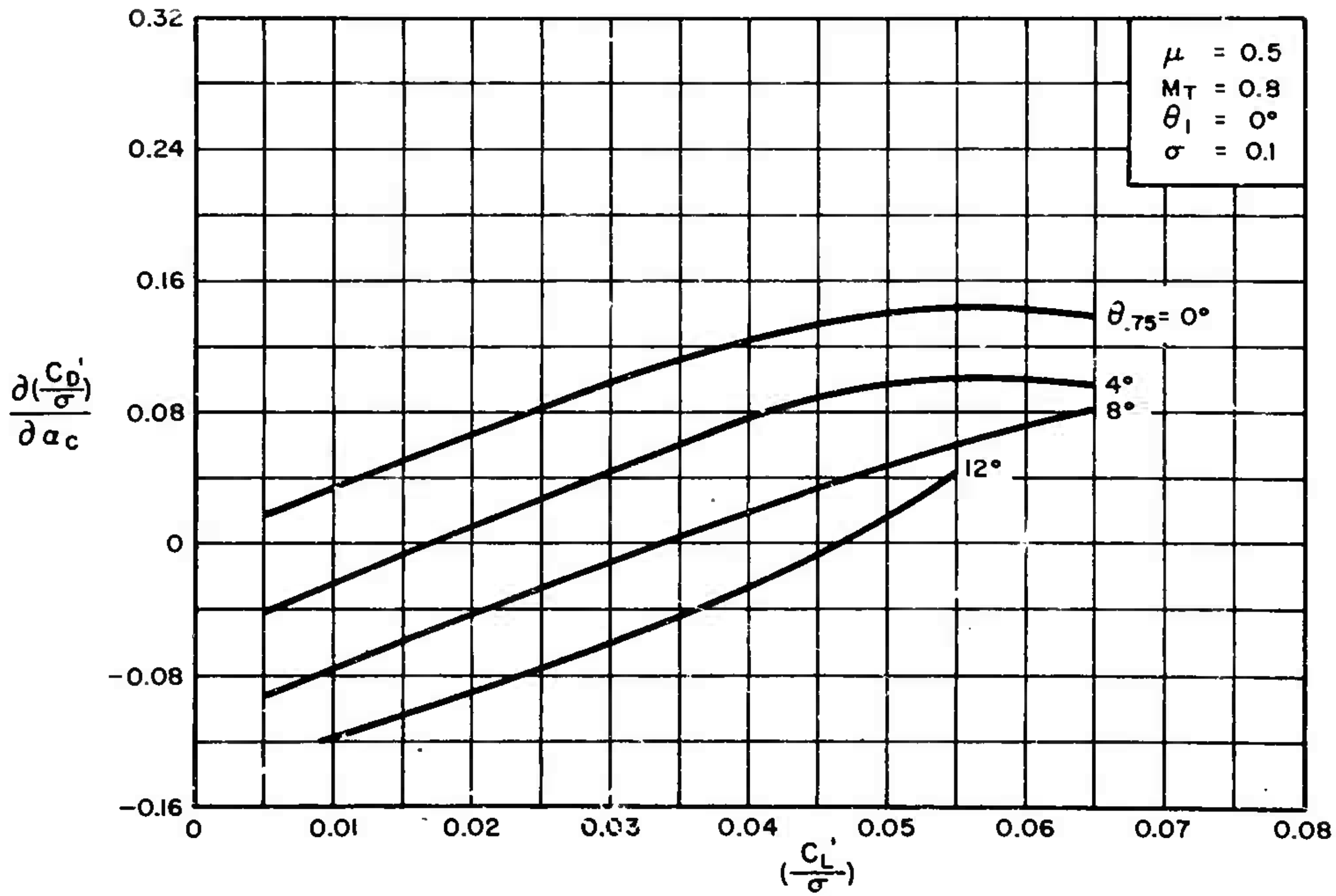

Figure 9. Continued

(e) $\mu = 0.5$

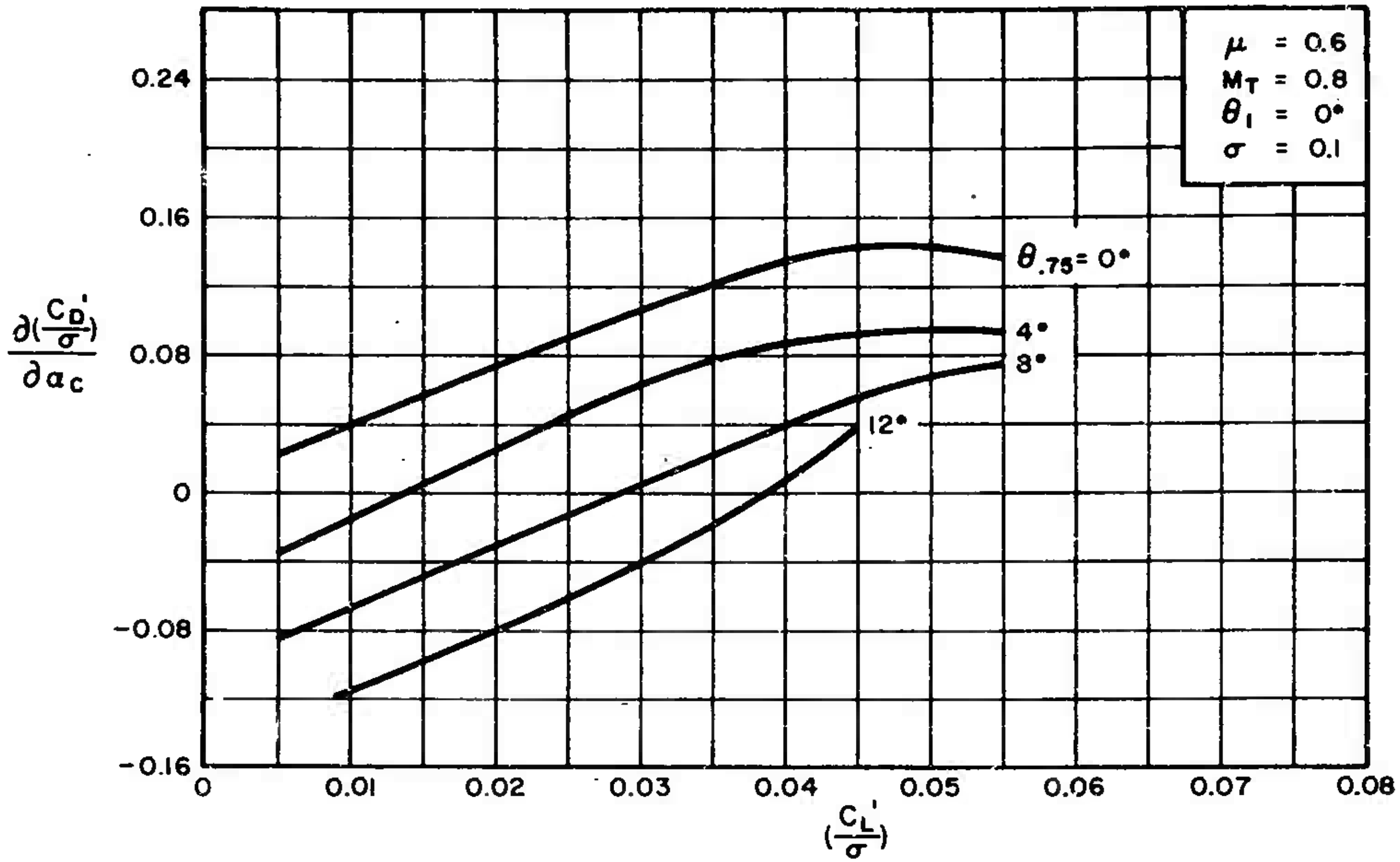

Figure 9. Continued

(f) $\mu = 0.6$

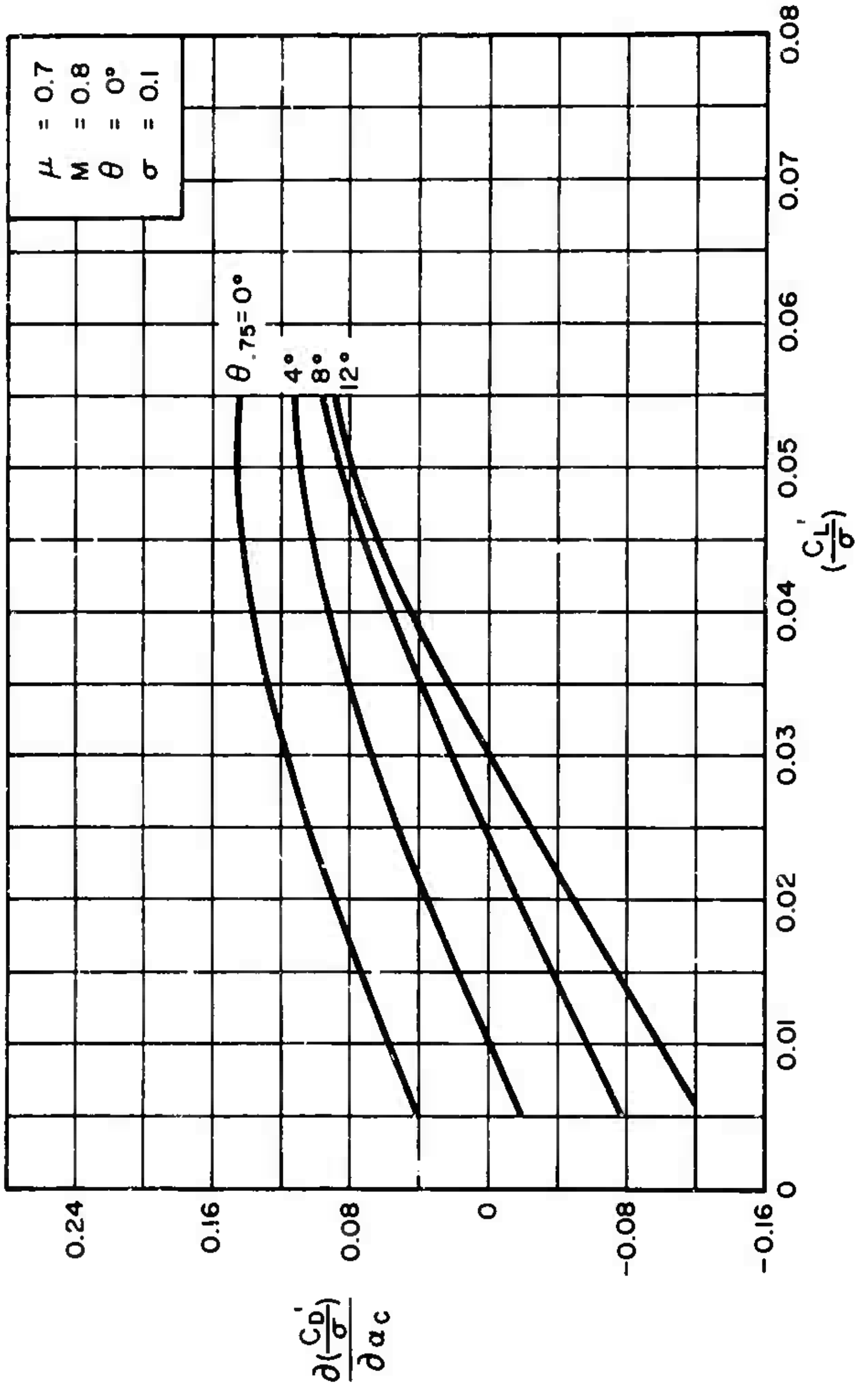

Figure 9. Continued

(g) $\mu = 0.7$

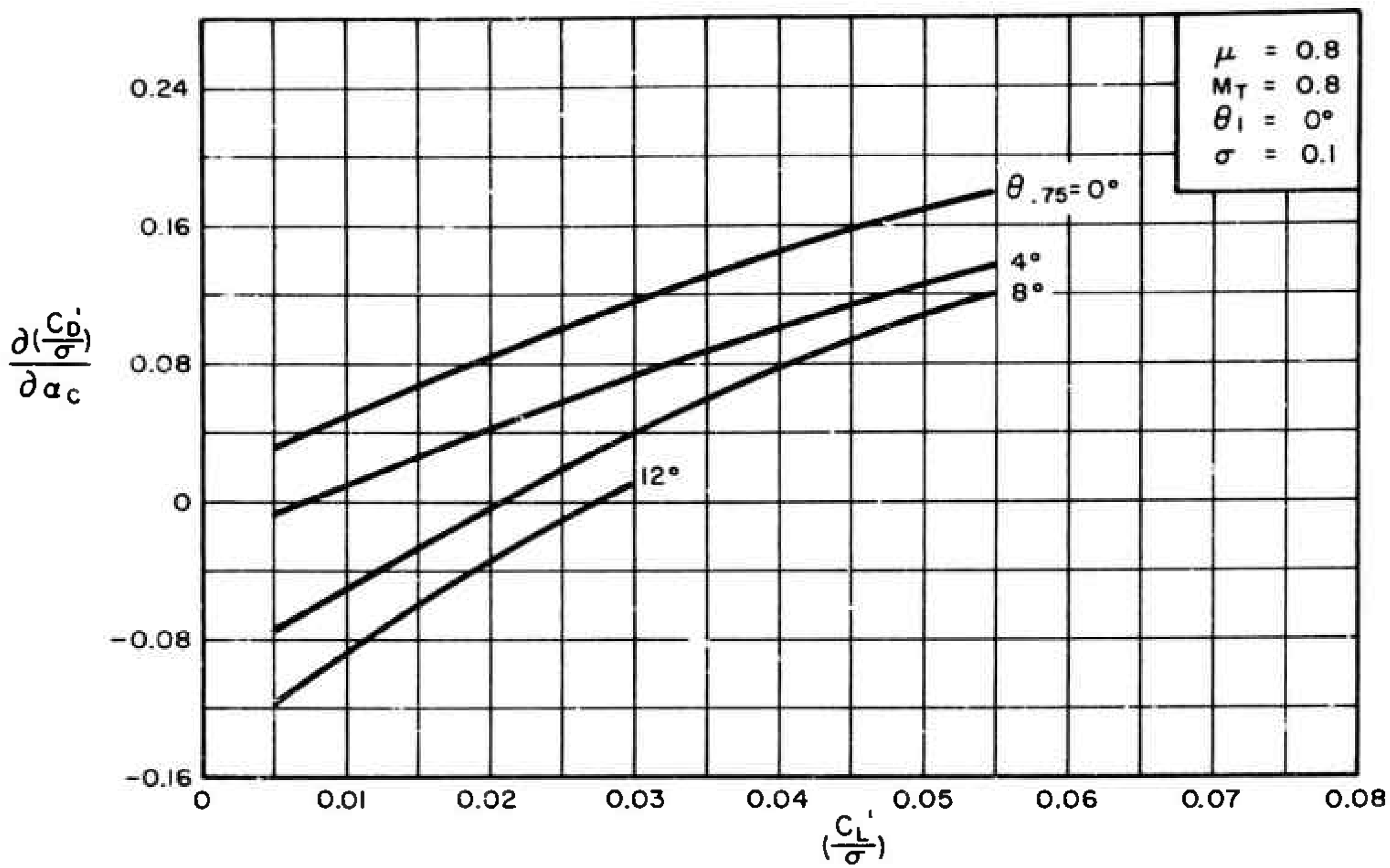

Figure 9. Continued

(h) $\mu = 0.8$

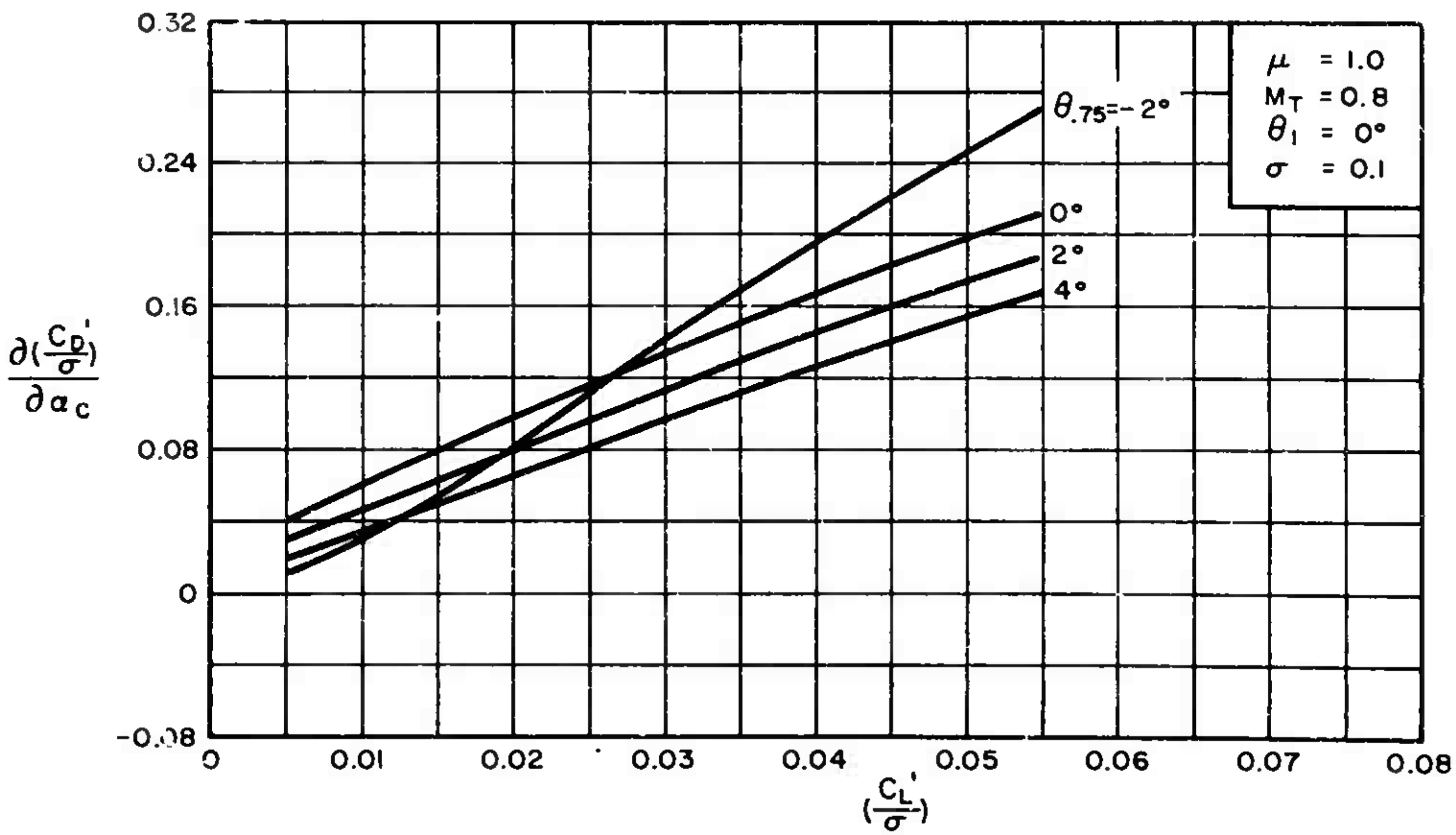

Figure 9. Concluded

(i) $\mu = 1.0$

7.5.2.3 $\dfrac{\partial(\frac{C_Q}{\sigma})}{\partial\alpha_C}$ for $\sigma = 0.1$, $\theta_1 = 0°$, and $M_T = 0.8$

Figures 10(a) through 10(g) present the isolated derivative $\partial(C_Q/\sigma)/\partial\alpha_C$ as a function of C_L'/σ for constant values of $\theta_{.75}$ and a range of μ values from $\mu = 0.3$ through $\mu = 1.0$. These were obtained from performance data of Reference 1 as slopes of the C_Q/σ vs. α_C relationships for constant values of μ and $\theta_{.75}$.

For the values of $\mu \leq 0.2$, the following expression may be used:

$$\frac{\partial(\frac{C_Q}{\sigma})}{\partial\alpha_C} = \left[\frac{\partial(\frac{C_Q}{\sigma})}{\partial\mu}\right]\left(\frac{\partial\mu}{\partial\alpha_C}\right) + \left[\frac{\partial(\frac{C_Q}{\sigma})}{\partial\lambda}\right]\left(\frac{\partial\lambda}{\partial\alpha_C}\right)$$

where $\partial(C_Q/\sigma)/\partial\mu$ and $\partial\lambda/\partial\alpha_C$ are obtained from Subsection 7.5.1.3 and 7.5.2.6, respectively, and

$$\frac{\partial\mu}{\partial\alpha_C} = -\mu \tan\alpha_C$$

$$\frac{\partial(\frac{C_Q}{\sigma})}{\partial\lambda} = \frac{1}{2}\left[(\delta_1 t_{52} + 2\lambda t_{55} + \delta_2 t_{56}\theta_{.75} - a(2\lambda t_{41} + t_{42}\theta_{.75})\right]$$

Values for δ_1, δ_2, t_{52}, t_{55}, t_{56}, t_{41}, and t_{42} may be obtained from Reference 3.

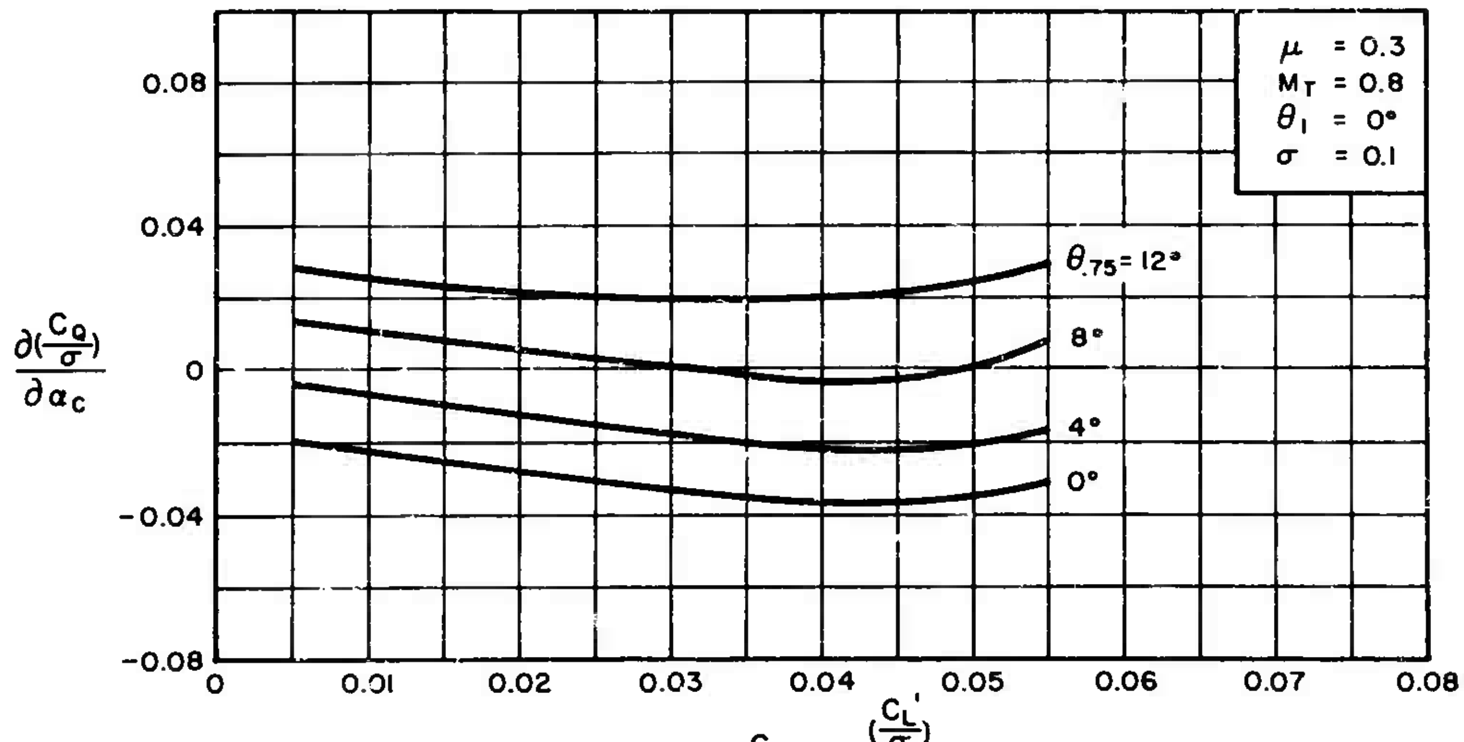

Figure 10. Variation of $\frac{\partial(\frac{C_Q}{\sigma})}{\partial \alpha_c}$ With $\frac{C_L'}{\sigma}$ for Constant Values of $\theta_{.75}$

(a) $\mu = 0.3$

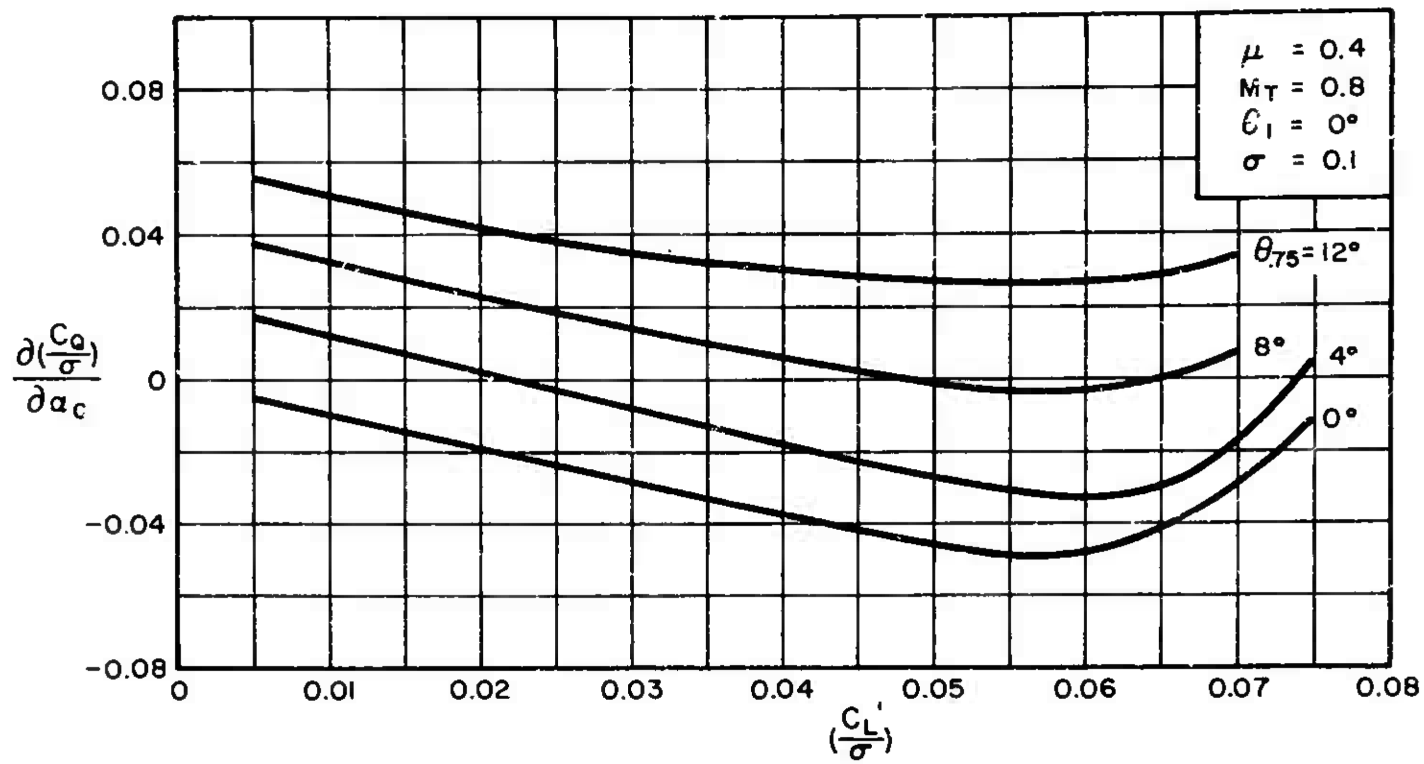

Figure 10. Continued

(b) μ = 0.4

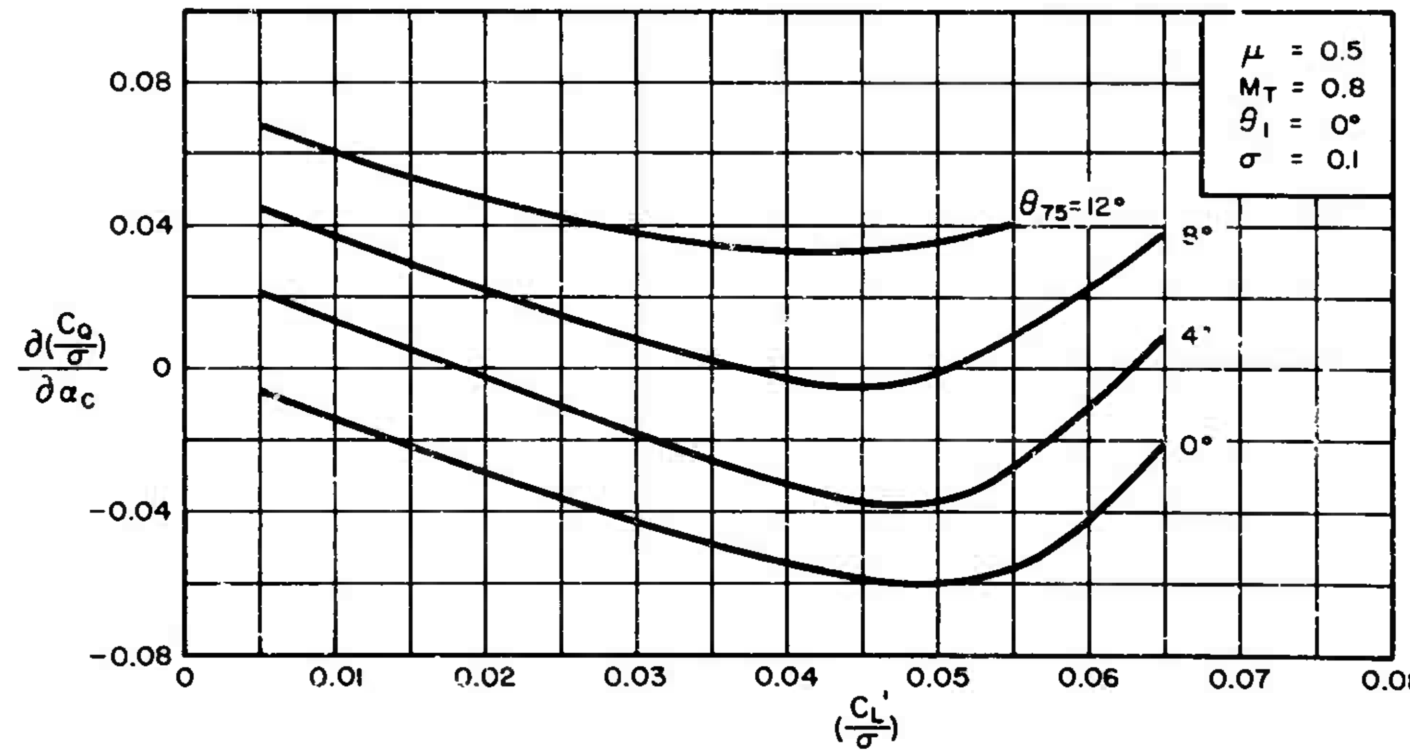

Figure 10. Continued

(c) $\mu = 0.5$

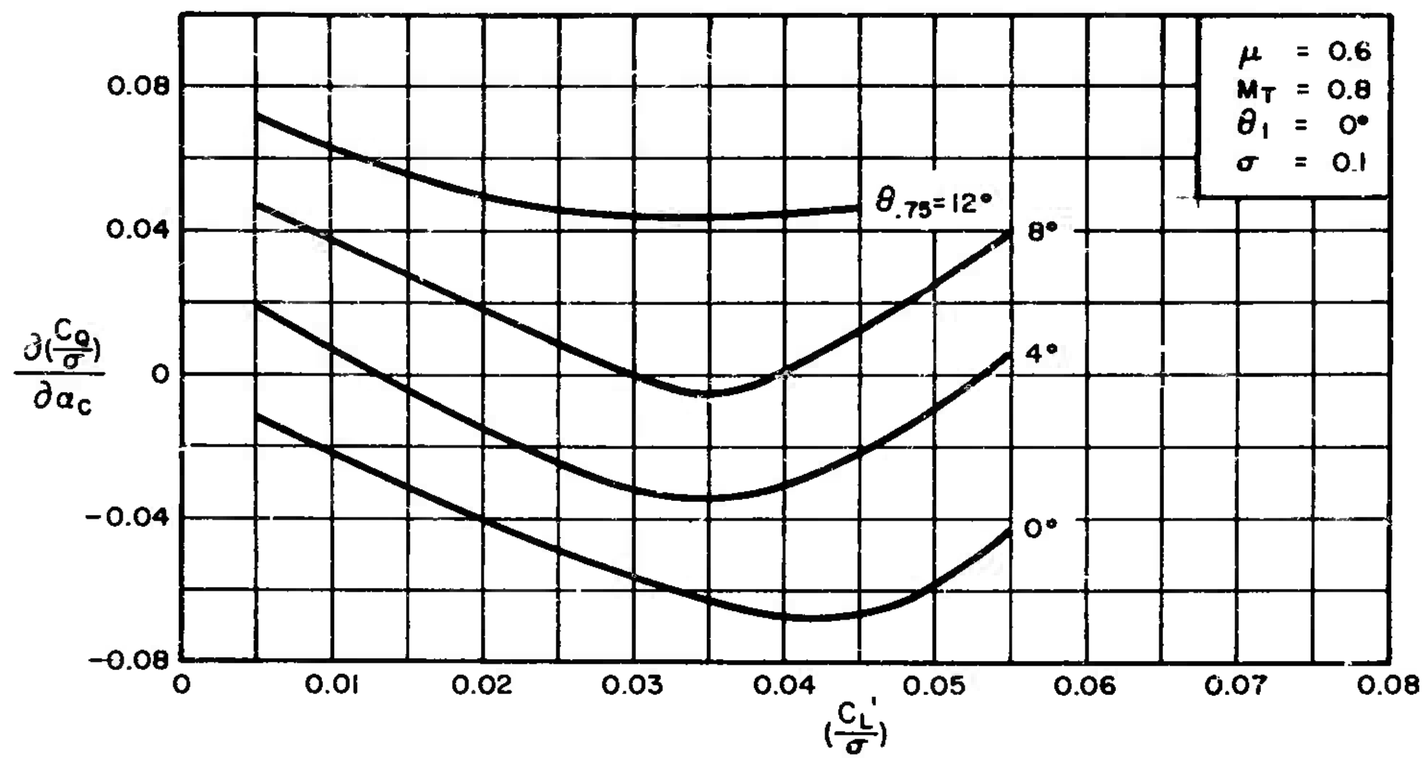

Figure 10. Continued

(d) $\mu = 0.6$

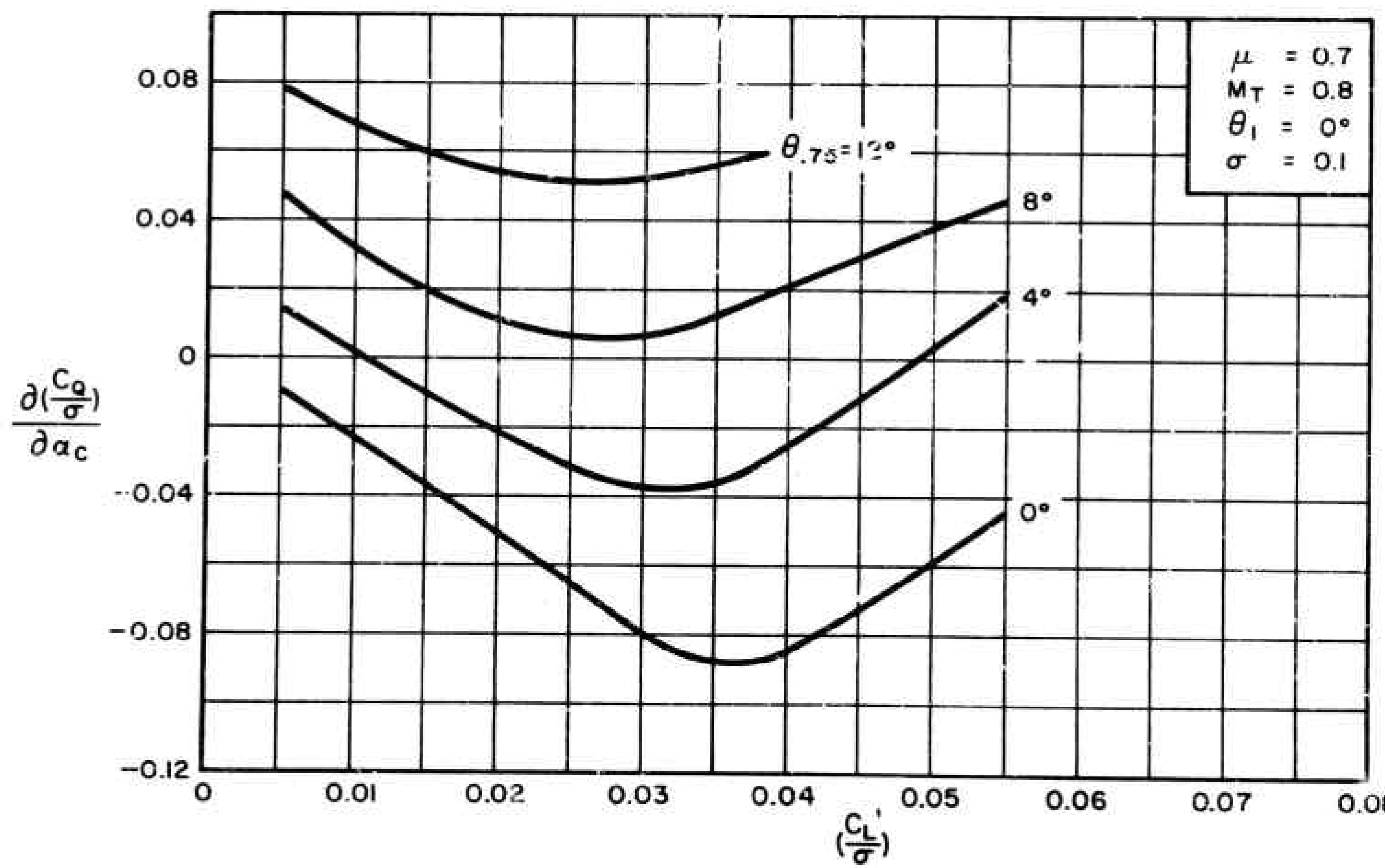

Figure 10. Continued

(e) $\mu = 0.7$

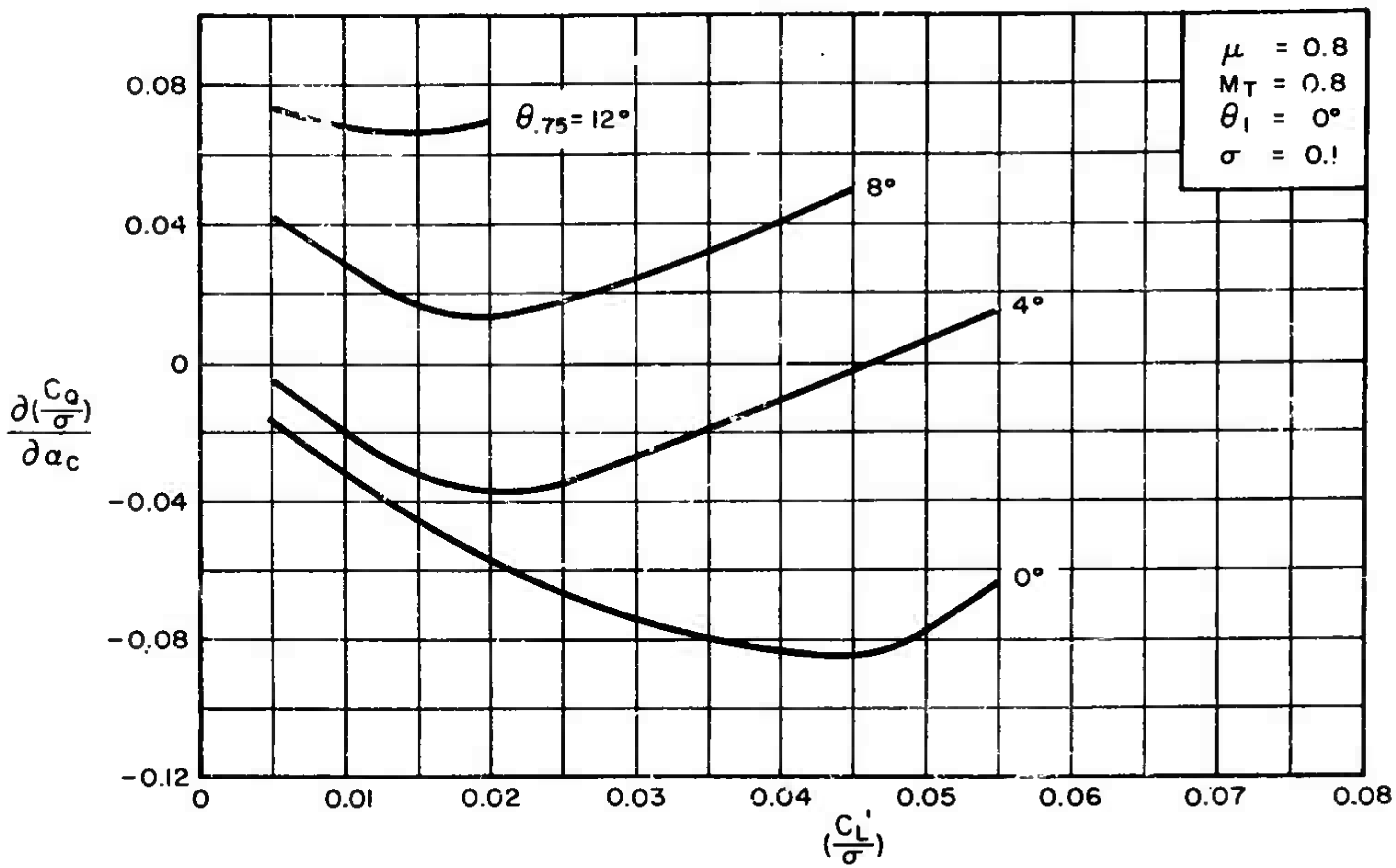

Figure 10. Continued

(f) $\mu = 0.8$

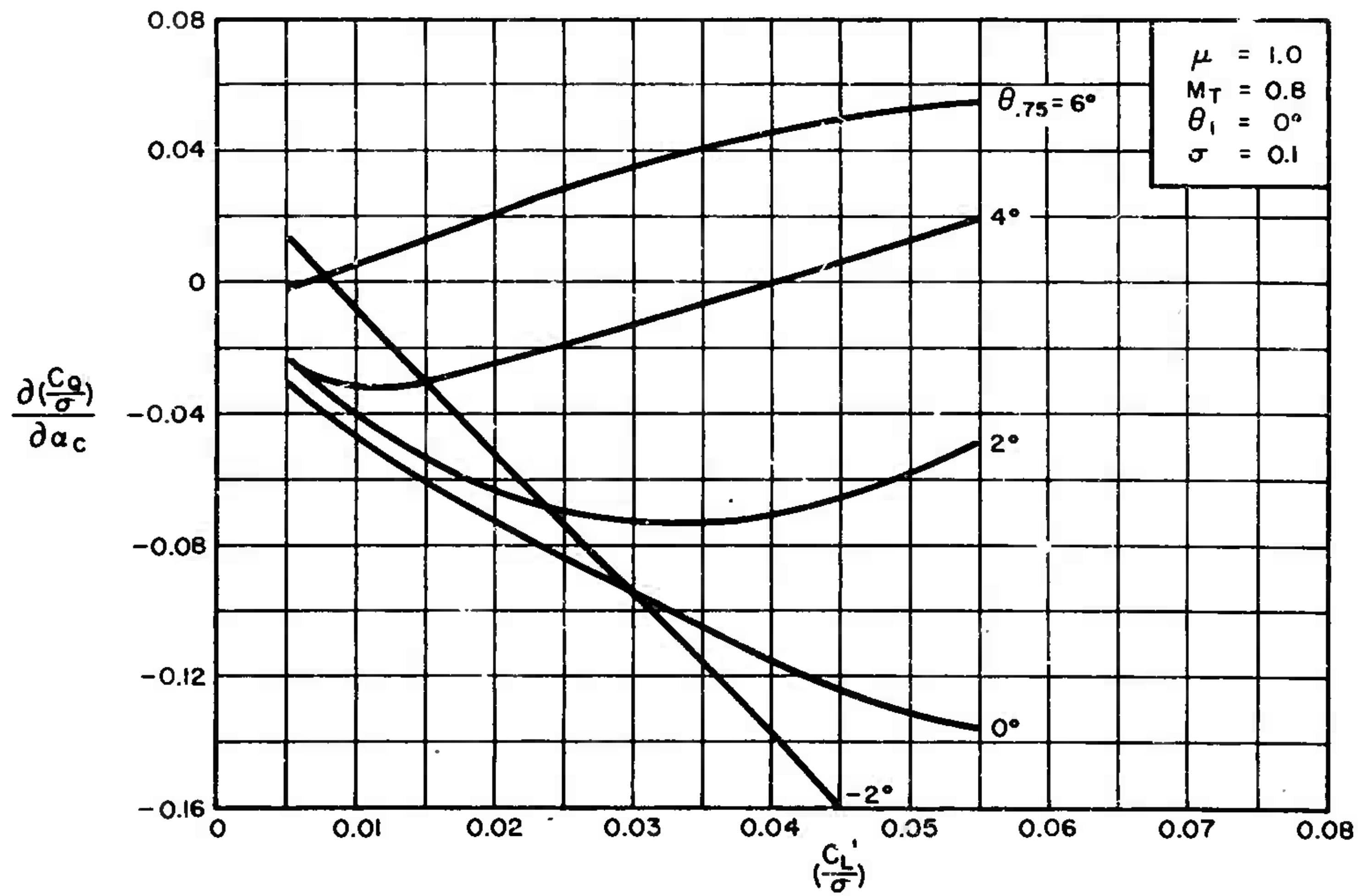

Figure 10. Concluded

(g) $\mu = 1.0$

7.5.2.4 $\frac{\partial a_1}{\partial \alpha_C}$ for $\sigma = 0.1$, $\theta_1 = 0°$, and $M_T = 0.8$

Figures 11(a) through 11(d) present the isolated rotor derivative $\partial a_1/\partial \alpha_C$ as a function of C_L'/σ for constant values of μ and a range of $\theta_{.75}$ values from $\theta_{.75} = 0°$ through $\theta_{.75} = 12°$.

The values of $\partial a_1/\partial \alpha_C$ for $\mu = 0.1$ and $\mu = 0.2$ were obtained directly from Reference 2.

The values of $\partial a_1/\partial \alpha_C$ for $\mu \geq 0.3$ were obtained from the theoretical rotor performance data of Reference 1 by graphically obtaining slopes of the a_1 vs. α_C relationships for constant values of μ and $\theta_{.75}$.

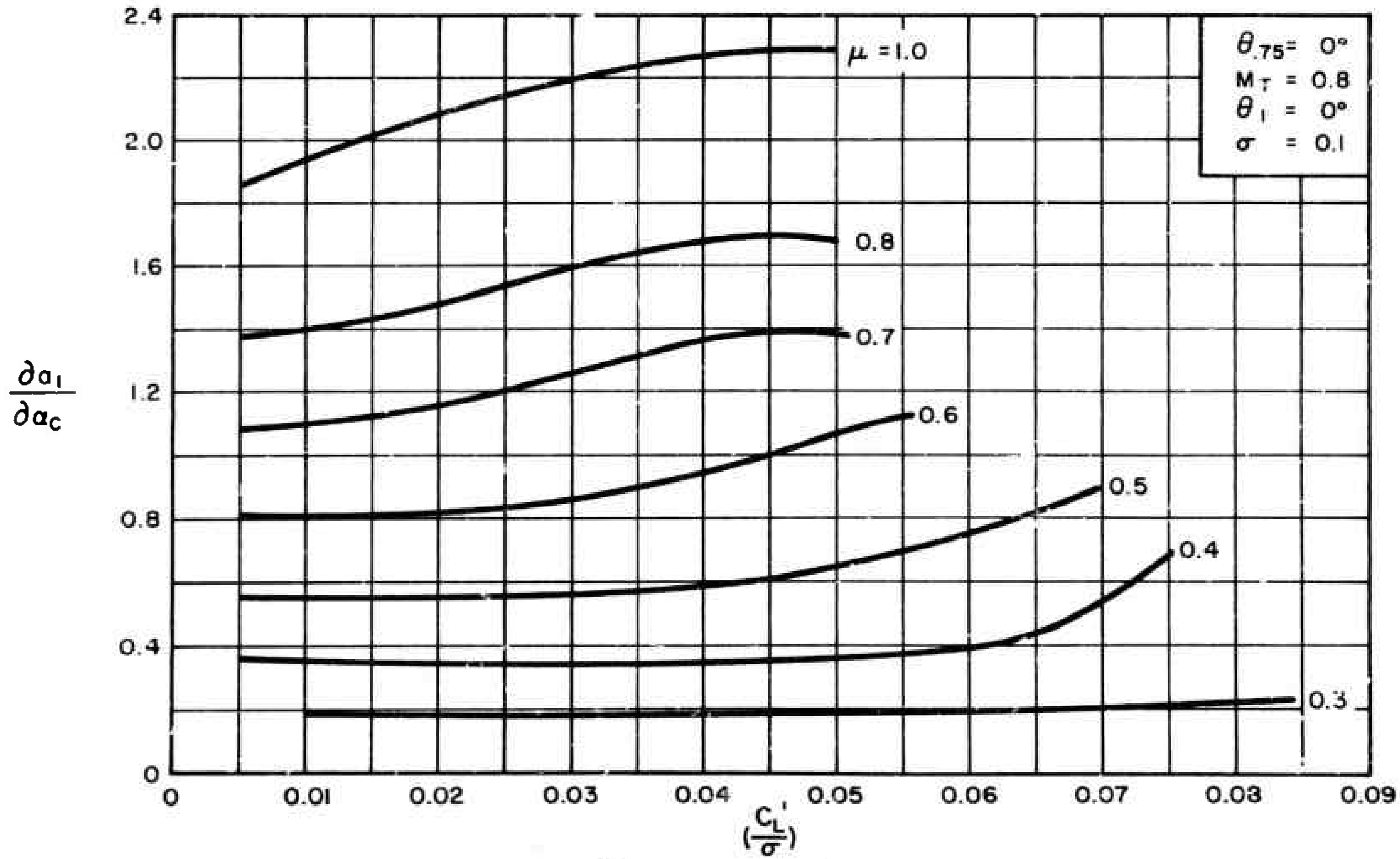

Figure 11. Variation of $\frac{\partial a_1}{\partial \alpha_c}$ With $\frac{C_L'}{\sigma}$ for Constant Values of μ

(a) $\theta_{.75}$ = 0°

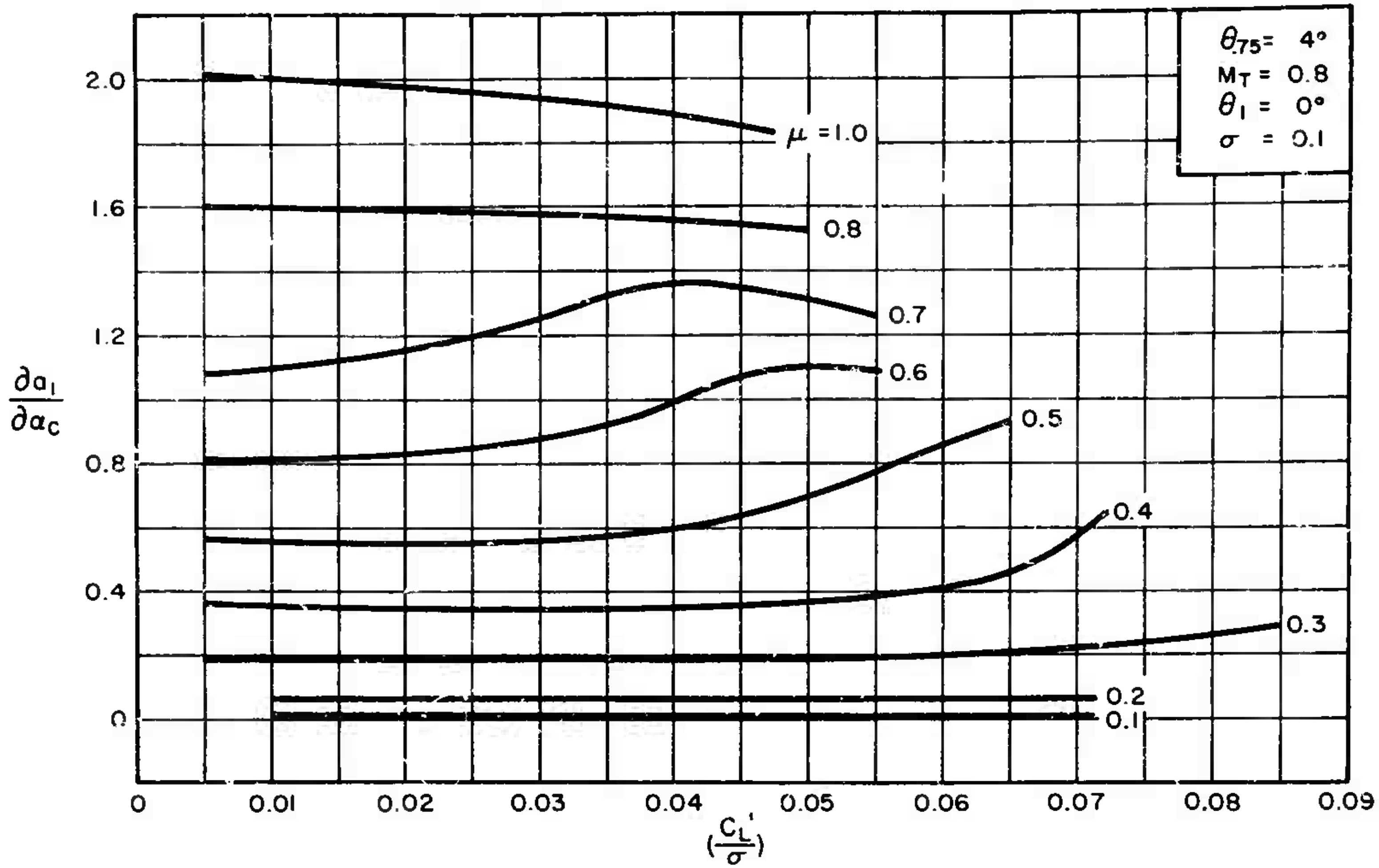

Figure 11 Continued

(b) $\theta_{.75} = 4°$

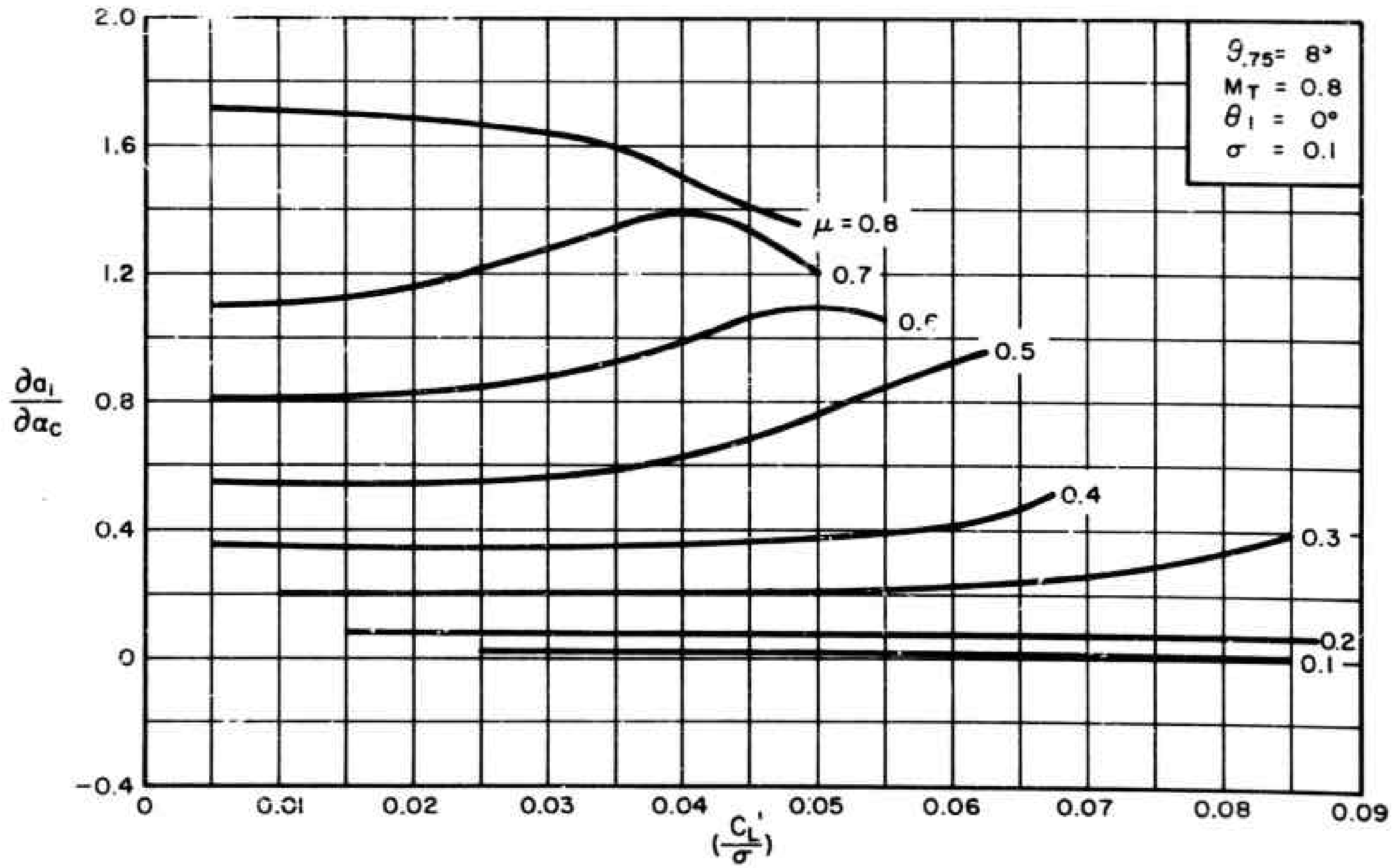

Figure 11. Continued

(c) $\theta_{.75} = 8°$

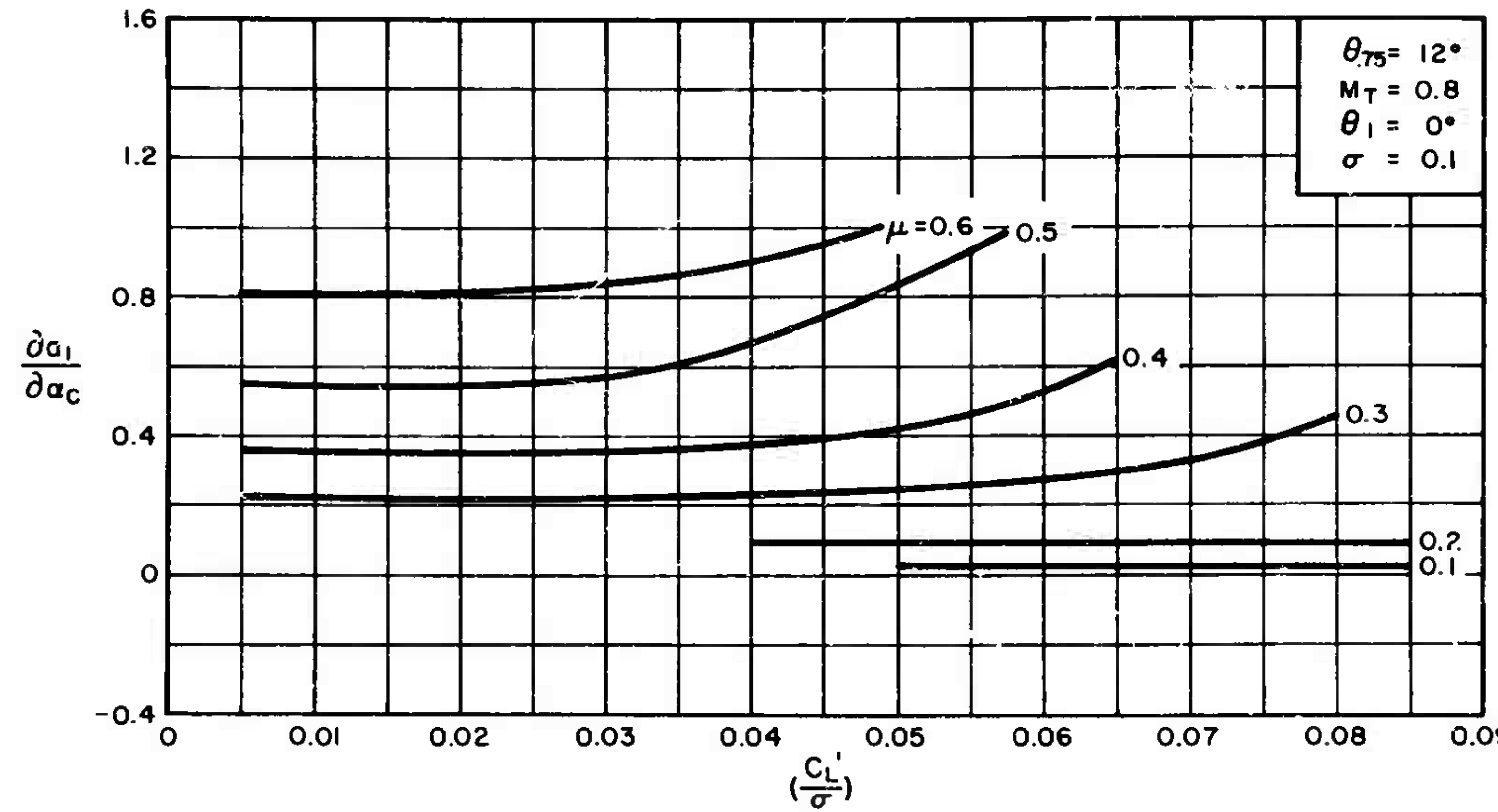

Figure 11. Concluded

(d) $\theta_{.75} = 12°$

7.5.2.5 $\dfrac{\partial b_1}{\partial a_c}$ for $\sigma = 0.1$, $\theta_1 = 0°$, and $M_T = 0.8$

Figures 12(a) through 12(d) present the isolated rotor derivative $\partial b_1/\partial a_c$ as a function of C_L'/σ for constant values of μ and a range of $\theta_{.75}$ values from $\theta_{.75} = 0$ through $\theta_{.75} = 12°$. These derivatives were obtained from the theoretical data of Reference 1 by graphically obtaining slopes of the b_1 vs. a_c relationships for constant values of μ and $\theta_{.75}$. These derivatives are specifically applicable to rotors having Lock inertia number $\gamma = 8.0$. However, since the lateral flapping angle b_1 is essentially proportional to γ, a correction factor of $\gamma/8.0$ may be utilized to compute $\partial b_1/\partial a_c$ derivatives for rotors having γ values other than 8.0. Thus:

$$\left(\frac{\partial b_1}{\partial a_c}\right)_{\gamma} = \frac{\gamma}{8.0}\left(\frac{\partial b_1}{\partial a_c}\right)_{\gamma=8.0}$$

The $\partial b_1/\partial a_c$ derivatives for $\mu \leq 0.2$ can be computed by using the following expression:

$$\left(\frac{\partial b_1}{\partial a}\right) = \gamma\left[\left(\frac{2}{9}B^2 - \frac{1}{3}\mu^2\right)\lambda + \frac{\partial \lambda}{\partial \mu}(t_{17}) + \left(\frac{B^3}{6} + \frac{B\mu^2}{4}\right)\theta_{.75}\right]$$

where $\partial\lambda/\partial\mu$ is presented in Subsection 7.5.1.6, and where values of t_{17} can be obtained from Reference 3.

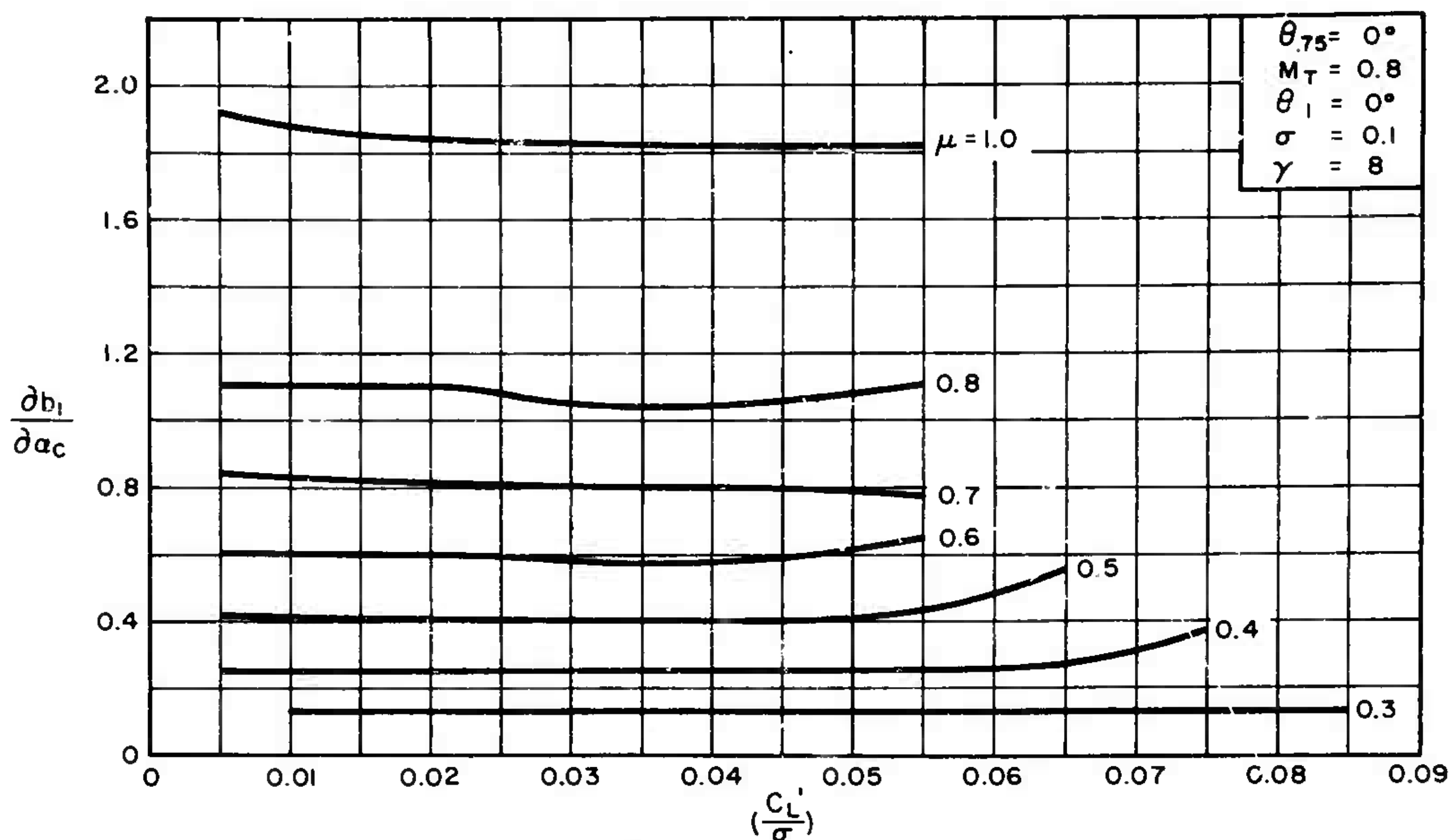

Figure 12. Variation of $\frac{\partial b_1}{\partial a_c}$ With $\frac{C_L'}{\sigma}$ for Constant Values of μ

(a) $\theta_{.75} = 0°$

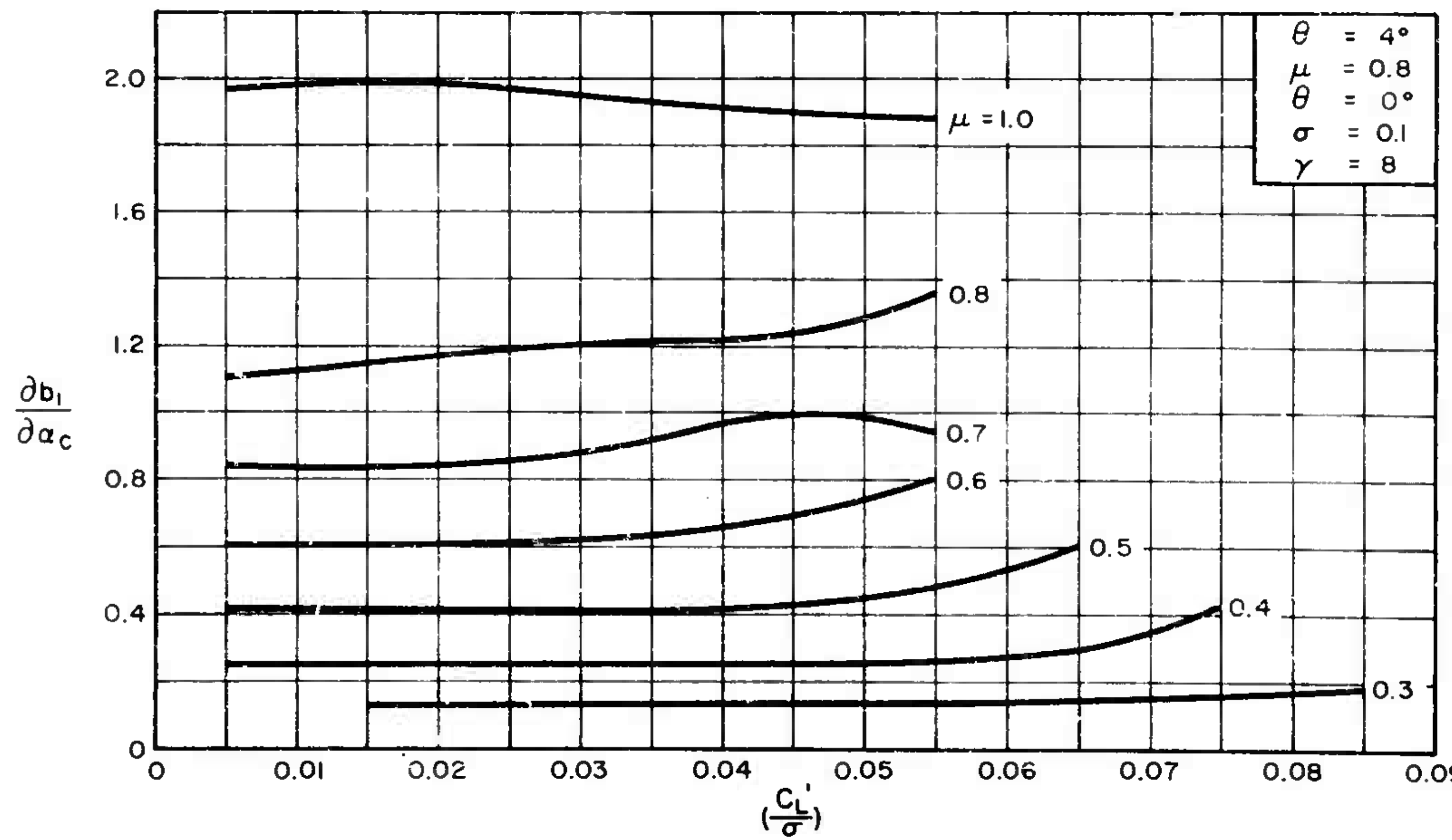

Figure 12. Continued

(b) $\theta_{.75} = 4°$

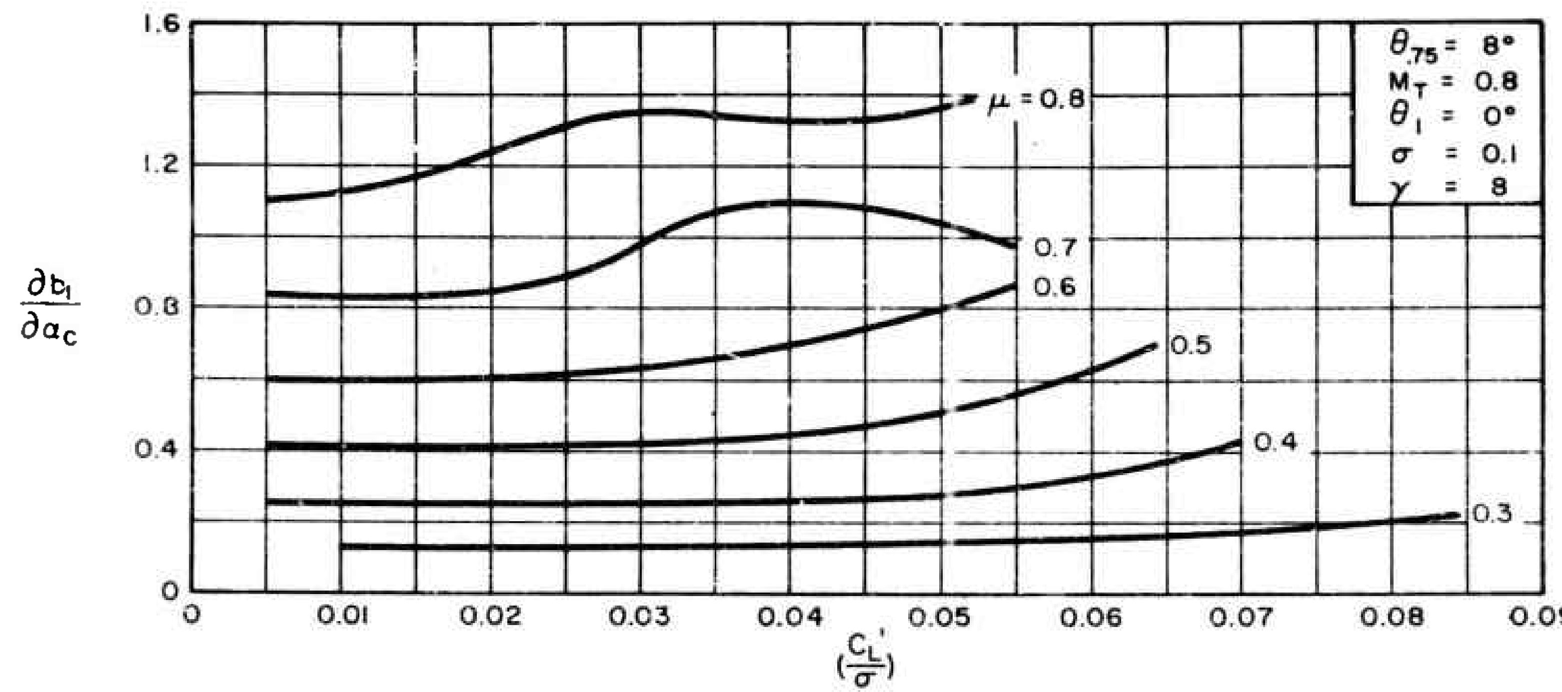

Figure 12. Continued

(c) $\theta_{.75} = 8°$

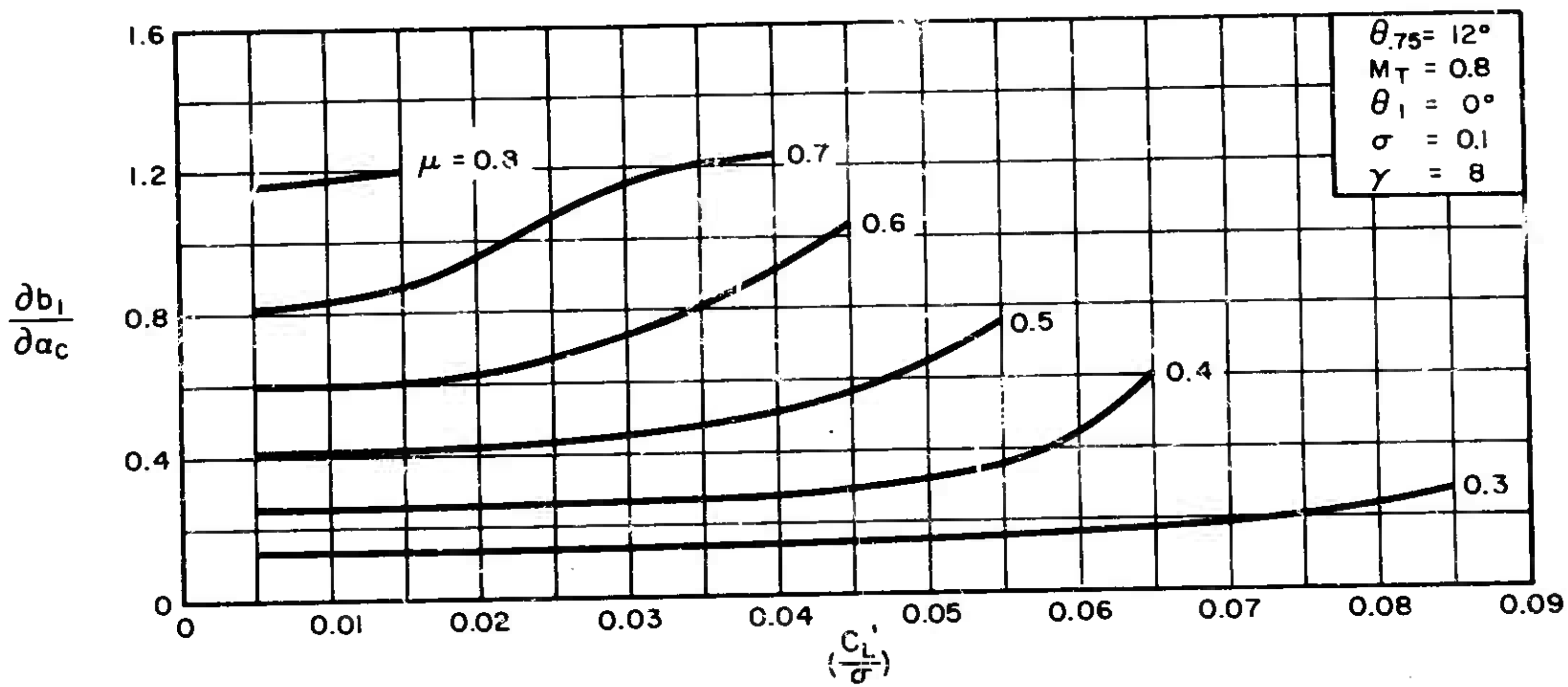

Figure 12. Concluded

(d) $\theta_{.75} = 12°$

7.5.2.6 $\frac{\partial \lambda}{\partial \alpha_c}$ for $\sigma = 0.1$, $\theta_1 = 0°$, and $M_T = 0.8$

The derivative $\partial\lambda/\partial\alpha_C$ is plotted in Figure 13 as a function of μ and is applicable for all values of $\theta_{.75}$ and C_L'/σ. The values of $\partial\lambda/\partial\alpha_C$ for $\mu = 0.1$ and $\mu = 0.2$ were obtained directly from Reference 2. For $\mu \geq 0.3$, the values were extracted graphically from the theoretical rotor performance data of Reference 1. The results obtained are applicable for all values of $\theta_{.75}$, C_L'/σ, and α_C.

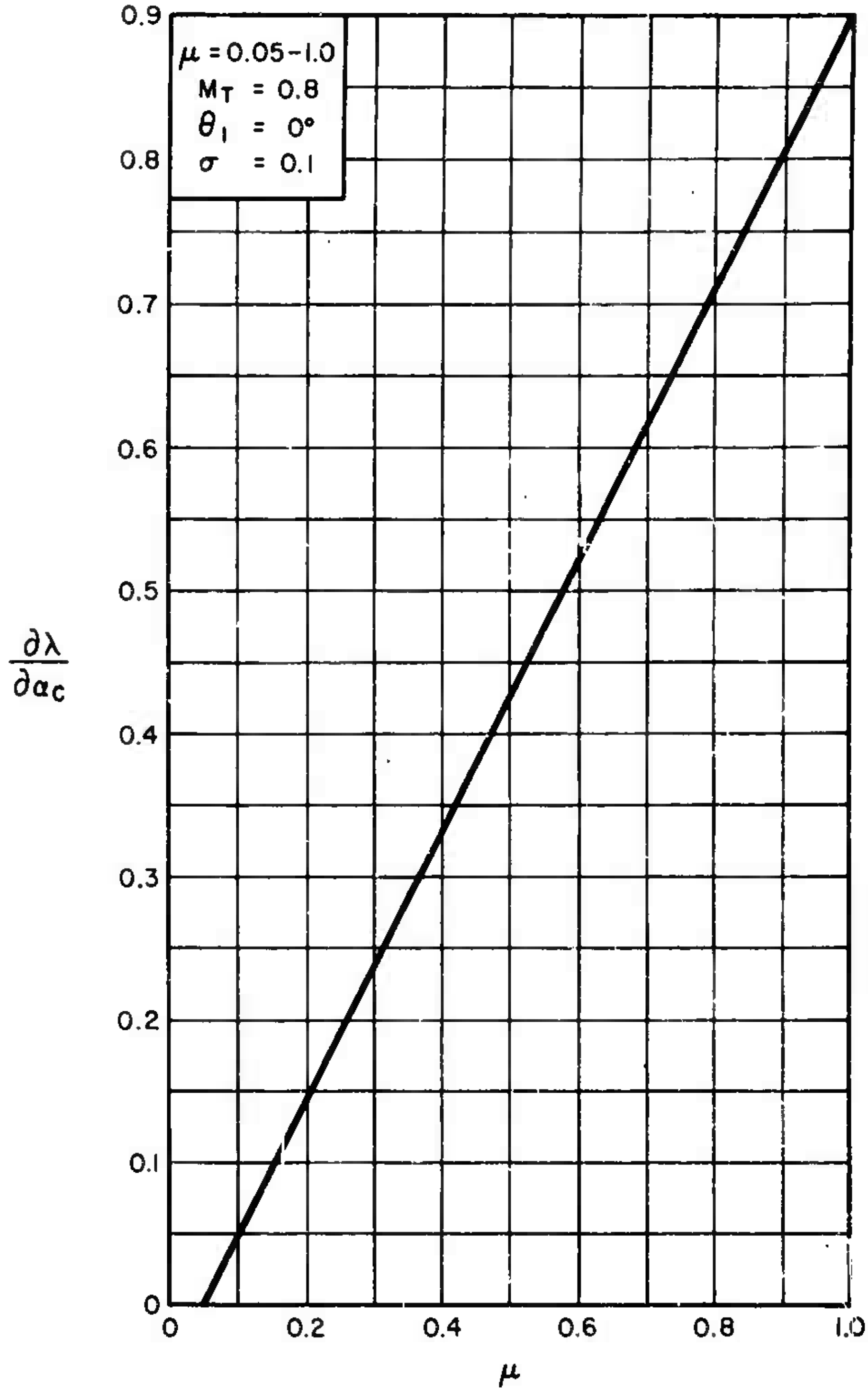

Figure 13. Variation of $\frac{\partial\lambda}{\partial\alpha_c}$ With μ for All Values of $\theta_{.75}$ and $\frac{C_L'}{\sigma}$

7.5.2.7 $\dfrac{\partial(\frac{C_Y'}{\sigma})}{\partial \alpha_c}$ for All Values of σ, θ_1, and M_T

Reference 1 and other reviewed reports do not include the calculated data required to obtain the rotor Y-force derivatives.

It is therefore suggested that the classical Bailey theory be utilized for this purpose. If the above theory is used, the following expression for $\partial(C_Y'/\sigma)/\partial\alpha_c$ can be derived:

$$\frac{\partial(\frac{C_Y'}{\sigma})}{\partial \alpha_c} = \frac{a}{2}\Bigg\{\frac{\partial a_0}{\partial \alpha_c}\left[a_0\left(\frac{1}{6}-\mu^2\right)-\frac{3}{4}\mu\left(\theta_{.75}+2\lambda\right)\right]$$

$$+\frac{\partial a_1}{\partial \alpha_c}\left[a_0\left(\frac{1}{6}-\mu^2\right)+\frac{1}{4}\mu b_1\right]$$

$$+\frac{\partial b_1}{\partial \alpha_c}\left[\theta_{.75}\left(\frac{1}{3}+\frac{3}{8}\mu^2\right)+\lambda\left(\frac{3}{4}+\frac{1}{8}\mu^2\right)+\frac{1}{4}\mu a_1\right]$$

$$+\frac{\partial \lambda}{\partial \alpha_c}\left[b_1\left(\frac{3}{4}+\frac{1}{8}\mu^2\right)-\frac{3}{2}\mu a_0\right]\Bigg\}$$

where

$$\frac{\partial a_0}{\partial \alpha_c} = \frac{\gamma}{2}\left[-\frac{\theta_{.75}}{4}\mu^2 \sin 2\alpha_c + \frac{1}{3}\frac{\partial \lambda}{\partial \alpha_c}\right]$$

and where $\partial a_1/\partial\alpha_c$, $\partial b_1/\partial\alpha_c$, and $\partial\lambda/\partial\alpha_c$ are given in Subsections 7.5.2.4, 7.5.2.5, and 7.4.2.6, respectively.

The expression for the isolated rotor derivative $\partial(C_Y'/\sigma)/\partial\alpha_c$ as given above is applicable for all values of σ, θ_1 and M_T, provided that the pertinent rotor parameters comprising the derivative are evaluated at the required conditions.

7.5.3 Isolated Rotor Derivatives With Respect to Rotor Collective Pitch at 75% Radius ($\theta_{.75}$)

7.5.3.1 $\frac{\partial(\frac{C_L'}{\sigma})}{\partial\theta_{.75}}$ for $\sigma = 0.1$, $\theta_1 = 0°$, and $M_T = 0.8$

Figures 14 through 15(g) present the isolated rotor derivative $\partial(C_L'/\sigma)/\partial\theta_{.75}$ as functions of C_L'/σ and α_C for $\mu = 0.1$ through 1.0.

The derivatives for low μ values, i.e., $\mu \leq 0.2$, were obtained by using the following equation:

$$\frac{\partial(\frac{C_L'}{\sigma})}{\partial\theta_{.75}} = \frac{\partial(\frac{C_T}{\sigma})}{\partial\theta_{.75}} \cos\alpha_C - \frac{\partial(\frac{C_H}{\sigma})}{\partial\theta_{.75}} \sin\alpha_C$$

where $\partial(C_T/\sigma)/\partial\theta_{.75}$ and $\partial(C_H/\sigma)/\partial\theta_{.75}$ were obtained from Reference 2. Values of $\partial(C_L'/\sigma)/\partial\theta_{.75}$ for $\mu \geq 0.3$ were extracted graphically from rotor performance data of Reference 1 by obtaining slopes of the C_L'/σ vs. $\theta_{.75}$ relationships for constant values of μ and α_C.

Figure 14 indicates that for $\mu \leq 0.2$ the derivatives are practically independent of α_C and C_L'/σ variations, whereas these for $\mu \geq 0.3$ presented in Figures 15(a) through 15(g) are functions of α_C and C_L'/σ.

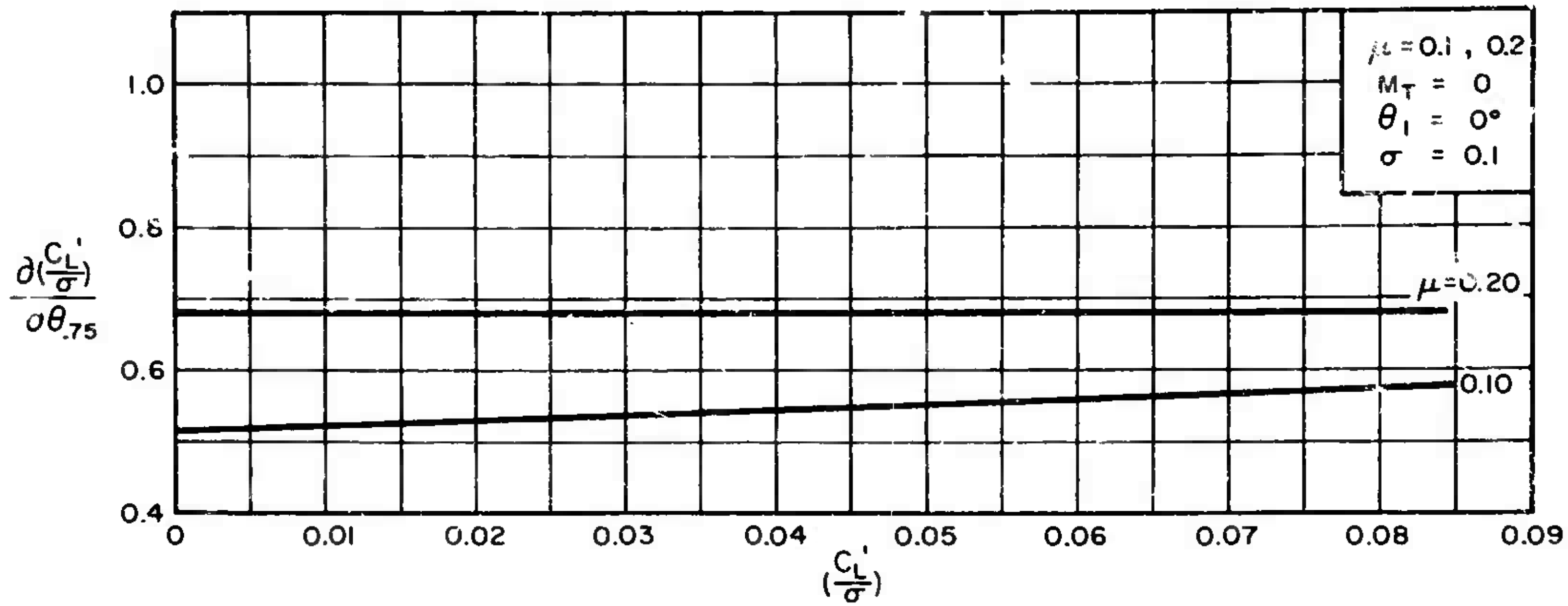

Figure 14. Variation of $\frac{\partial(\frac{C_L'}{\sigma})}{\partial\theta_{.75}}$ With $\frac{C_L'}{\sigma}$ for $\mu = 0.1$ and 0.2

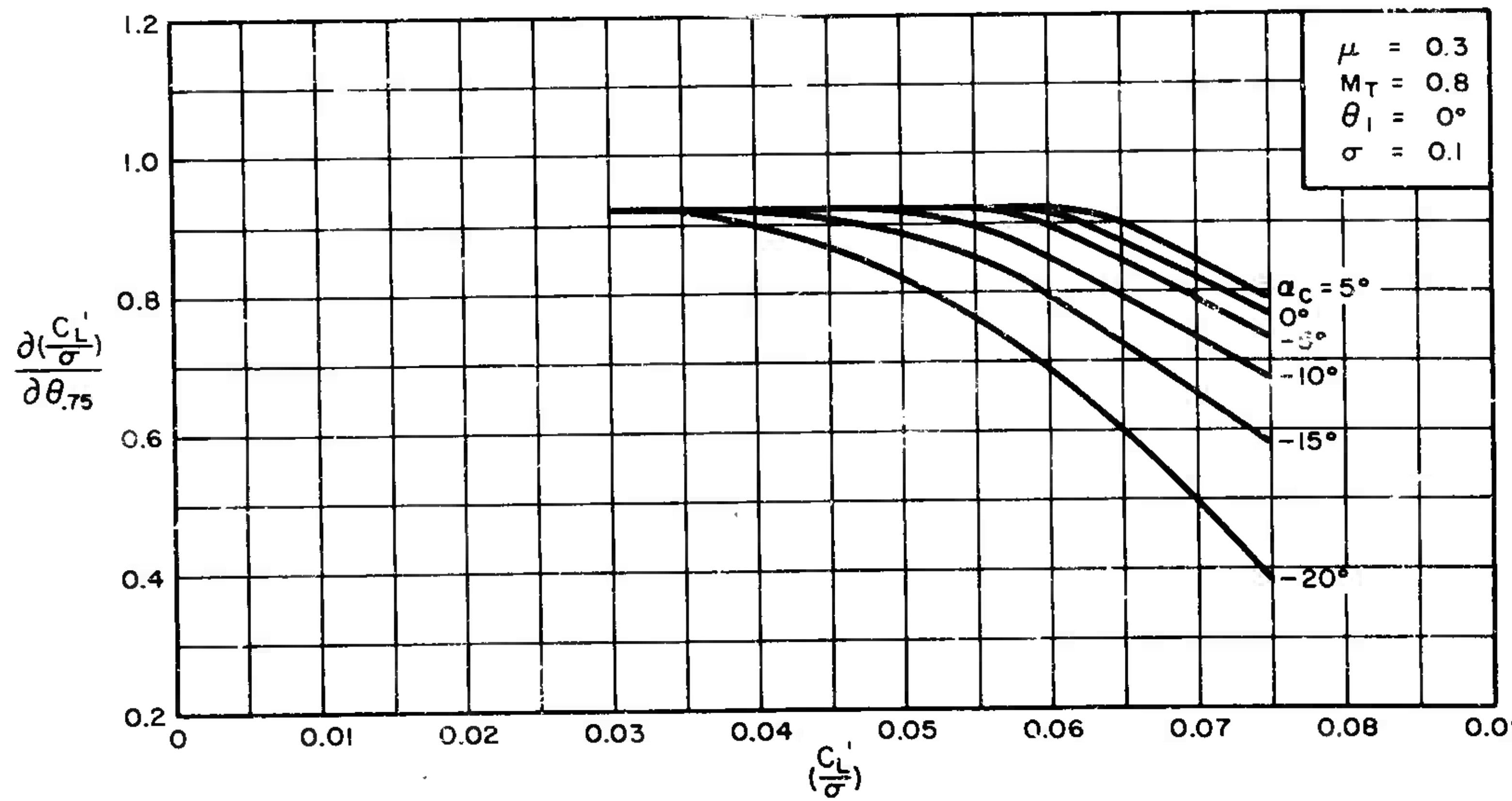

Figure 15. Variation of $\frac{\partial(\frac{C_L'}{\sigma})}{\partial\theta_{.75}}$ With $\frac{C_L'}{\sigma}$ for Constant Values of α_c

(a) $\mu = 0.3$

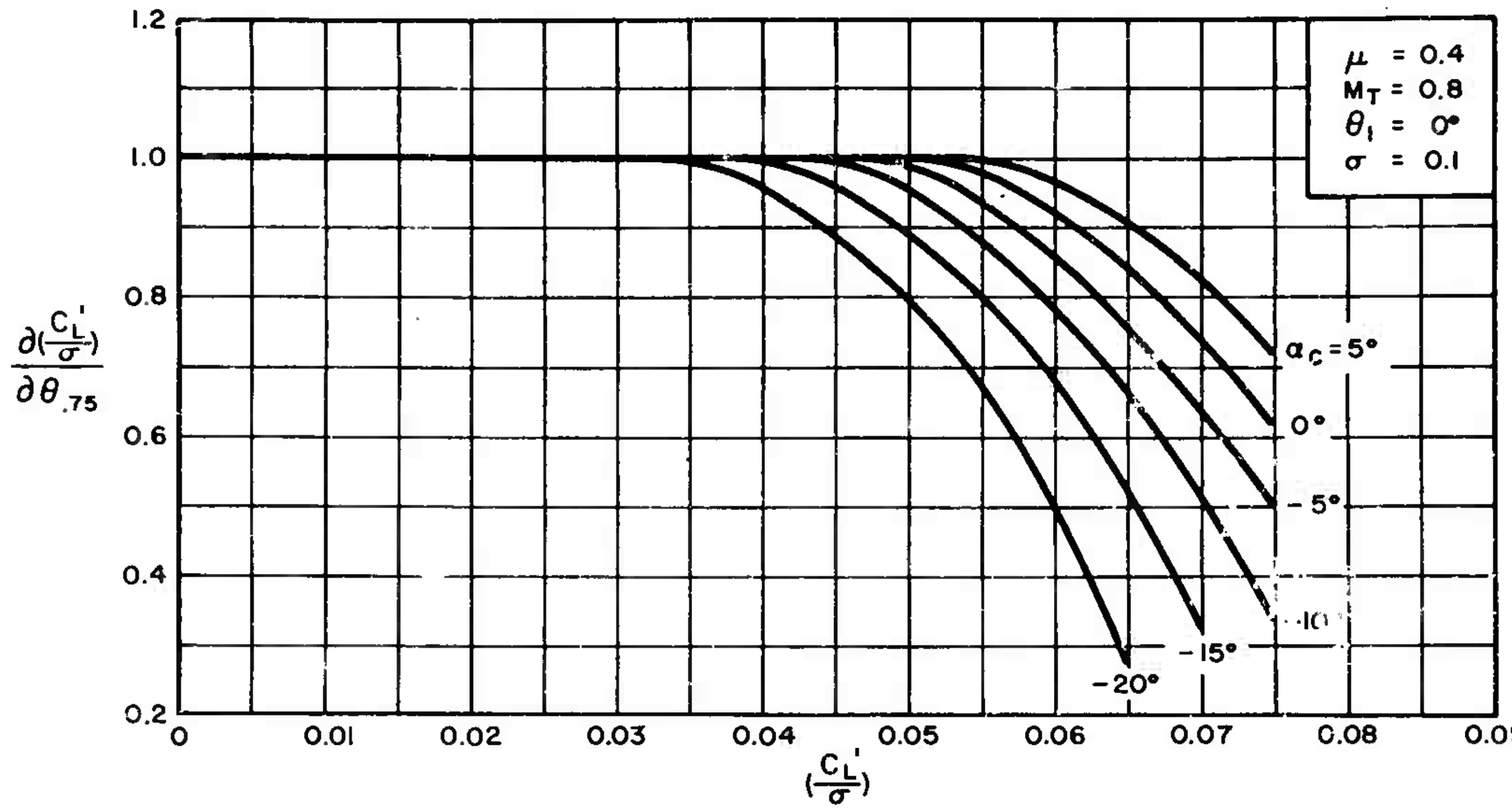

Figure 15. Continued

(b) μ = 0.4

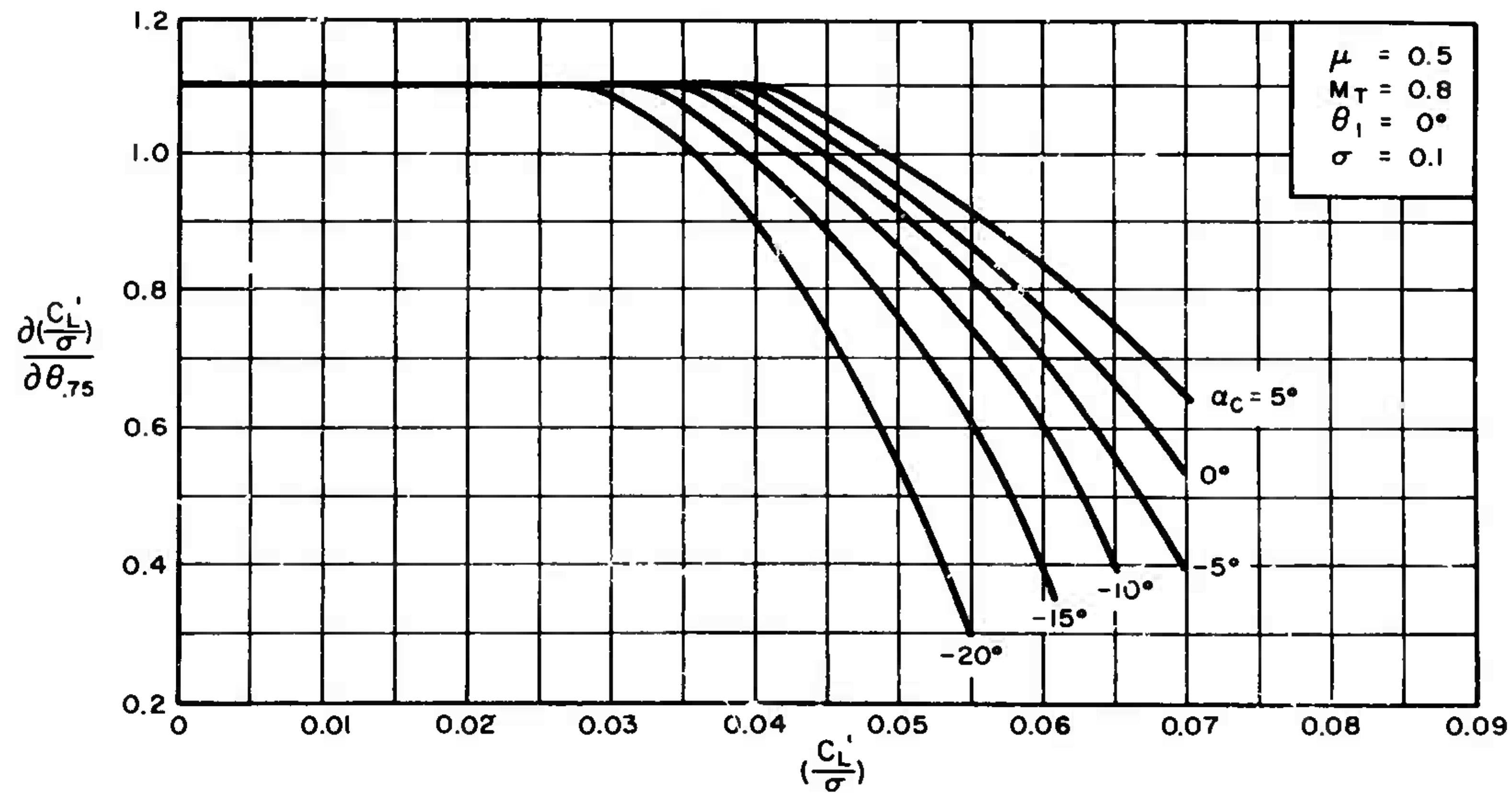

Figure 15. Continued

(c) μ = 0.5

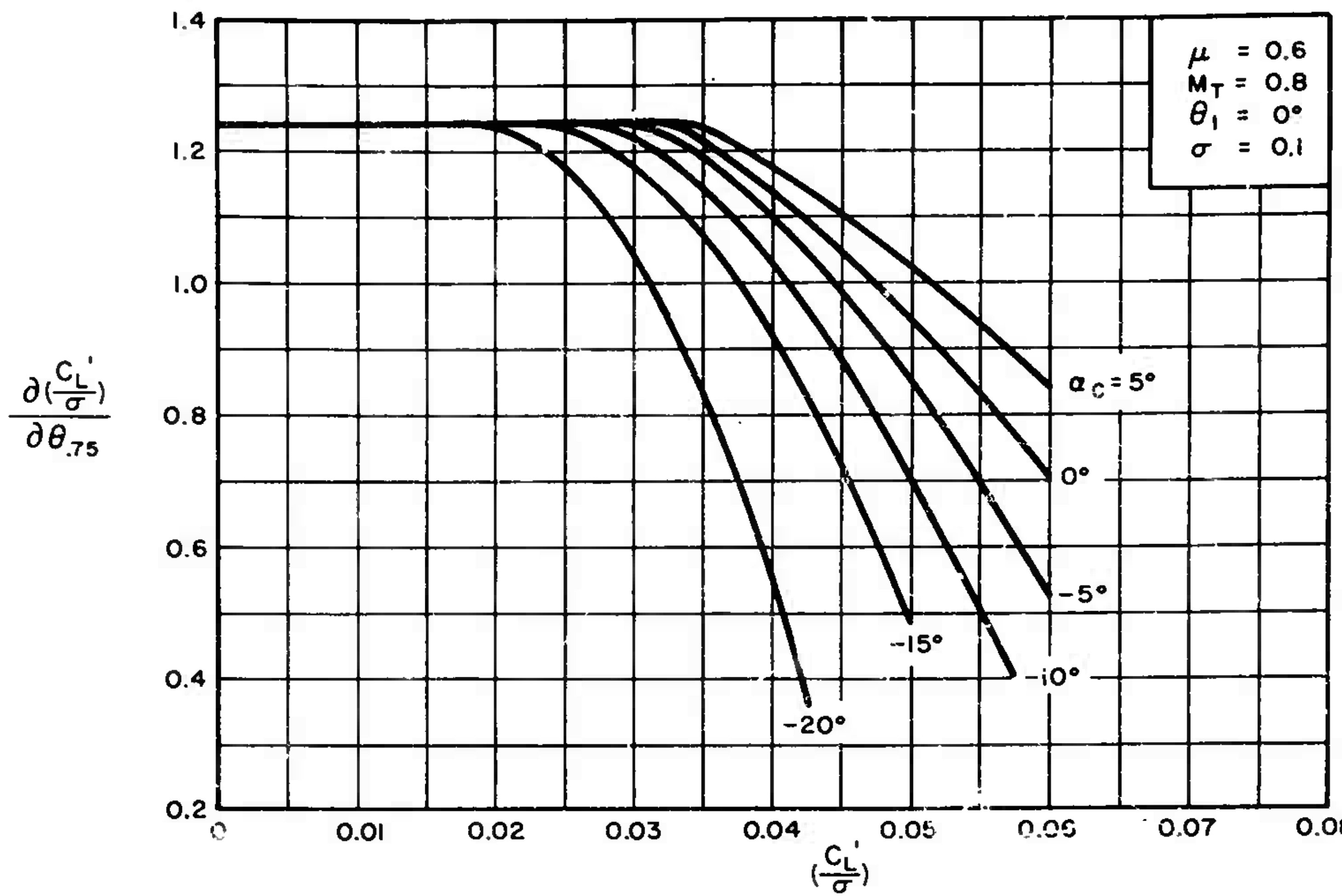

Figure 15. Continued

(d) $\mu = 0.6$

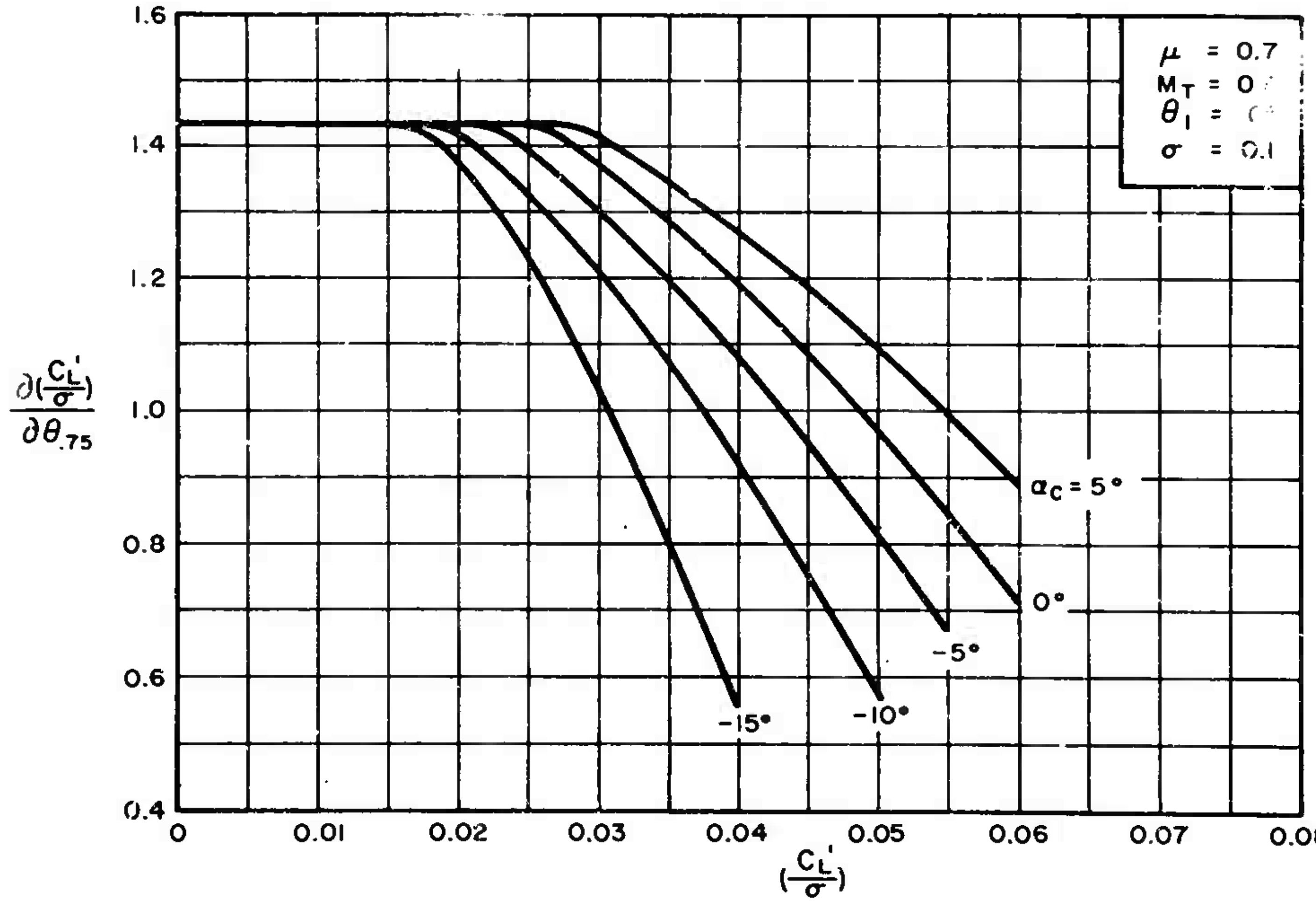

Figure 15. Continued

(e) $\mu = 0.7$

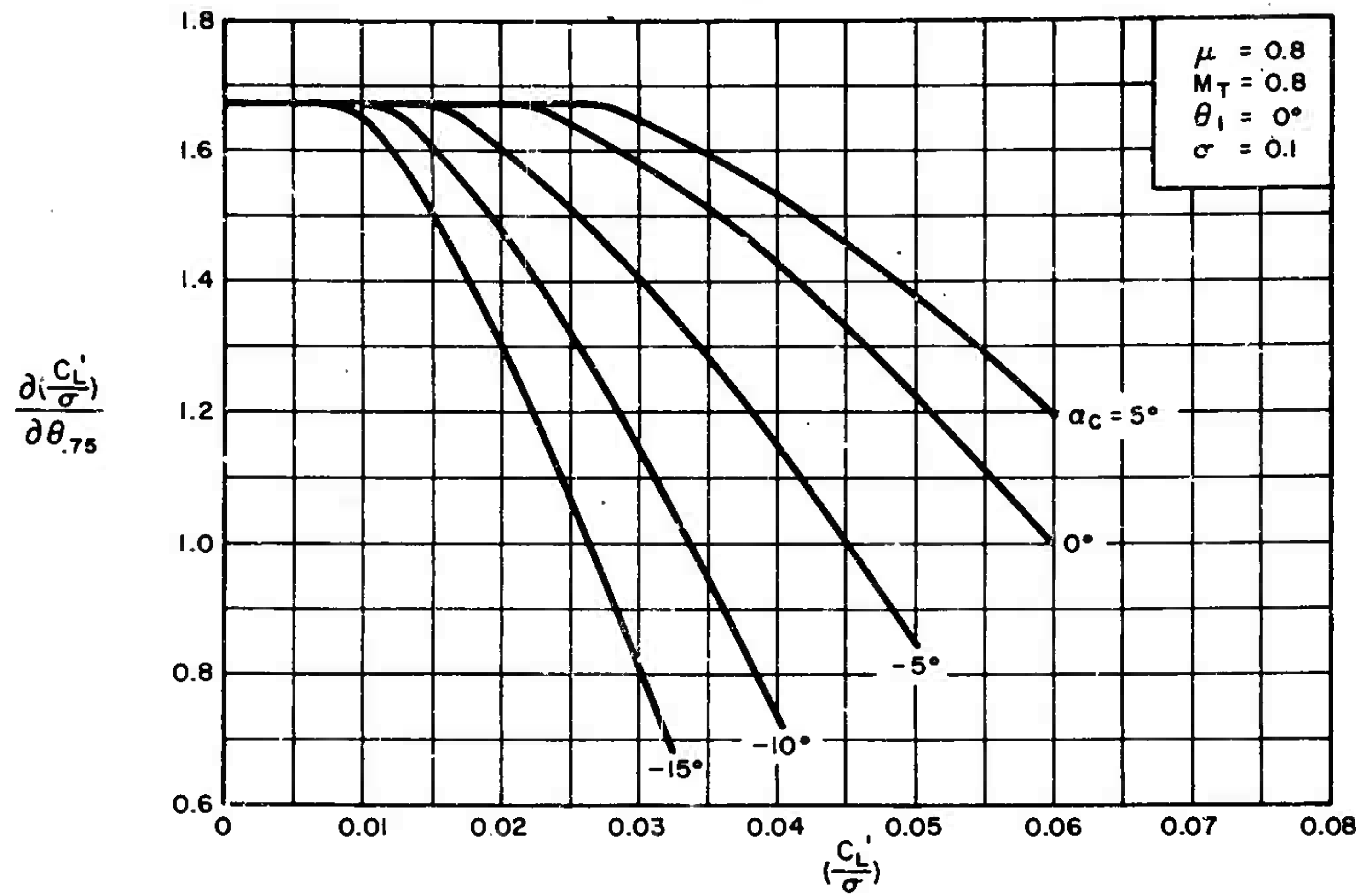

Figure 15. Continued

(f) $\mu = 0.8$

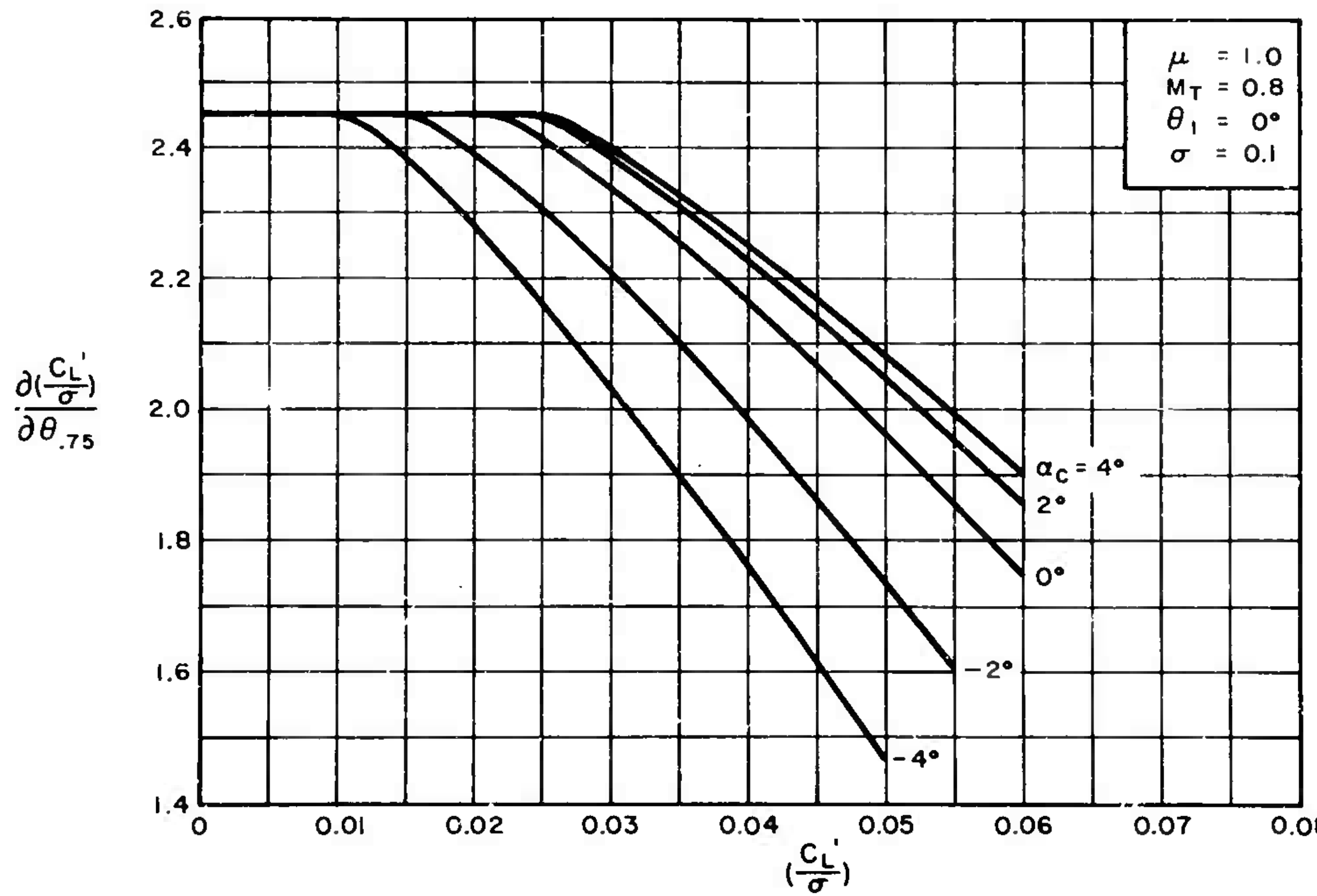

Figure 15. Concluded

(g) $\mu = 1.0$

7.5.3.2 $\dfrac{\partial(\frac{C_D'}{\sigma})}{\partial\theta_{.75}}$ for $\sigma = 0.1$, $\theta_1 = 0°$, and $M_T = 0.8$

Figures 16(a) through 16(i) present the isolated rotor derivative $\partial(C_D'/\sigma)/\partial\theta_{.75}$ as a function of C_L'/σ for constant values of α_C and a range of μ from $\mu = 0.1$ through $\mu = 1.0$.

The above derivatives for $\mu \leq 0.2$ were obtained by using the following equation:

$$\frac{\partial(\frac{C_D'}{\sigma})}{\partial\theta_{.75}} = \frac{\partial(\frac{C_T}{\sigma})}{\partial\theta_{.75}} \sin\alpha_C + \frac{\partial(\frac{C_H}{\sigma})}{\partial\theta_{.75}} \cos\alpha_C$$

where $\partial(C_T/\sigma)/\partial\theta_{.75}$ and $\partial(C_H'/\sigma)/\partial\theta_{.75}$ were obtained from Reference 2. For $\mu \geq 0.3$, $\partial(C_D'/\sigma)/\partial\theta_{.75}$ was extracted graphically from the theoretical rotor performance data of Reference 1.

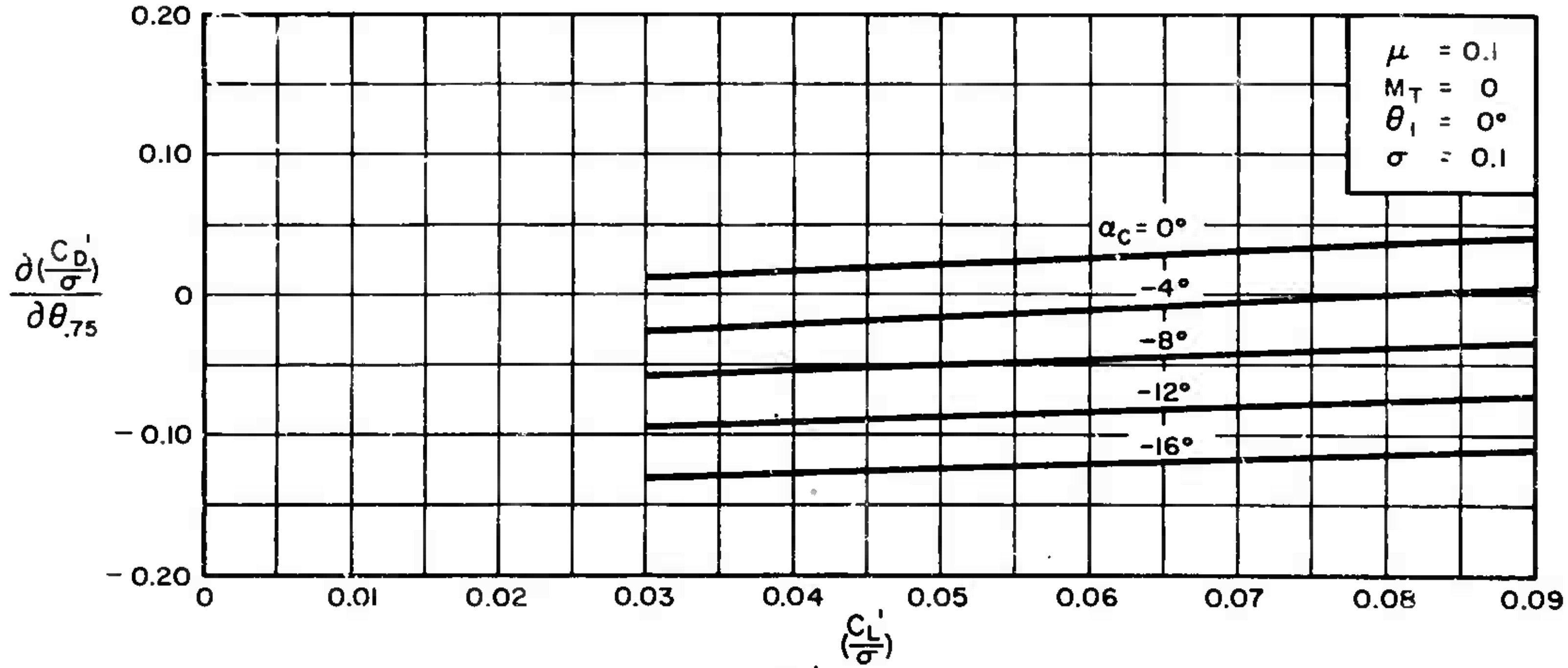

Figure 16. Variation of $\frac{\partial(\frac{C_D'}{\sigma})}{\partial\theta_{.75}}$ With $\frac{C_L'}{\sigma}$ for Constant Values of α_c

(a) $\mu = 0.1$

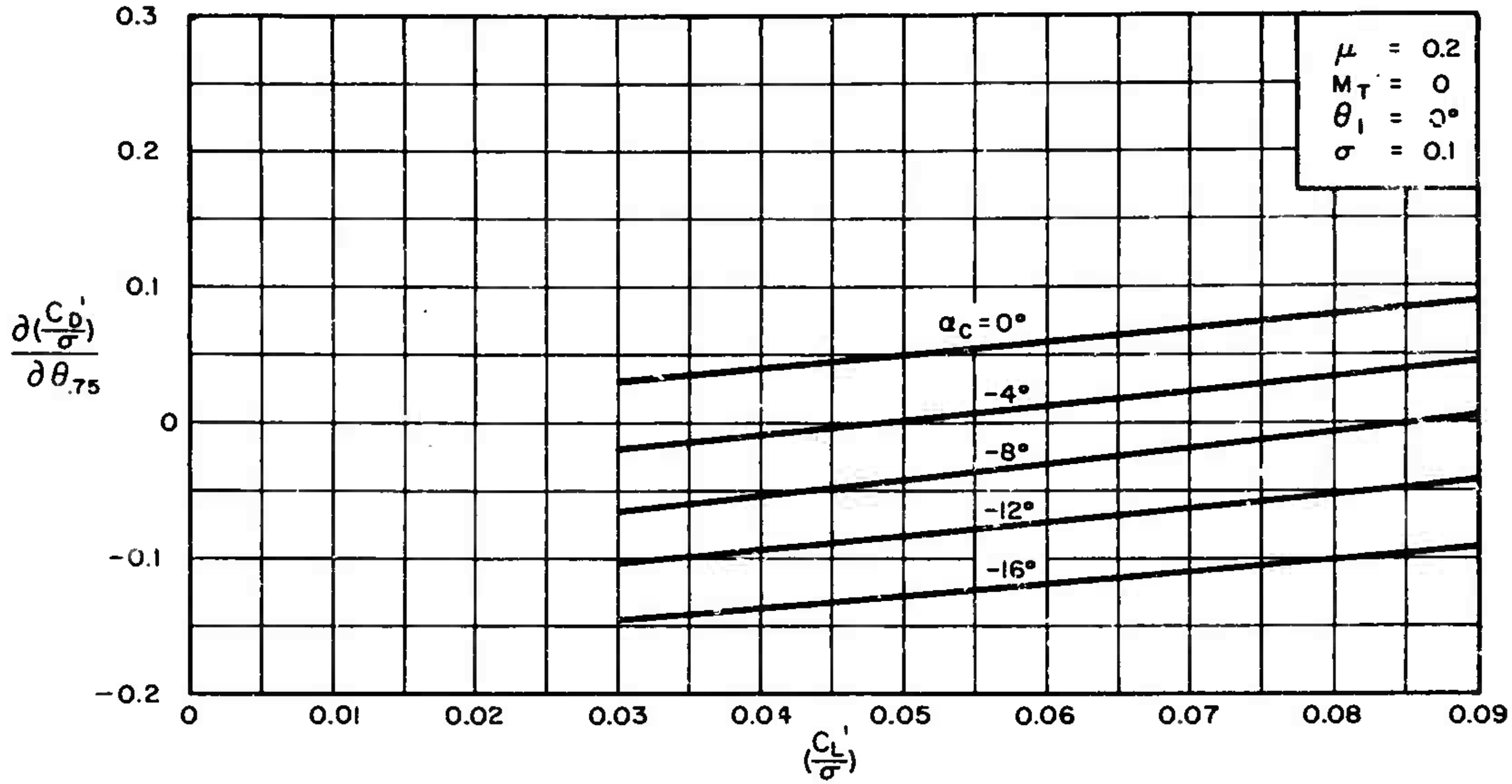

Figure 16. Continued

(b) μ = 0.2

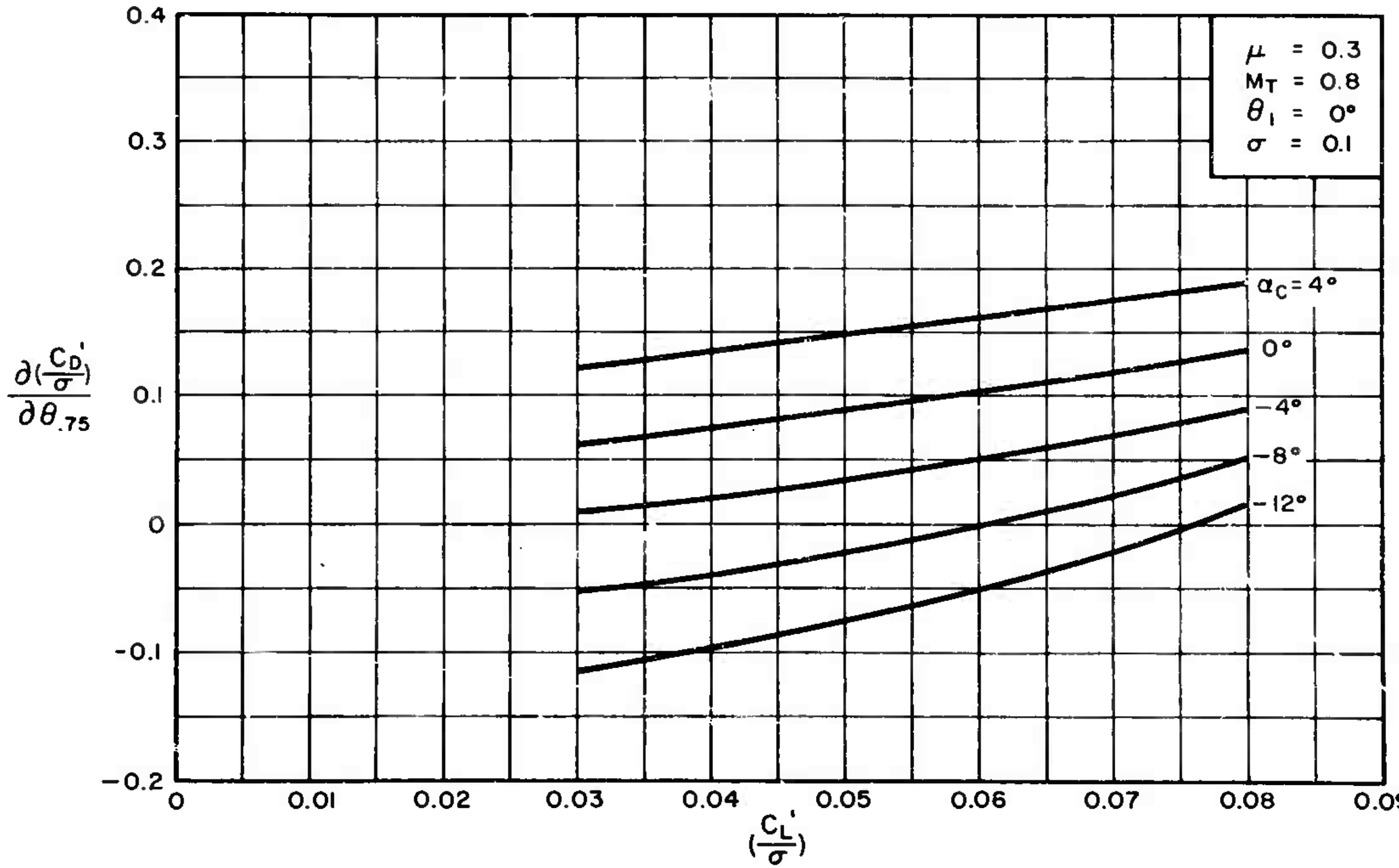

Figure 16. Continued

(c) $\mu = 0.3$

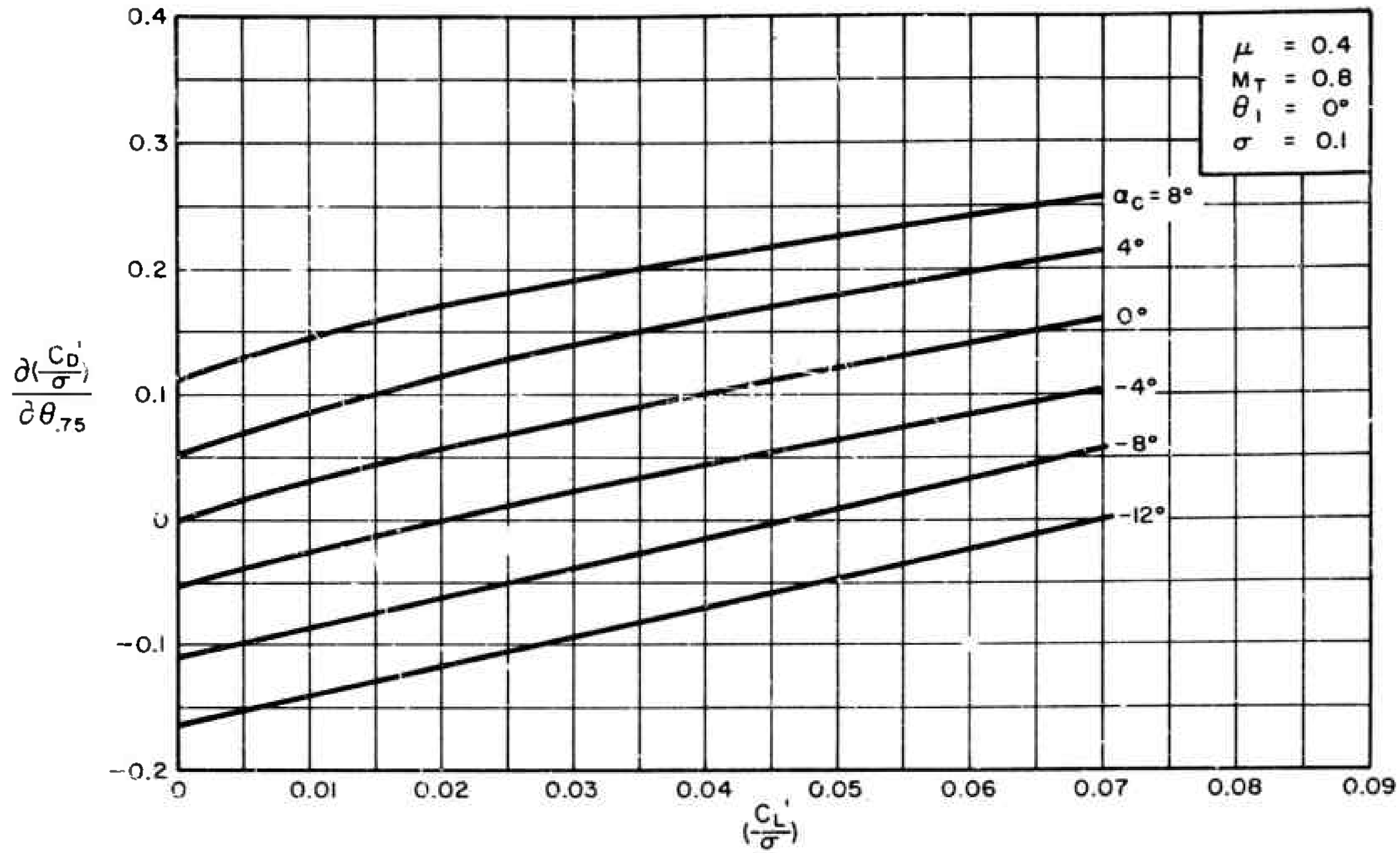

Figure 16. Continued

(d) $\mu = 0.4$

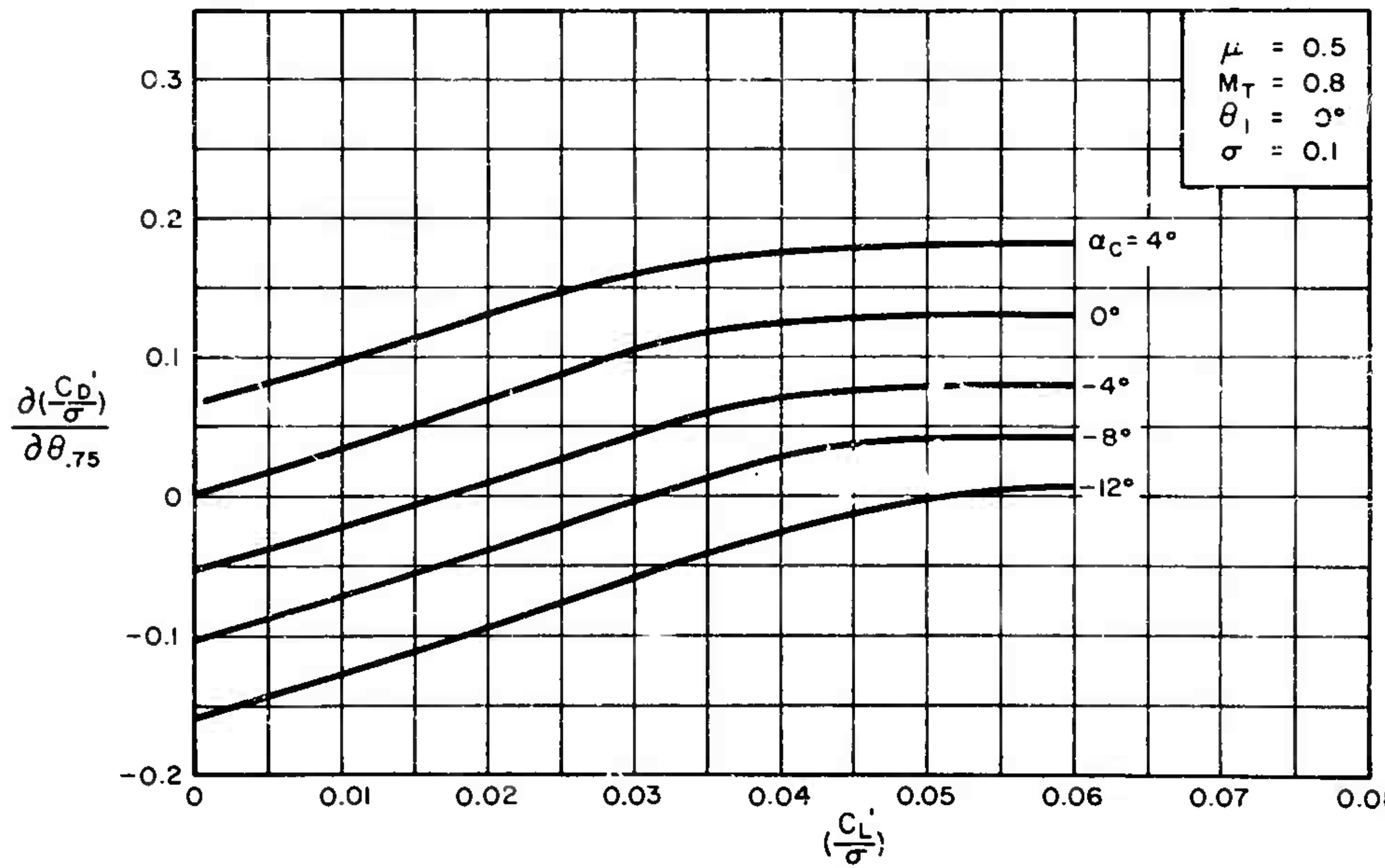

Figure 16. Continued

(e) $\mu = 0.5$

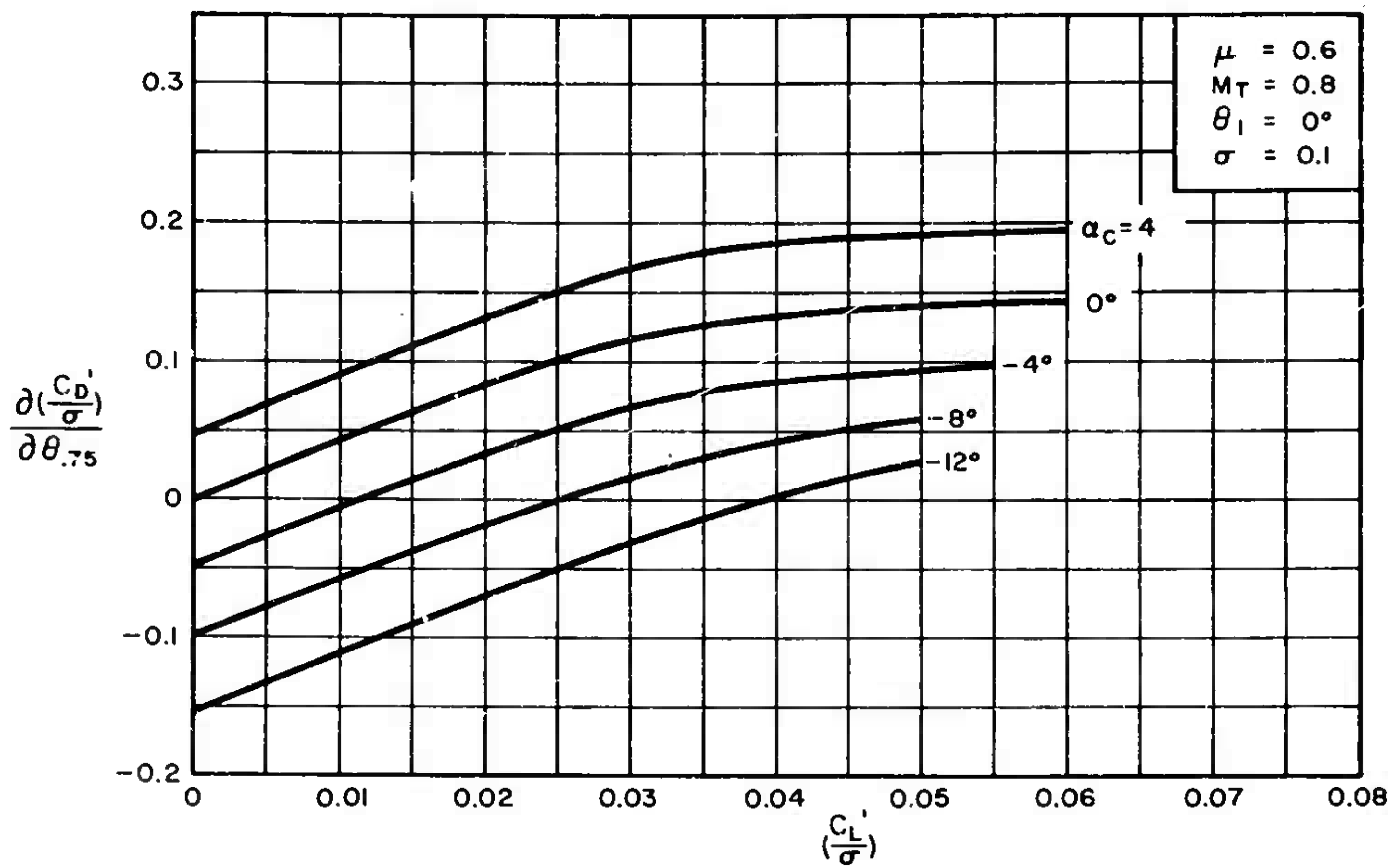

Figure 16. Continued

(f) $\mu = 0.6$

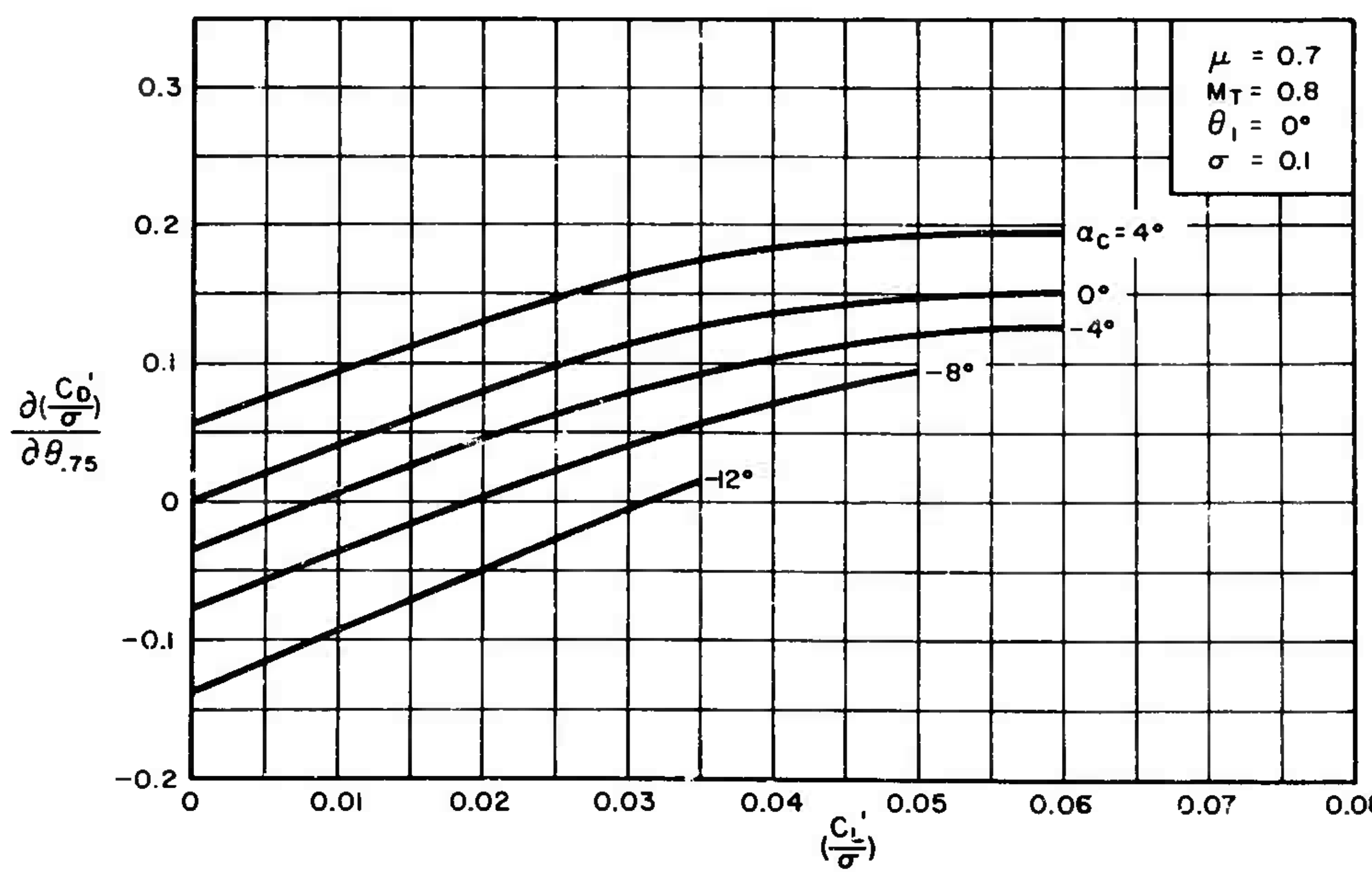

Figure 16. Continued

(g) μ = 0.7

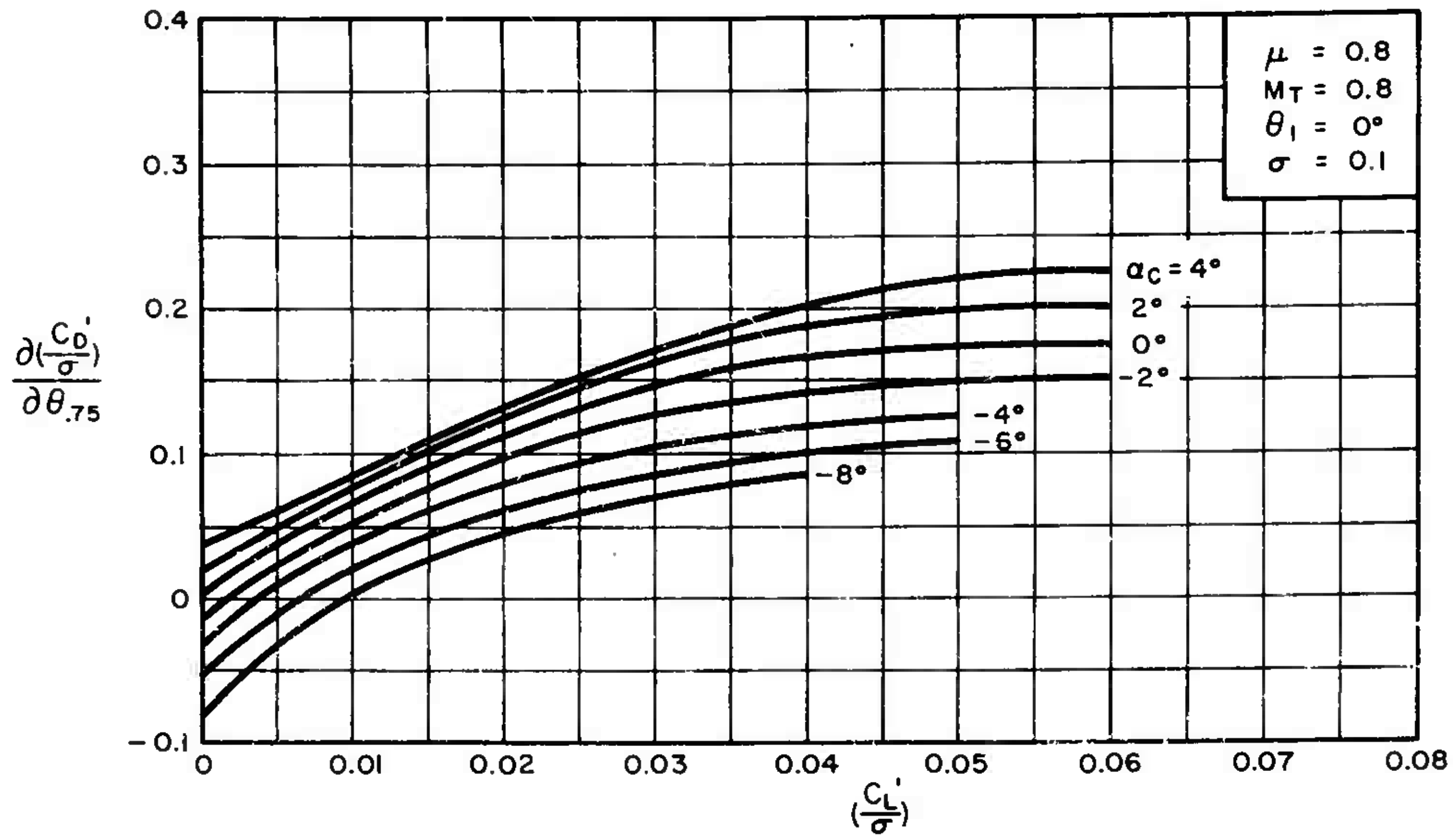

Figure 16. Continued

(h) $\mu = 0.8$

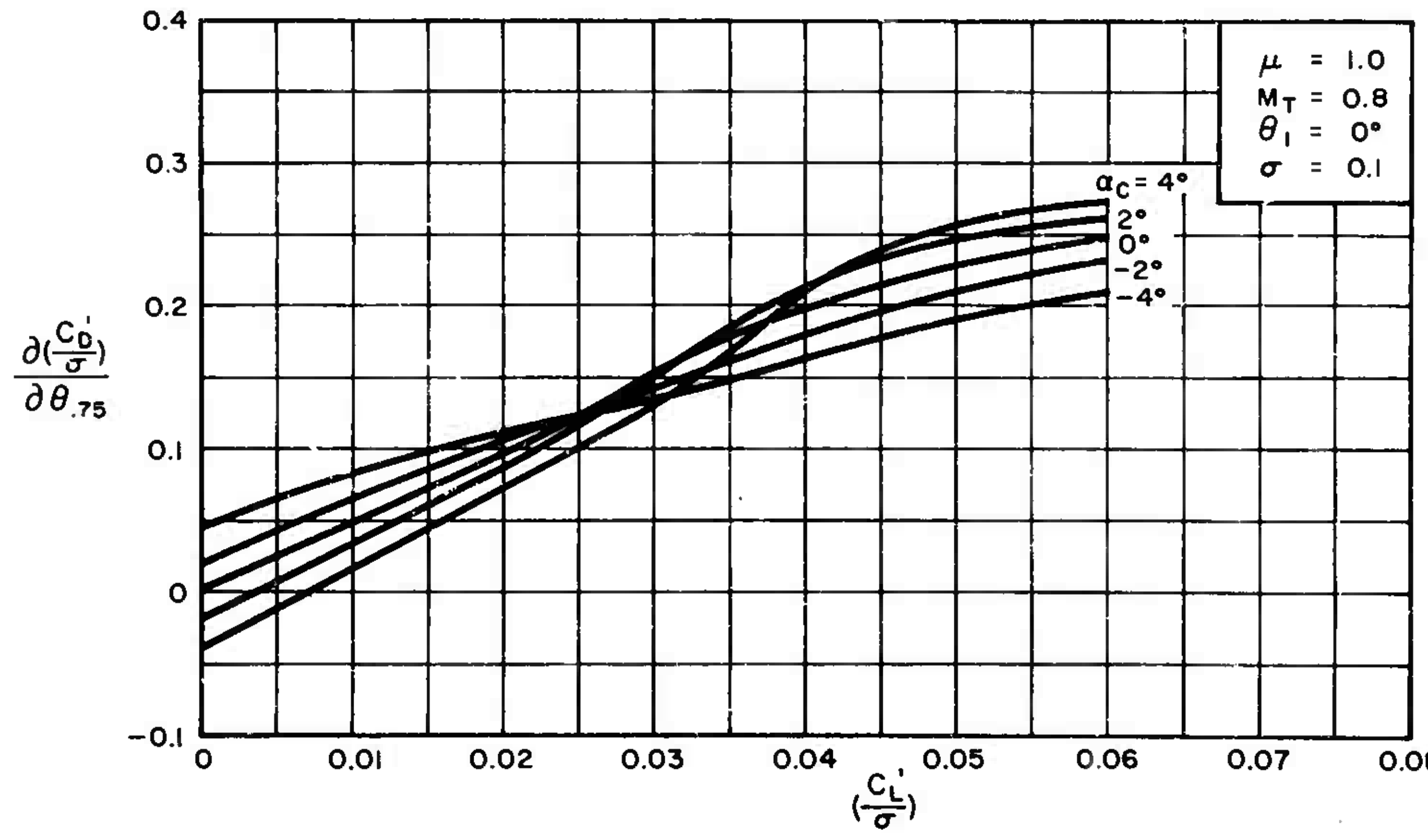

Figure 16. Concluded

(i) μ = 1.0

7.5.3.3 $\frac{\partial(\frac{C_Q}{\sigma})}{\partial\theta_{.75}}$ for $\sigma = 0.1$, $\theta_1 = 0°$, and $M_T = 0.8$

Figures 17(a) through 17(g) present the isolated rotor derivative $\partial(C_Q/\sigma)/\partial\theta_{.75}$ as a function of C_L'/σ for constant values of α_C and a range of tip speed ratios from $\mu = 0.3$ through $\mu = 1.0$. The values of the above derivatives for $\mu \geq 0.3$ were extracted graphically from the theoretical rotor performance data of Reference 1.

For $\mu \leq 0.2$, the following expression was used:

$$\frac{\partial(\frac{C_Q}{\sigma})}{\partial\theta_{.75}} = \frac{1}{2}\left\{\left[\delta_1 t_{53} + \lambda(\delta_2 t_{56} - a t_{42}) + 2\theta_{.75}(\delta_2 t_{58} - a t_{44})\right]\right.$$

$$\left. + \frac{\partial\lambda}{\partial\theta_{.75}}\left[\delta_1 t_{52} + 2\lambda(\delta_2 t_{55} - a t_{41}) + \theta_{.75}(\delta_2 t_{56} - a t_{42})\right]\right\}$$

where $\partial\lambda/\partial\theta_{.75}$ is presented in Subsection 7.5.3.6, and where δ_1, δ_2, t_{53}, t_{56}, t_{58}, t_{52}, ... can be obtained from Reference 3.

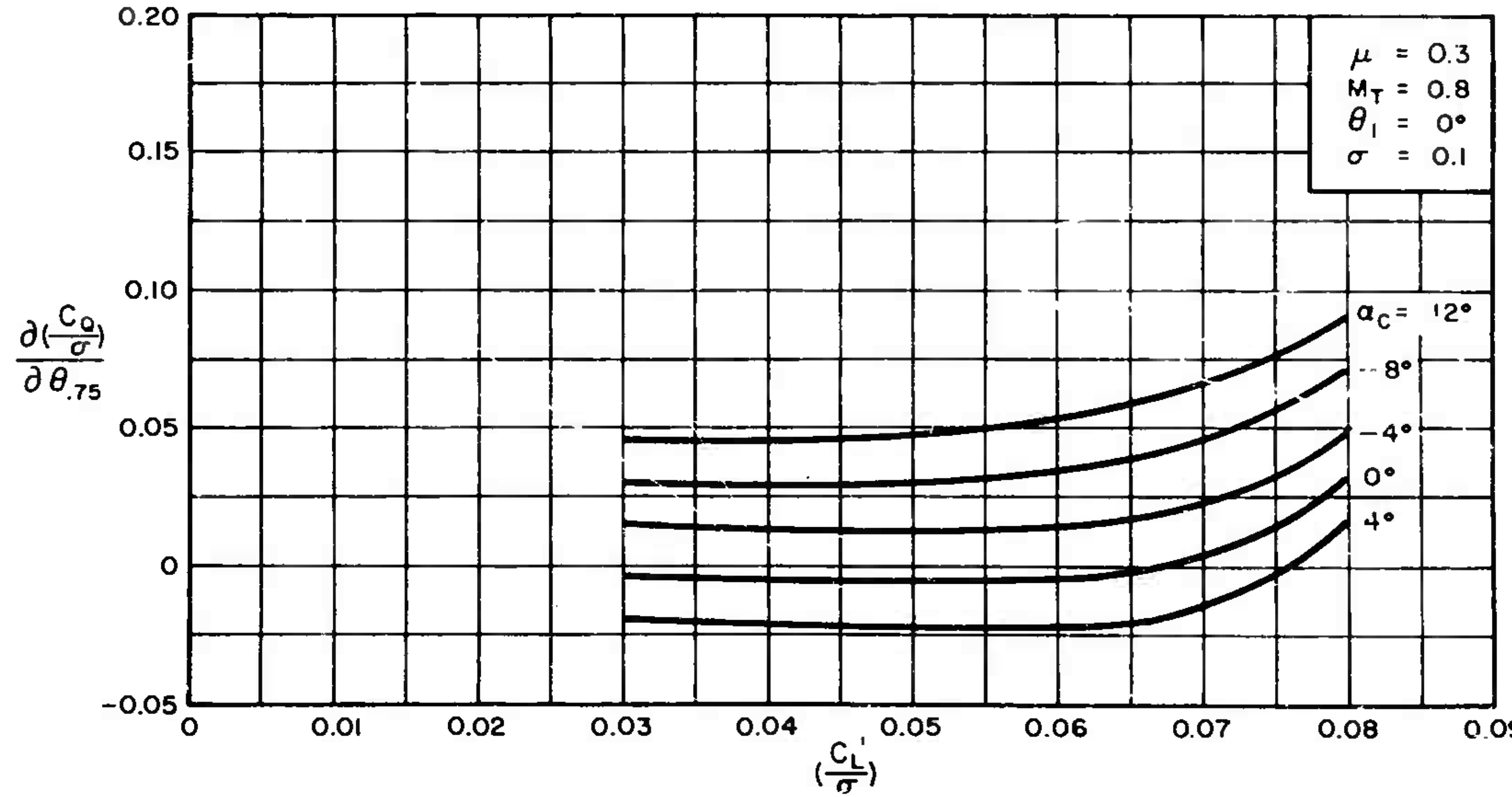

Figure 17. Variation of $\frac{\partial(\frac{C_Q}{\sigma})}{\partial \theta_{.75}}$ With $\frac{C_L'}{\sigma}$ for Constant Values of α_C

(a) $\mu = 0.3$

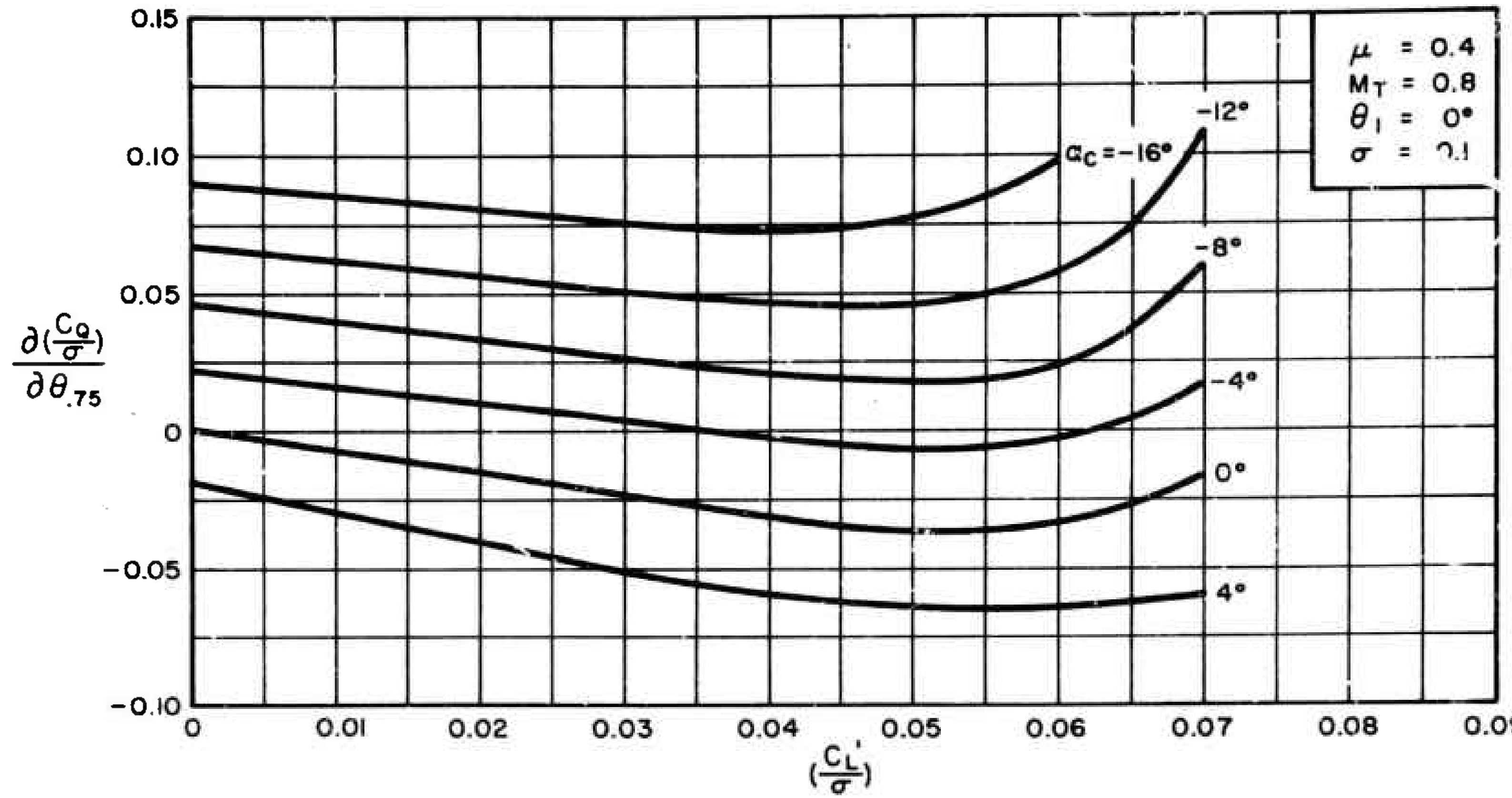

Figure 17. Continued

(b) $\mu = 0.4$

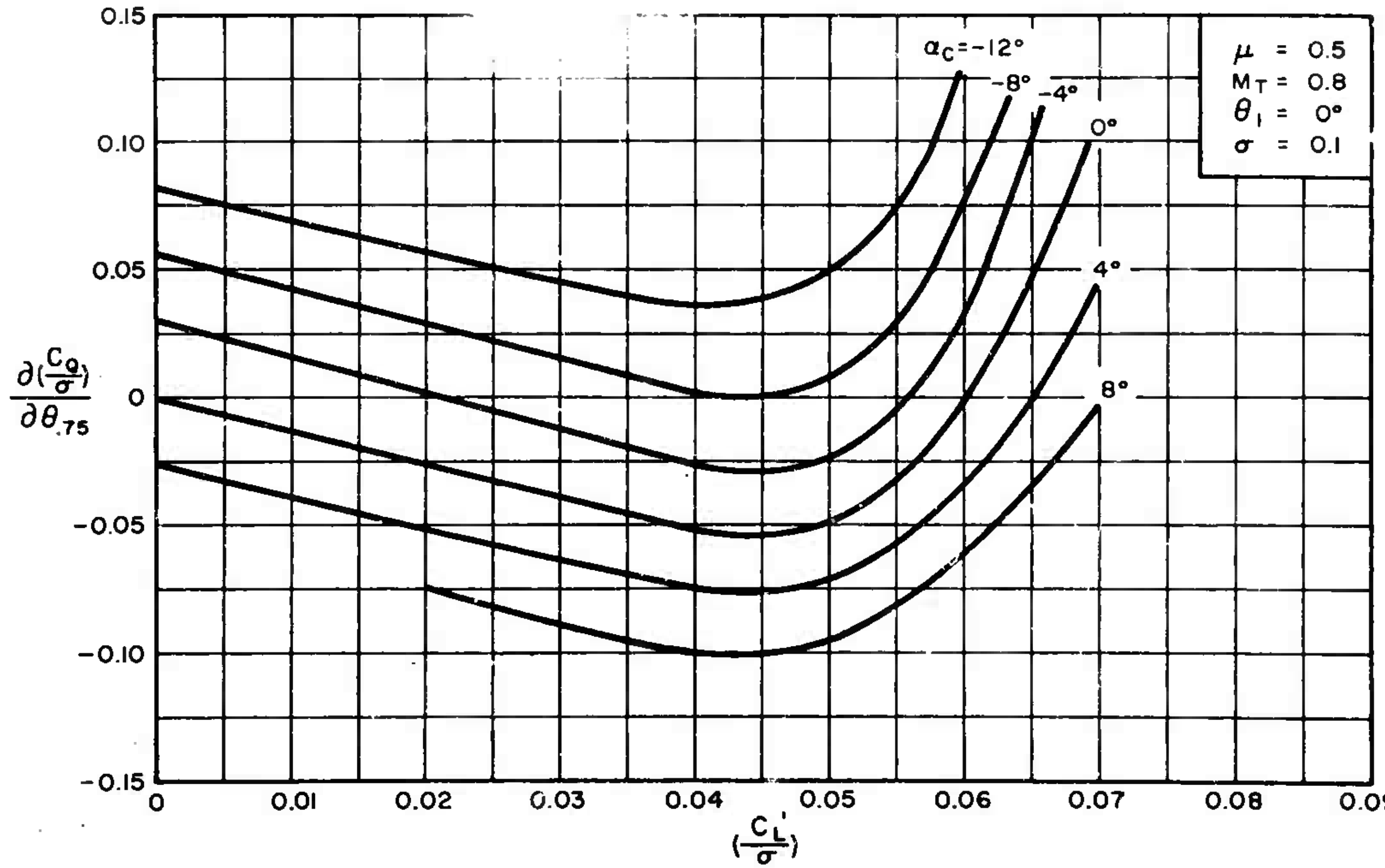

Figure 17. Continued

(c) $\mu = 0.5$

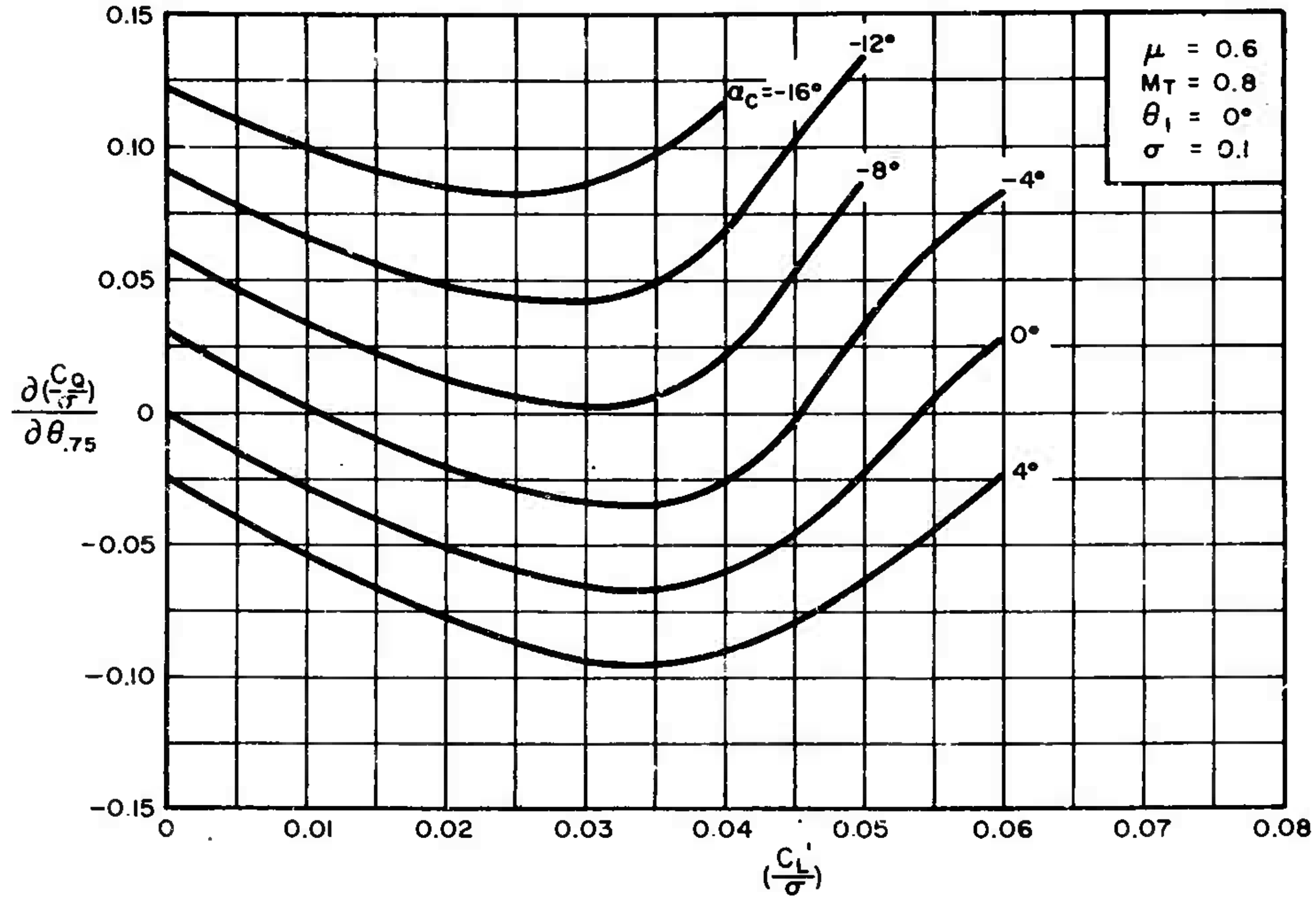

Figure 17. Continued

(d) $\mu = 0.6$

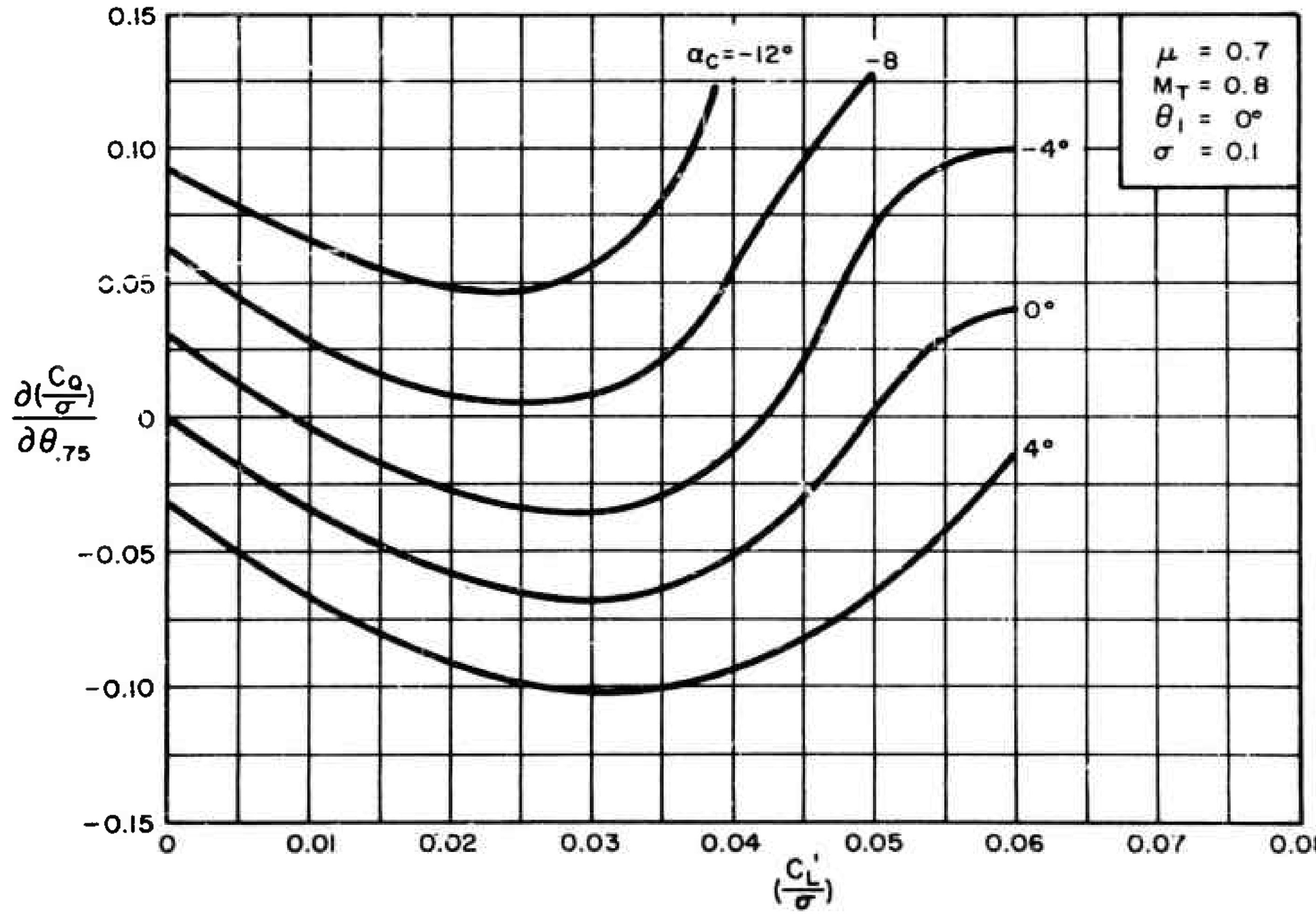

Figure 17. Continued

(e) μ = 0.7

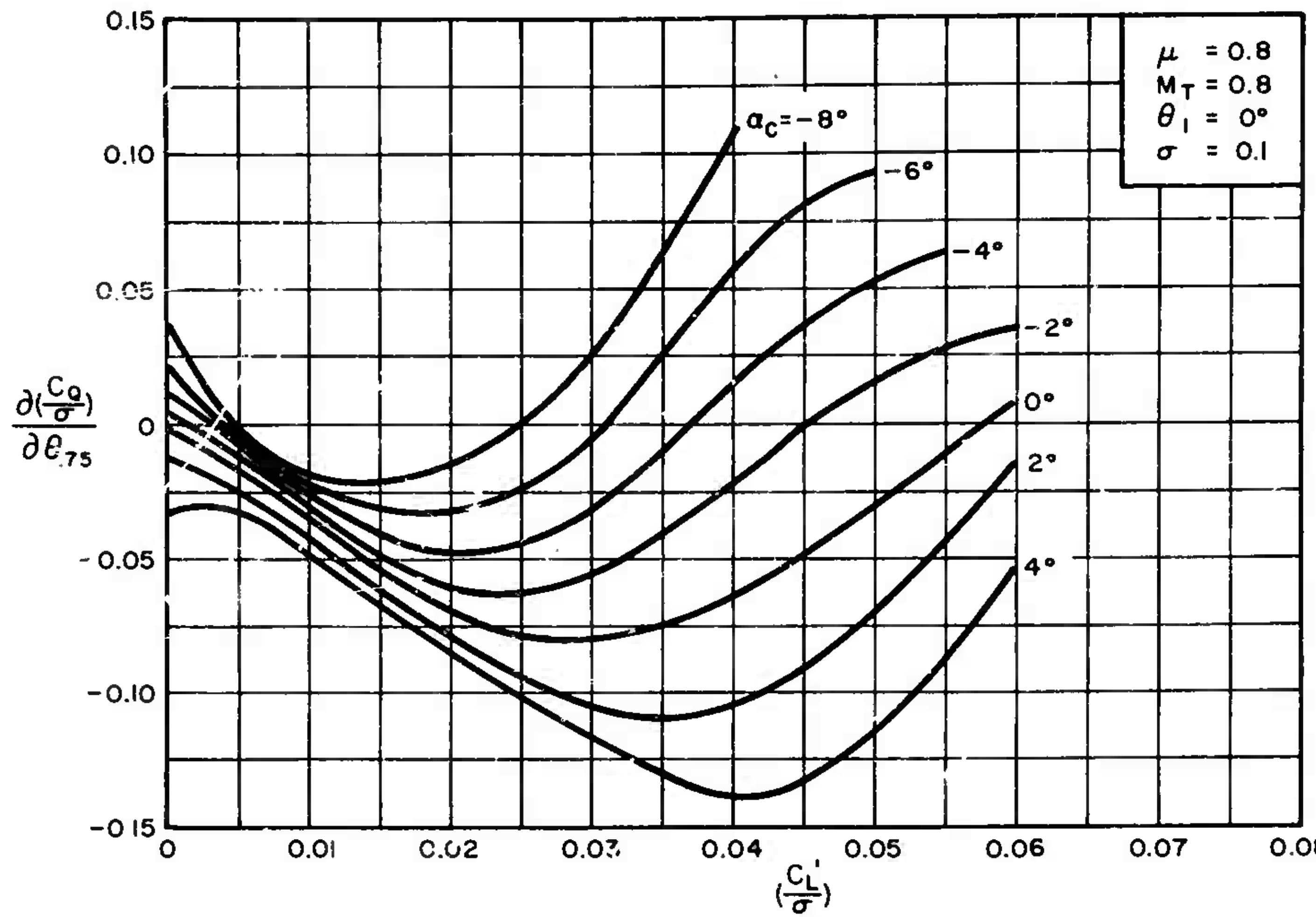

Figure 17. Continued

(f) μ = 0.8

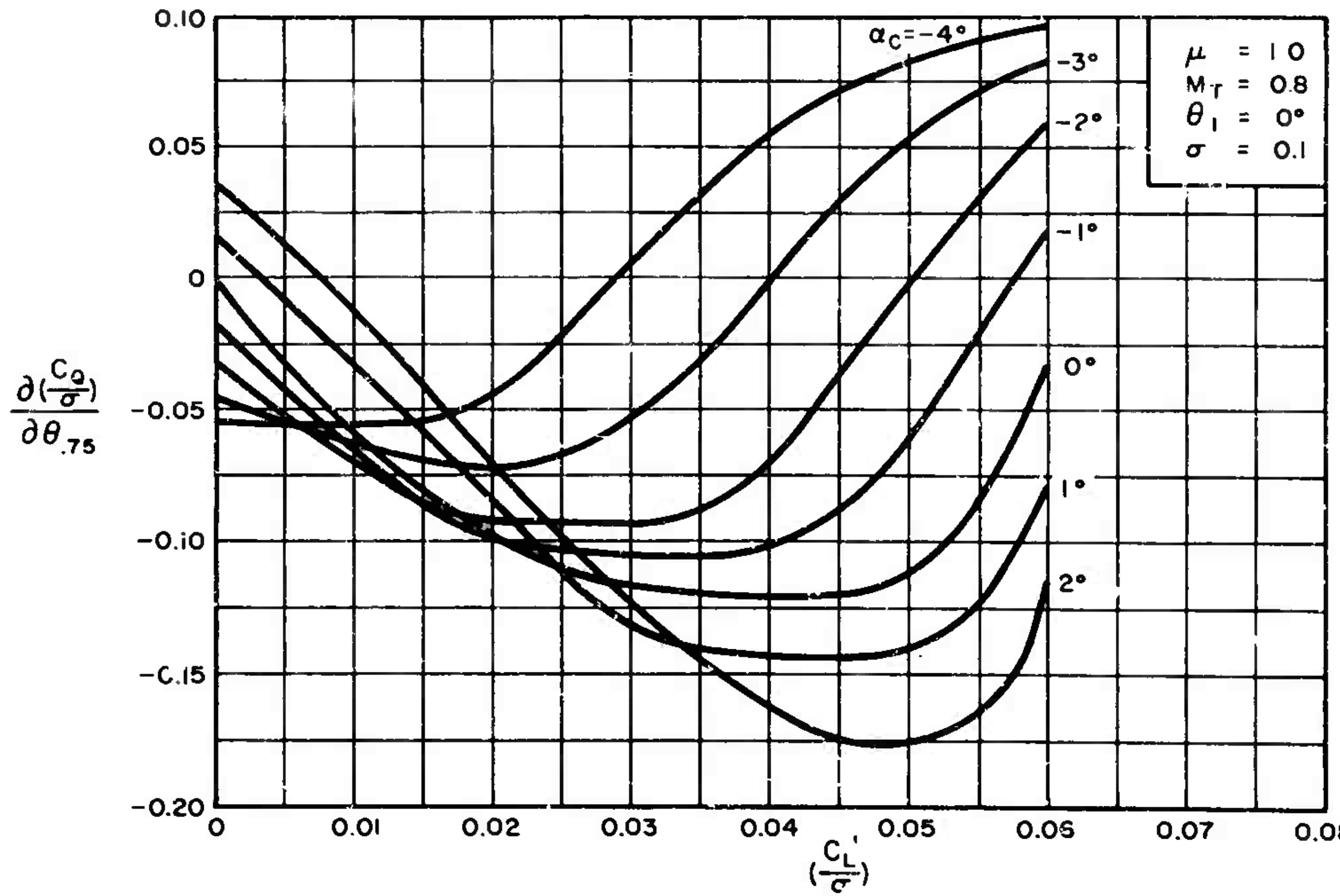

Figure 17. Concluded

(g) μ = 1.0

7.5.3.4 $\dfrac{\partial a_1}{\partial \theta_{.75}}$ for $\sigma = 0.1$, $\theta_1 = 0°$, and $M_T = 0.8$

Figure 18 presents the variation of the isolated rotor derivative $\partial a_1/\partial \theta_{.75}$ as a function of rotor tip speed ratio μ .

For values of $\mu \leq 0.2$, the above derivative was obtained by using the following expression:

$$\frac{\partial a_1}{\partial \theta_{.75}} = t_{14} \frac{\partial \lambda}{\partial \theta_{.75}} + (t_{15})$$

where $\partial \lambda / \partial \theta_{.75}$ is presented in Subsection 7.5.3.6 and where t_{14}, t_{15} can be obtained from Reference 3.

For values of $\mu \geq 0.3$, the $\partial a_1/\partial \theta_{.75}$ derivative was obtained graphically by using the theoretical data of Reference 1.

The results obtained are applicable for all values of $\theta_{.75}$, C_L'/σ, and α_C.

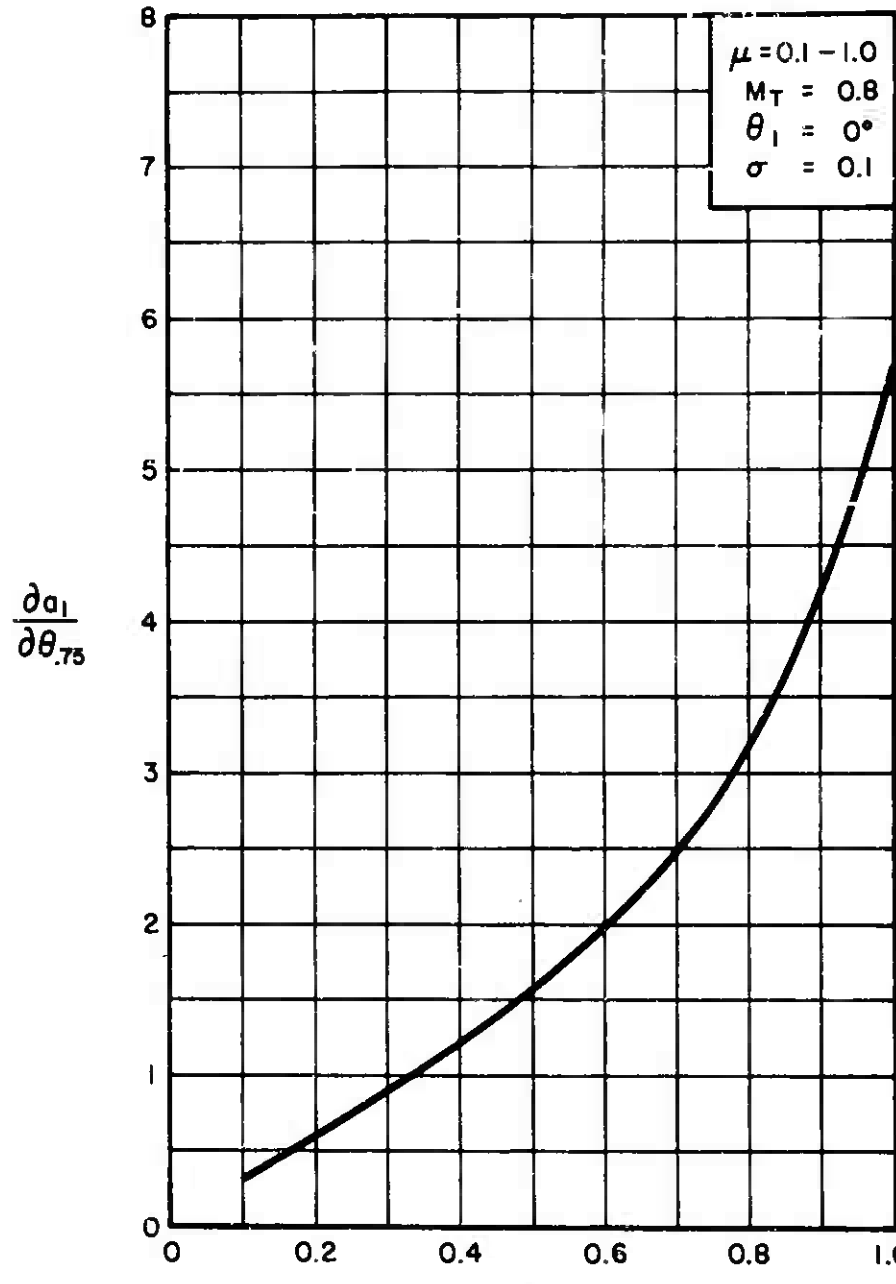

Figure 18. Variation of $\frac{\partial a_1}{\partial \theta_{.75}}$ With μ for All Values of $\theta_{.75}$, $\frac{C_L}{\sigma}$ and α_C.

7.5.3.5 $\frac{\partial b_1}{\partial \theta_{.75}}$ for $\sigma = 0.1$, $\theta_1 = 0°$, and $M_T = 0.8$

Figure 19 presents the variation of the isolated rotor derivative $\partial b_1/\partial \theta_{.75}$ as a function of rotor tip speed ratio μ.

For values of $\mu \leq 0.2$, the above derivative was obtained by using the following expression:

$$\frac{\partial b_1}{\partial \theta_{.75}} = \gamma \left[t_{17} \frac{\partial \lambda}{\partial \theta} + t_{18} \right]$$

where $\partial \lambda/\partial \theta_{.75}$ is presented in Subsection 7.5.3.6 and where t_{17}, t_{18} can be obtained from Reference 3.

For values of $\mu \geq 0.3$, the values of $\partial b_1/\partial \theta_{.75}$ were extracted graphically from the theoretical data of Reference 1.

The results obtained are applicable for all values of $\theta_{.75}$, C_L'/σ, α_C, and $\gamma = 8.0$.

As explained previously for γ values other than 8.0, the $\partial b_1/\partial \theta_{.75}$ derivatives can be obtained as follows:

$$\left(\frac{\partial b_1}{\partial \theta_{.75}}\right)_{\gamma} = \frac{\gamma}{8.0} \left(\frac{\partial b_1}{\partial \theta_{.75}}\right)_{\gamma = 8.0}$$

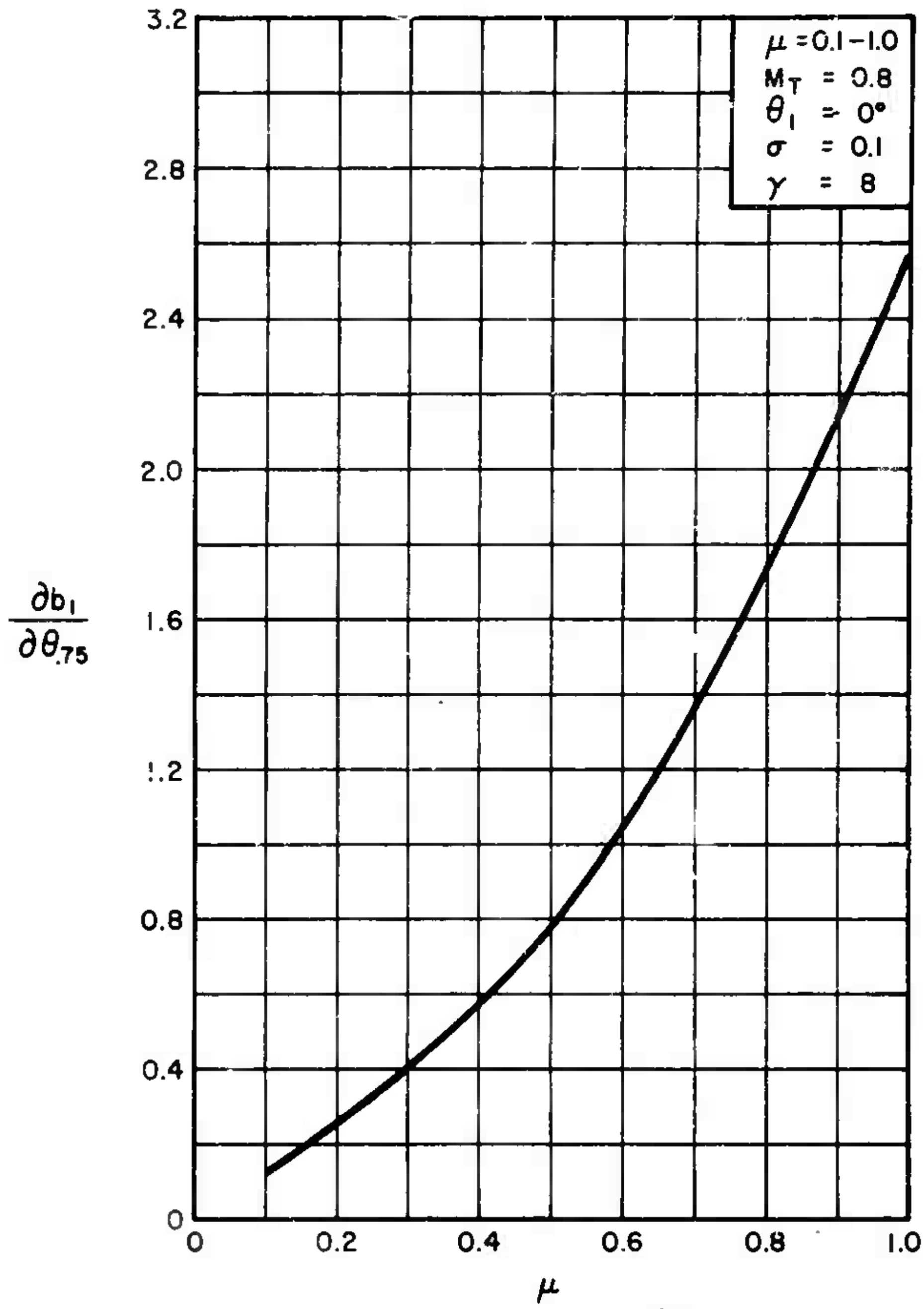

Figure 19. Variation of $\frac{\partial b_1}{\partial \theta_{.75}}$ With μ for All Values of $\theta_{.75}$, $\frac{C_L'}{\sigma}$ and α_C.

7.5.3.6 $\frac{\partial\lambda}{\partial\theta_{.75}}$ for $\sigma = 0.1$, $\theta_1 = 0°$, and $M_T = 0.8$

Figure 20 presents the variation of the isolated rotor derivative $\partial\lambda/\partial\theta_{.75}$ as a function of rotor tip speed ratio μ .

For values of $\mu \leq$ 0.2, the above derivative was obtained by using the following expression:

$$\frac{\partial\lambda}{\partial\theta_{.75}} = \frac{\frac{2}{a}\left[\frac{\partial(\frac{C_L'}{\sigma})}{\partial\theta_{.75}}\right] - t_{32}}{t_{31}}$$

where $\partial(C_L'/\sigma)/\partial\theta_{.75}$ is presented in Subsection 7.5.3.1 and where t_{31}, t_{32} can be obtained from Reference 3.

For $\mu \geq$ 0.3, the values of the $\partial\lambda/\partial\theta_{.75}$ derivative were extracted graphically from the theoretical data of Reference 1.

The results obtained are applicable for all values of $\theta_{.75}$, C_L'/σ, and α_C .

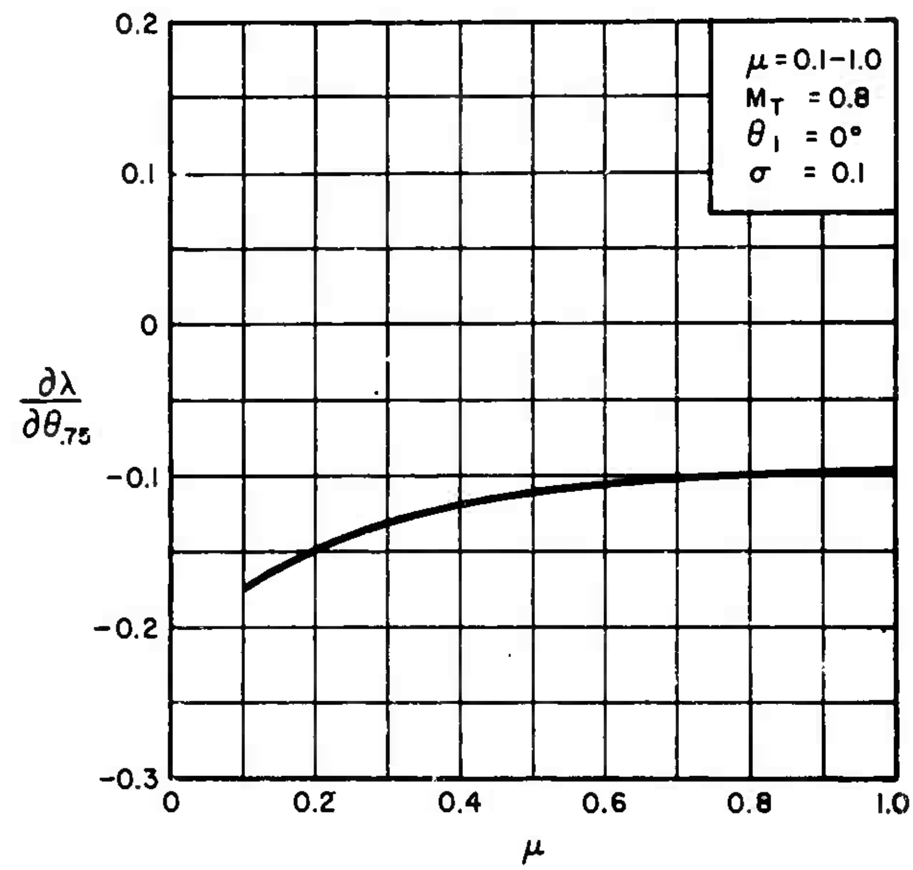

Figure 20. Variation of $\frac{\partial\lambda}{\partial\theta_{.75}}$ With μ for All Values of $\theta_{.75}$, $\frac{C_L'}{\sigma}$ and a_c.

7.5.3.7 $\dfrac{\partial(\frac{C_Y'}{\sigma})}{\partial\theta_{.75}}$ for All Values of σ, θ_1, and M_T

Reference 1 and other reviewed reports do not include the calculated data required to evaluate the rotor Y-force derivatives. It is therefore suggested that the classical Bailey theory be utilized for this purpose. If the above theory is used, the following expression for the isolated rotor derivative $\partial(C_Y'/\sigma)/\partial\theta_{.75}$ can be derived:

$$\frac{\partial(\frac{C_Y'}{\sigma})}{\partial\theta_{.75}} = \frac{a}{2}\left\{\frac{\partial a_0}{\partial\theta_{.75}}\left[\mu(-\frac{3}{4}\theta_{.75}-\frac{3}{2}\lambda-\mu a_1)+\frac{a_1}{6}\right]\right.$$

$$+\frac{\partial a_1}{\partial\theta_{.75}}\left[\frac{a_0}{6}+\mu(-\mu a_0+\frac{b_1}{4})\right]$$

$$+\frac{\partial b_1}{\partial\theta_{.75}}\left[\theta_{.75}(\frac{1}{3}+\frac{3}{8}\mu^2)+\lambda(\frac{3}{4}+\mu^2)+\frac{\mu a_1}{4}\right]$$

$$+\frac{\partial\lambda}{\partial\theta_{.75}}\left[b_1(\frac{3}{4}+\mu^2)-\frac{3}{2}\mu a_0\right]$$

$$\left.-\frac{3}{4}\mu a_0+b_1(\frac{1}{3}+\frac{3}{8}\mu^2)\right\}$$

where

$$\frac{\partial a_0}{\partial\theta_{.75}} = \frac{\gamma}{2}\left[\frac{1}{4}(1-\mu^2)+\frac{1}{3}\frac{\partial\lambda}{\partial\theta_{.75}}\right],$$

and where $\partial a_1/\partial\theta_{.75}$, $\partial b_1/\partial\theta_{.75}$, and $\partial\lambda/\partial\theta_{.75}$ are given in Subsections 7.5.3.4, 7.5.3.5, and 7.5.3.6, respectively.

The above derivative is applicable for all values of σ, θ_1, and M_T, provided that the pertinent rotor parameters comprising this derivative are evaluated at the required condition.

7.5.4 Effect of Blade Twist on the Isolated Rotor Derivatives

This section presents the effect of linear blade twist on various rotor isolated derivatives with respect to the basic variables μ , α_C , and $\theta_{.75}$.

The effect of blade twist on each isolated rotor derivative for a selected range of pertinent rotor parameters is shown on the comparison plots of Figures 21 through 23. The plots present the derivatives for zero blade twists together with the corresponding derivatives obtained for linear twists of -8°. These plots are based on the theoretical data of Reference 1.

7.5.4.1 Effect of Blade Twist on the Isolated Rotor Derivatives With Respect to μ

Figures 21(a) through 21(f) give an indication of the effect of blade twist on the derivatives $\partial(C_L'/\sigma)/\partial\mu$, $\partial(C_D'/\sigma)/\partial\mu$, $\partial(C_Q/\sigma)/\partial\mu$, $\partial a_1/\partial\mu$, $\partial b_1/\partial\mu$, $\partial\lambda/\partial\mu$, respectively. A rotor tip speed ratio of $\mu = 0.4$ and an advancing tip Mach number of $M_T = 0.8$ are selected for this presentation. The collective pitch range covers values from $\theta_{.75} = -4°$ to $\theta_{.75} = 12°$.

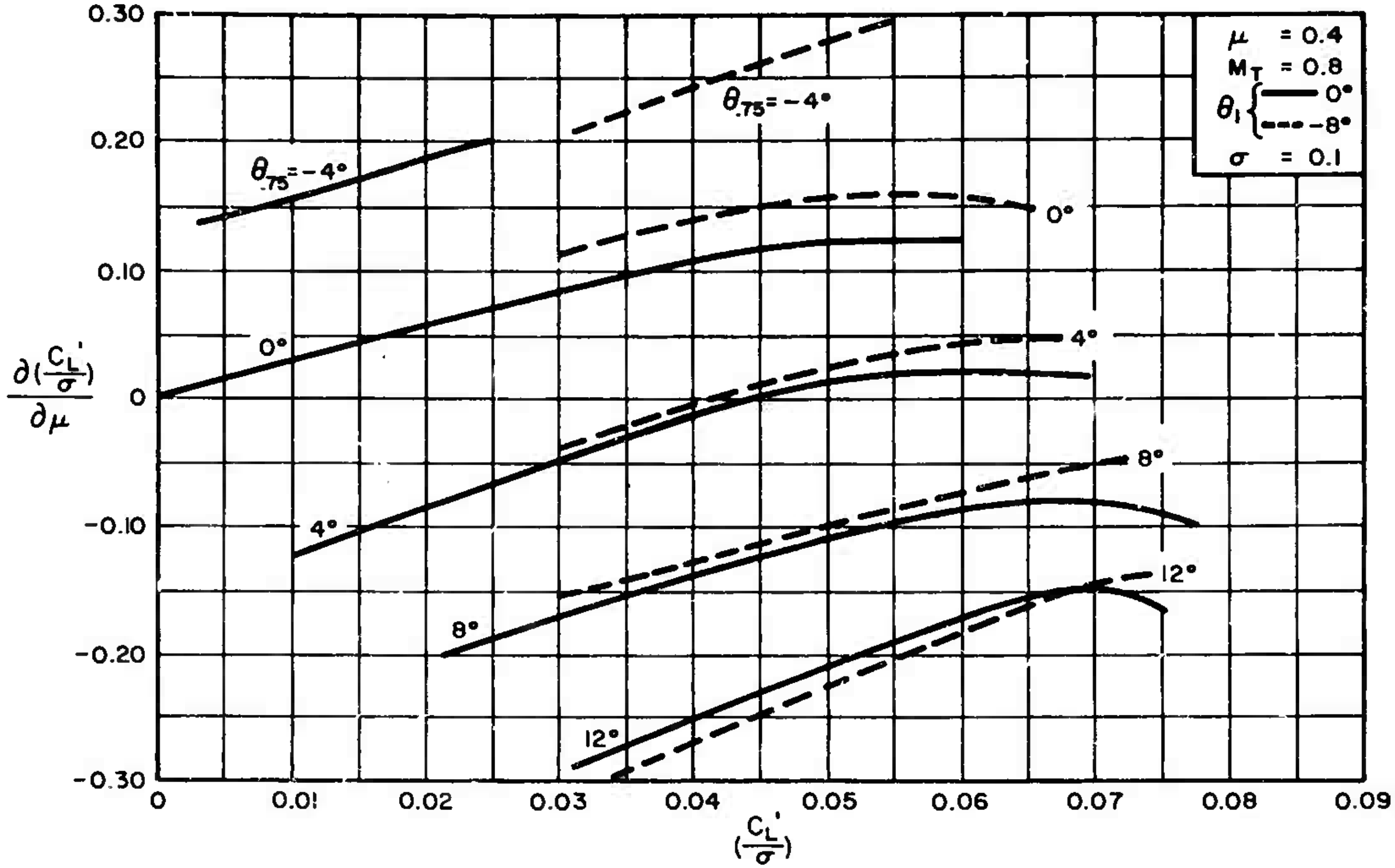

Figure 21. Effect of Blade Twist on μ Derivatives

(a) $\partial(\frac{C_L'}{\sigma})/\partial\mu$

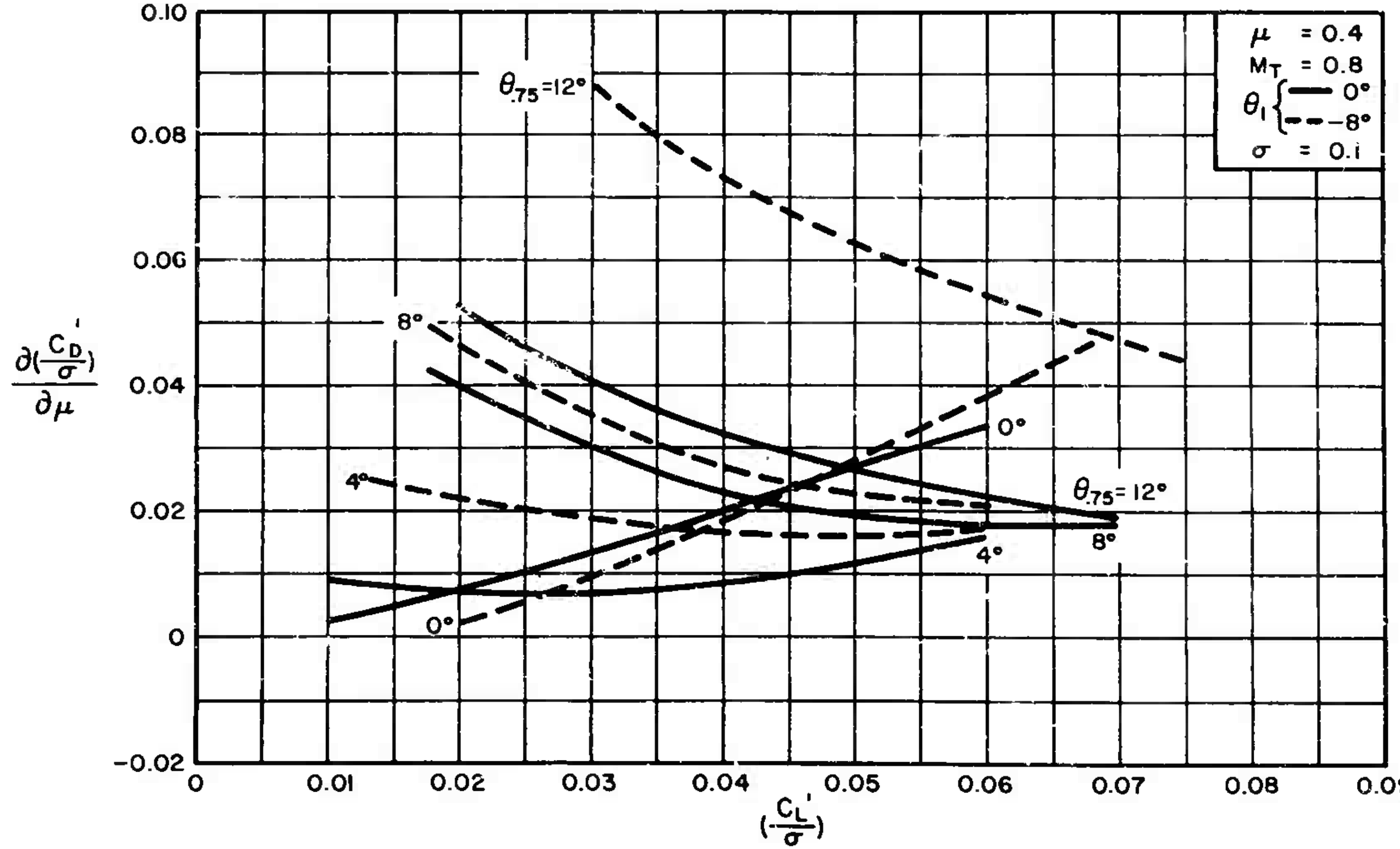

Figure 21. Continued

(b) $\partial(\frac{C_D'}{\sigma})/\partial\mu$

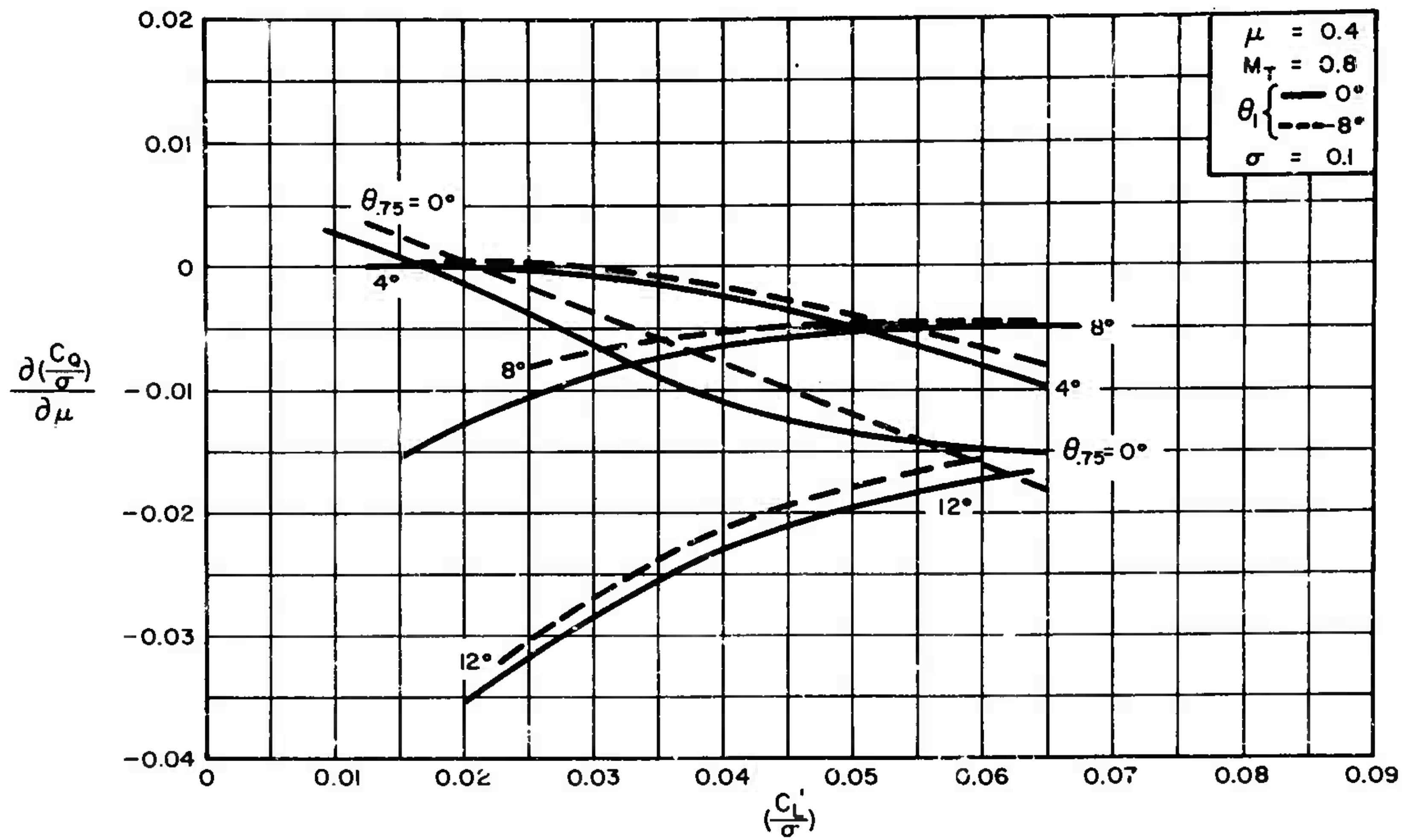

Figure 21. Continued

(c) $\partial(\frac{C_Q}{\sigma})/\partial\mu$

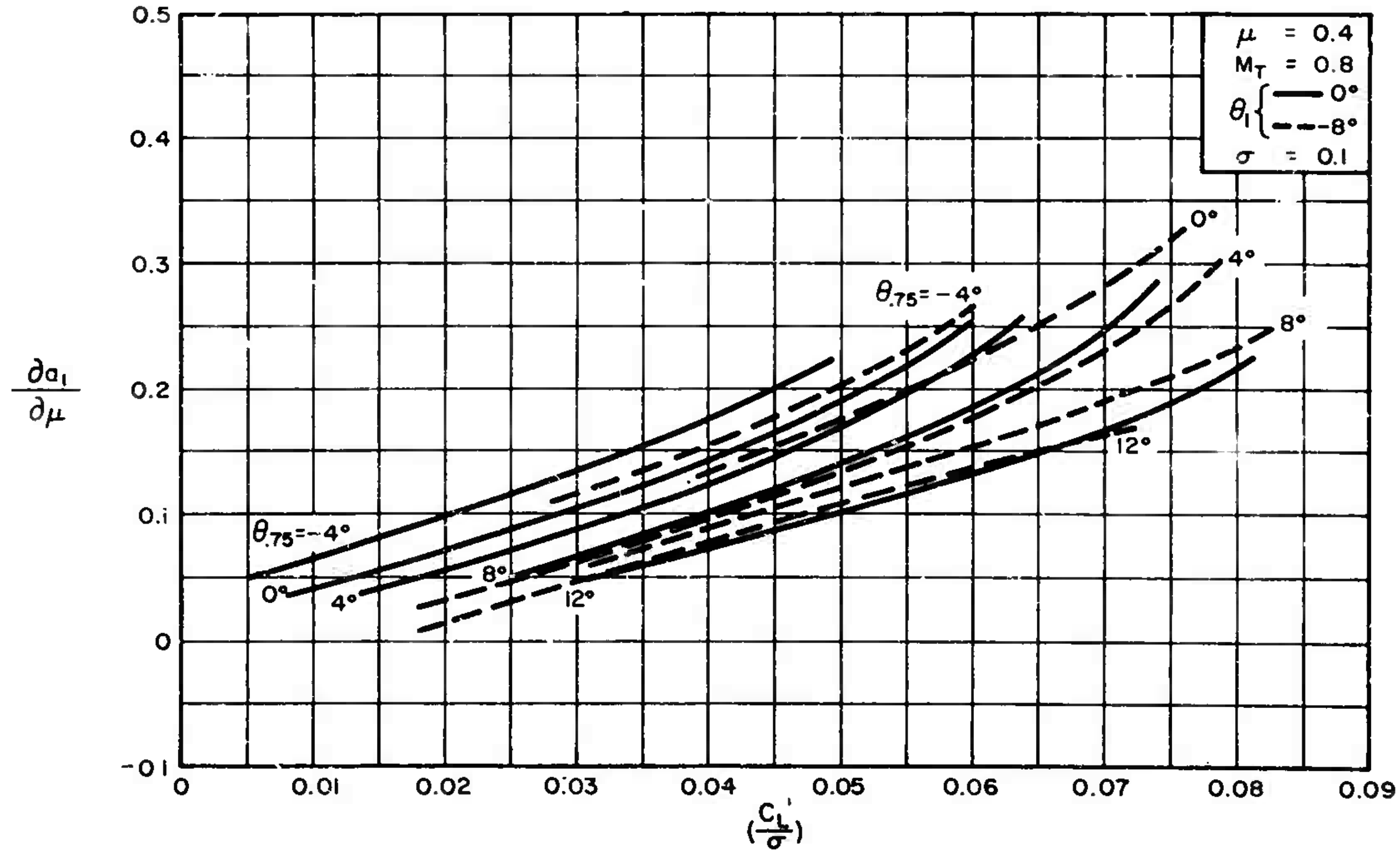

Figure 21. Continued

(d) $\partial a_1/\partial \mu$

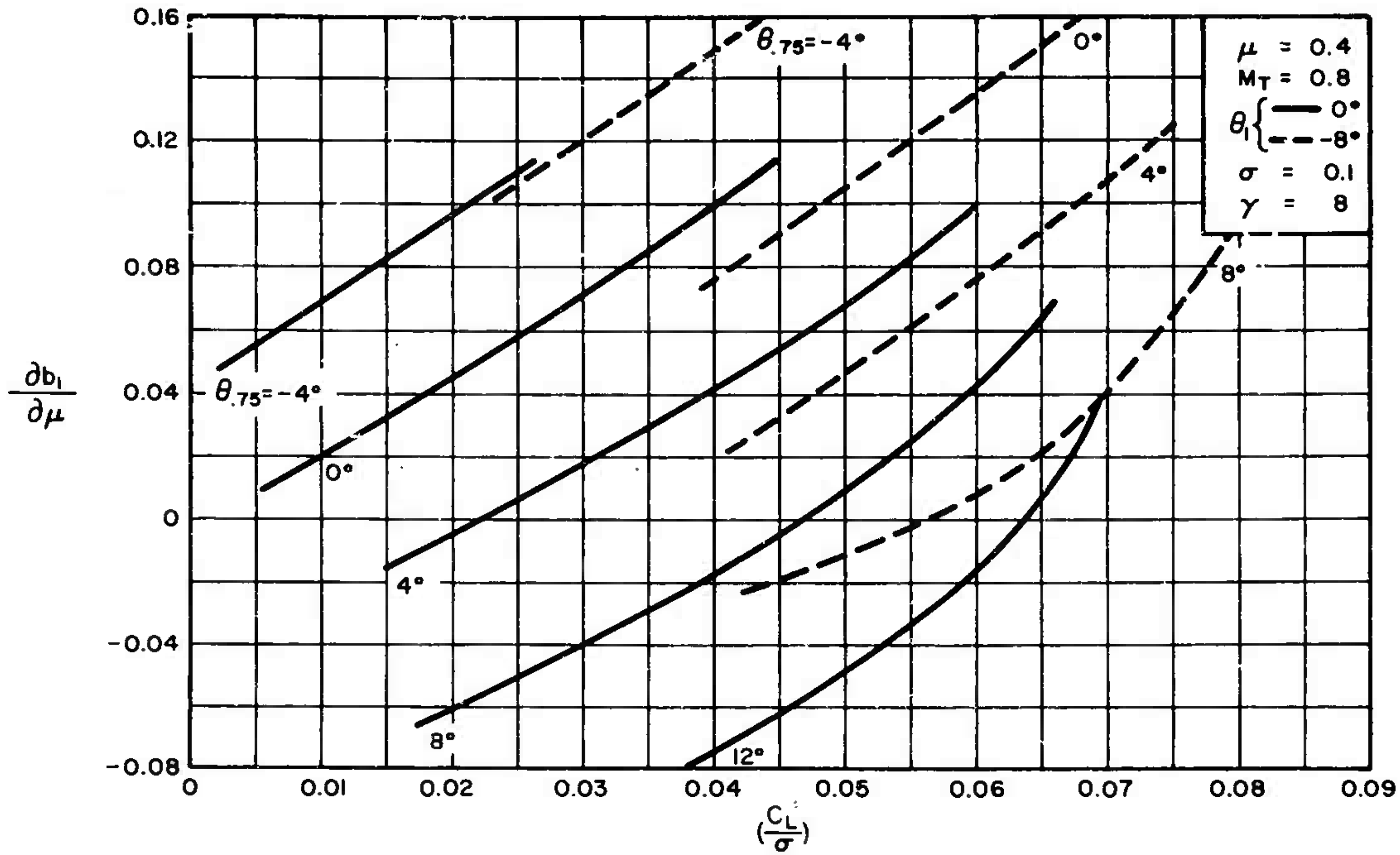

Figure 21. Continued

(e) $\partial b_1/\partial \mu$

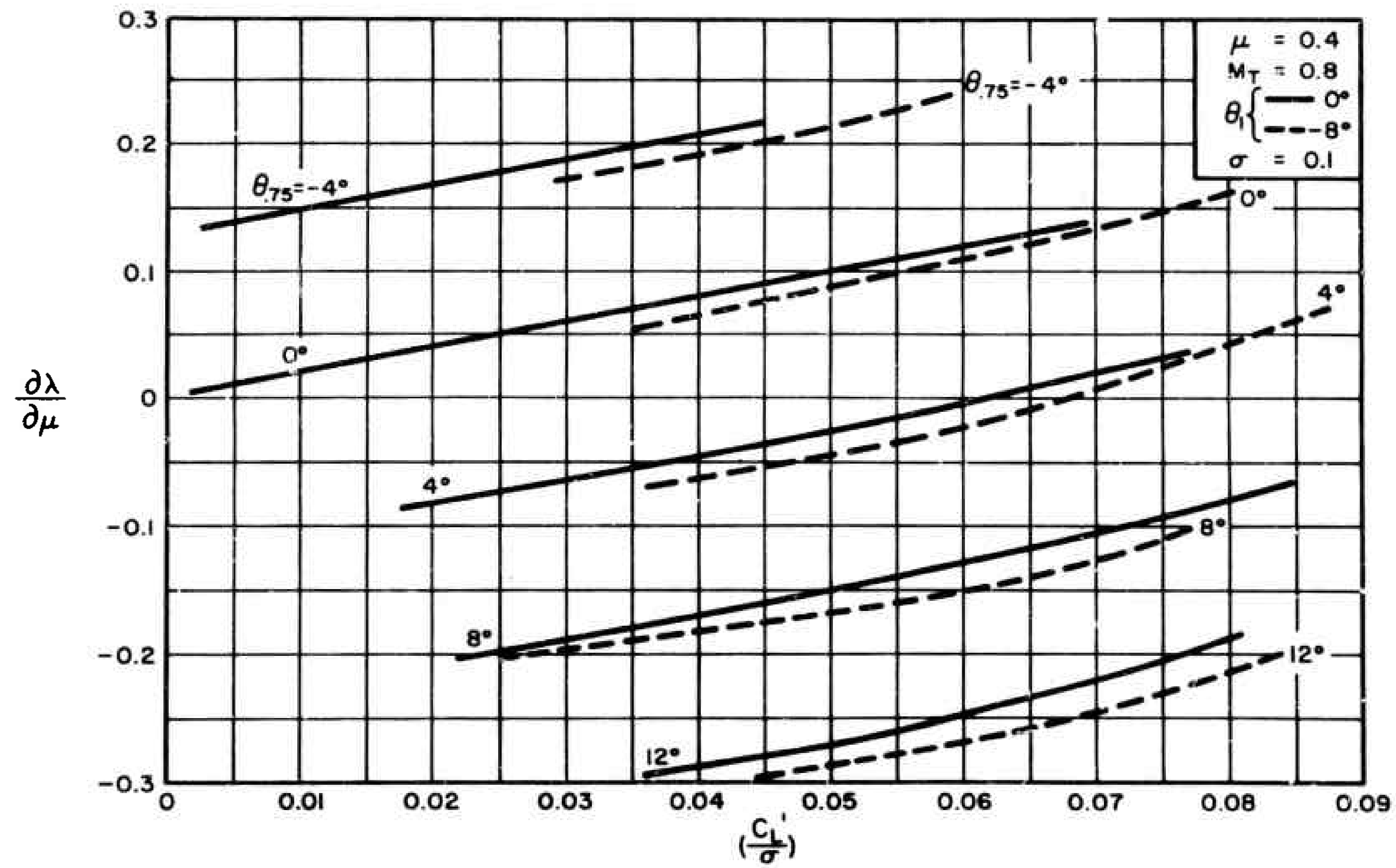

Figure 21. Concluded

(f) $\partial\lambda/\partial\mu$

7.5.4.2 Effect of Blade Twist on the Isolated Rotor Derivatives With Respect to α_C

Figures 22(a) through 22(f) present an indication of the effect of blade twist on the isolated rotor derivatives $\partial(C_L'/\sigma)/\partial\alpha_C$, $\partial(C_D'/\sigma)/\partial\alpha_C$, $\partial(C_Q/\sigma)/\partial\alpha_C$, $\partial a_1/\partial\alpha_C$, $\partial b_1/\partial\alpha_C$, $\partial\lambda/\partial\alpha_C$, respectively. A rotor tip speed ratio of μ = 0.4 and an advancing tip Mach number of M_T = 0.8 are selected for this presentation. The collective pitch range covers values from $\theta_{.75}$ = -4° to $\theta_{.75}$ = 12°.

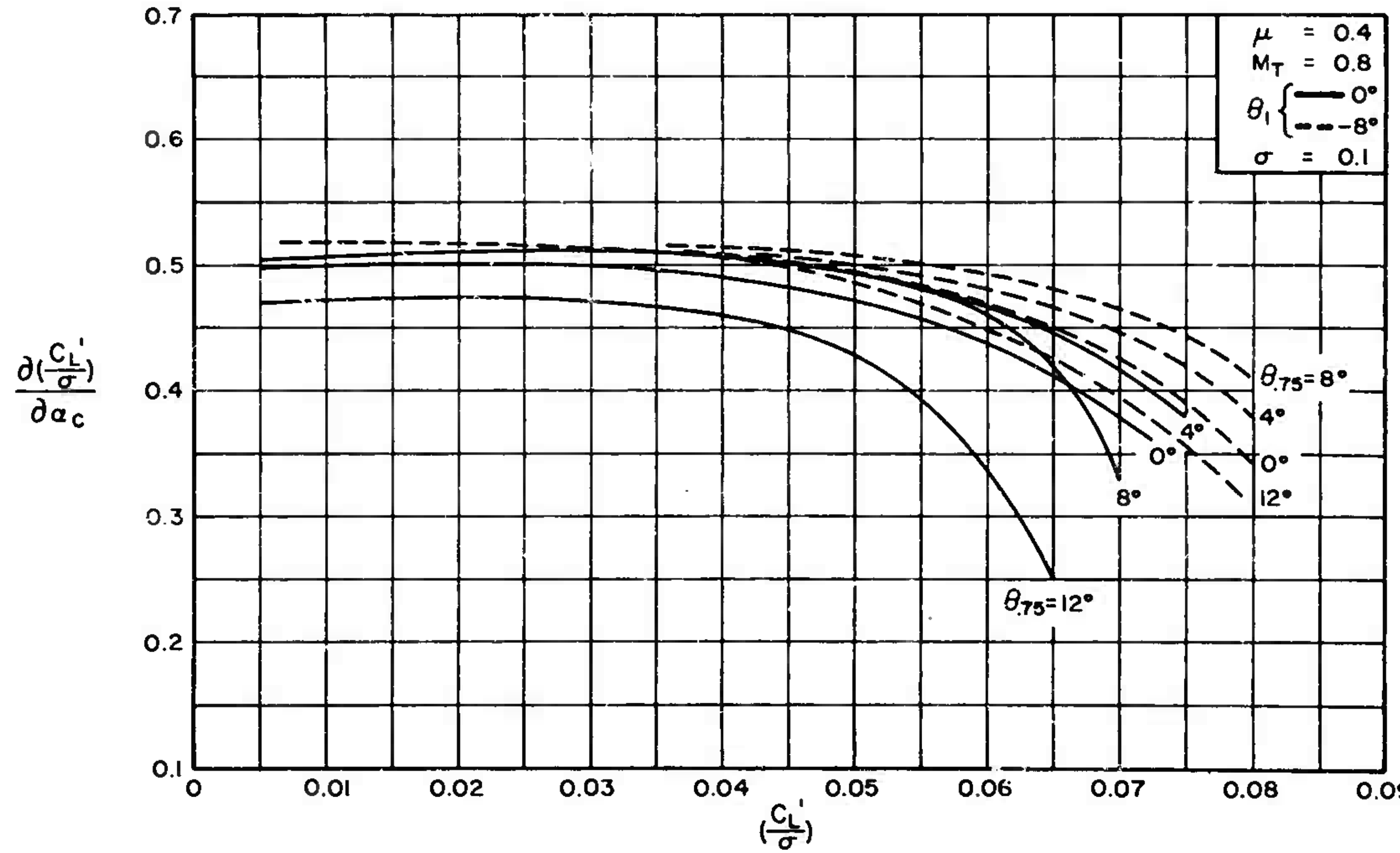

Figure 22. Effect of Blade Twist on α_c Derivatives

(a) $\partial(\frac{C_L'}{\sigma})/\partial \alpha_c$

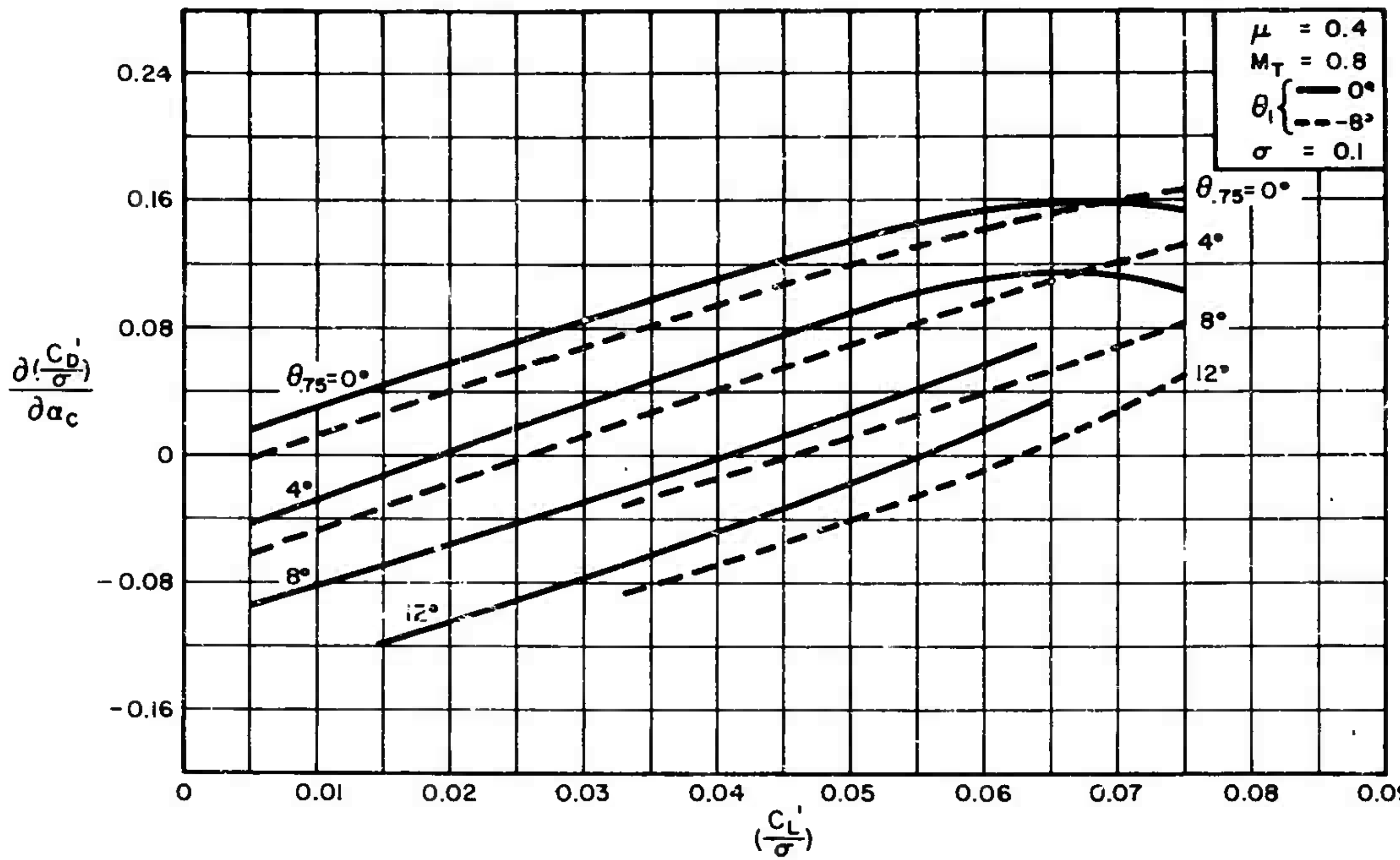

Figure 22. Continued

(b) $\partial(\frac{C_D'}{\sigma})/\partial \alpha_c$

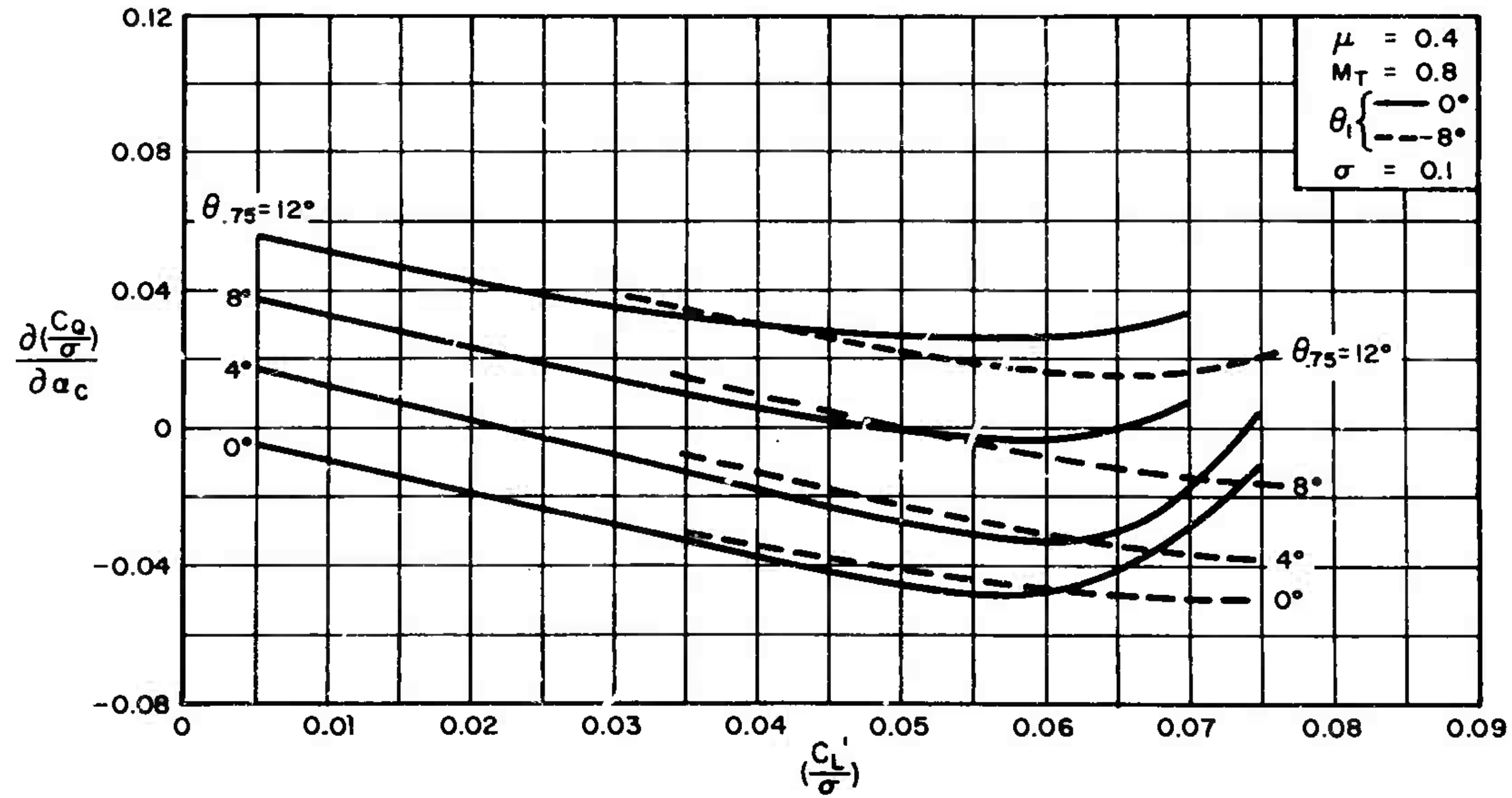

Figure 22. Continued

(c) $\partial(\frac{C_Q}{\sigma})/\partial \alpha_c$

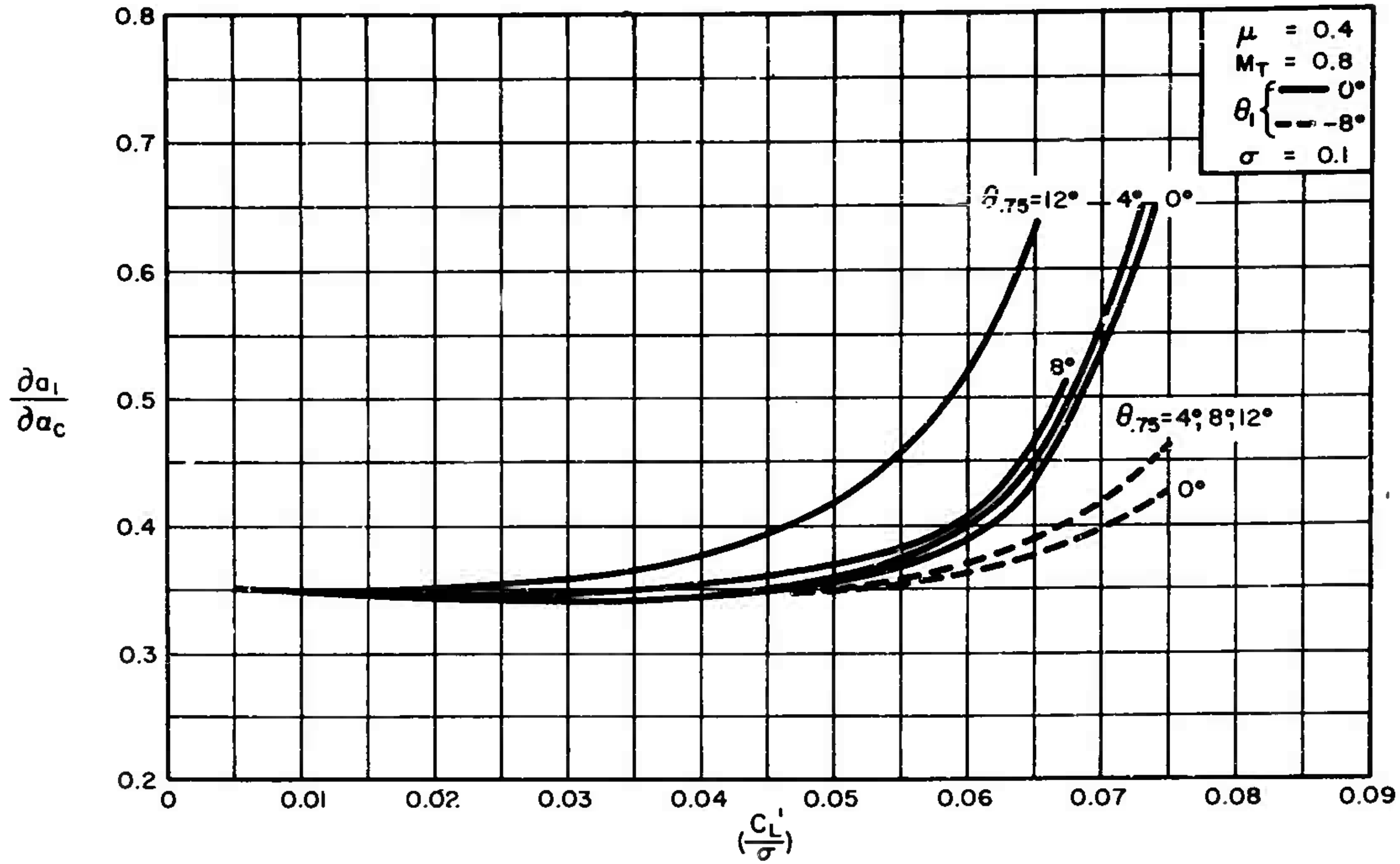

Figure 22. Continued

(d) $\partial a_1/\partial a_c$

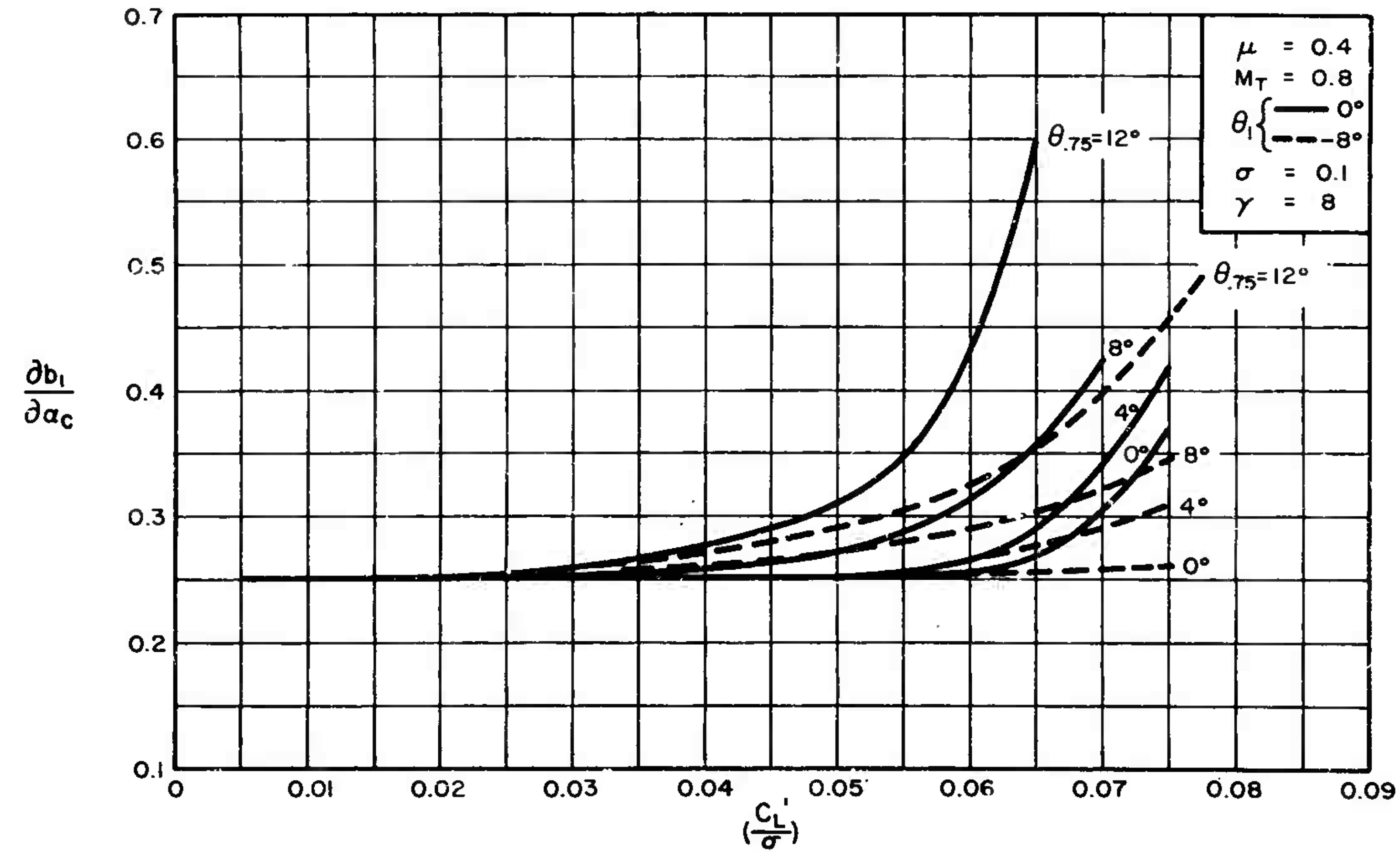

Figure 22. Continued

(e) $\partial b_1/\partial a_c$

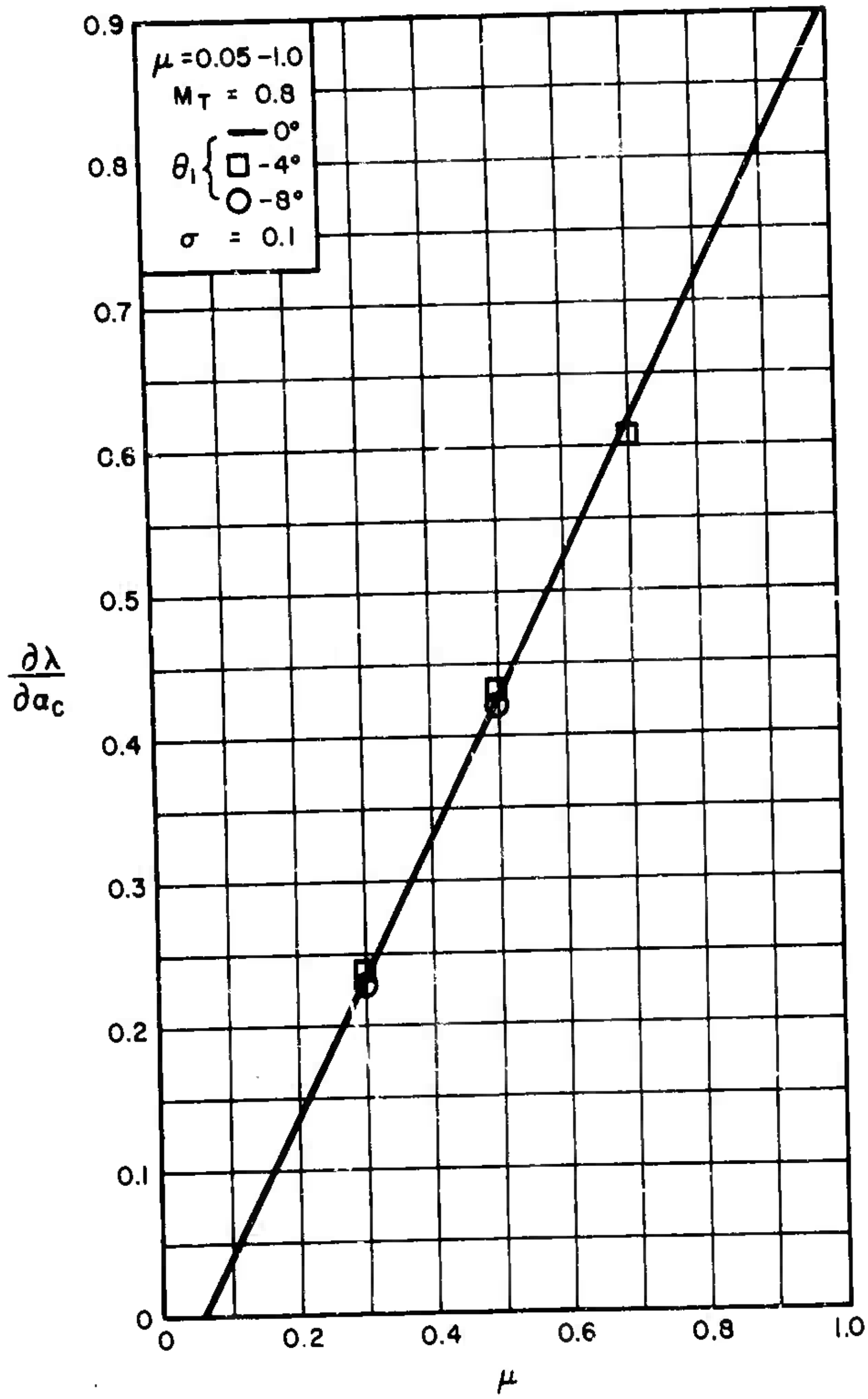

Figure 22. Concluded

(f) $\partial\lambda/\partial a_c$

7.5.4.3 Effect of Blade Twist on the Isolated Rotor Derivatives With Respect to $\theta_{.75}$

Figures 23(a) through 23(f) present an indication of the effect of blade twist on the isolated rotor derivatives $\partial(C_L'/\sigma)/\partial\theta_{.75}$, $\partial(C_D'/\sigma)/\partial\theta_{.75}$, $\partial(C_Q/\sigma)/\partial\theta_{.75}$, $\partial a_1/\partial\theta_{.75}$ $\partial b_1/\partial\theta_{.75}$, $\partial\lambda/\partial\theta_{.75}$, respectively. A representative rotor tip speed ratio of $\mu = 0.3$ and an advancing tip Mach number of $M_T = 0.8$ are selected for this presentation. The range of rotor angle of attack extends from $\alpha_C = +5°$ to $\alpha_C = -20°$.

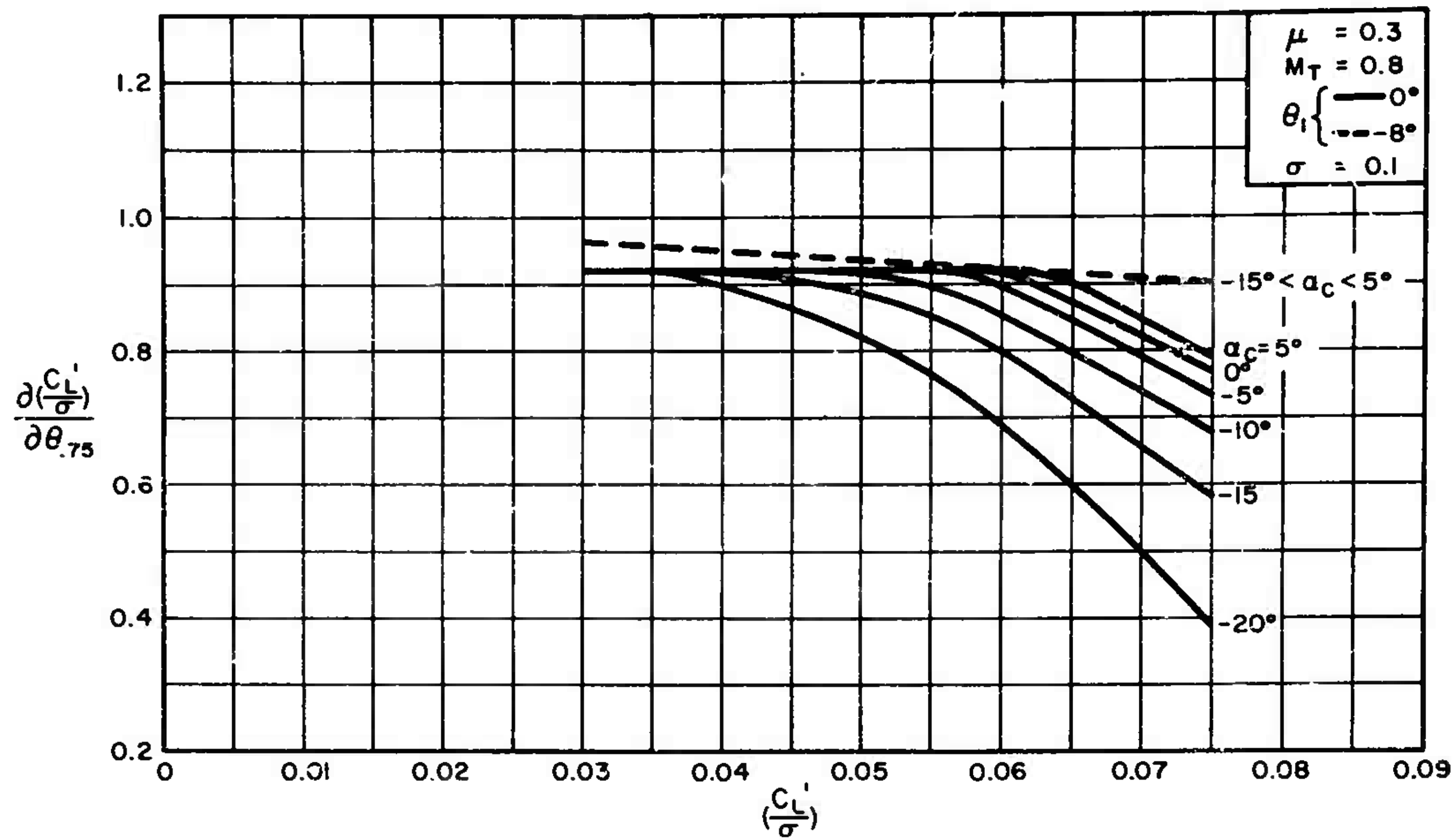

Figure 23. Effect of Blade Twist on $\theta_{.75}$ Derivatives

(a) $\partial(\frac{C_L'}{\sigma})/\partial\theta_{.75}$

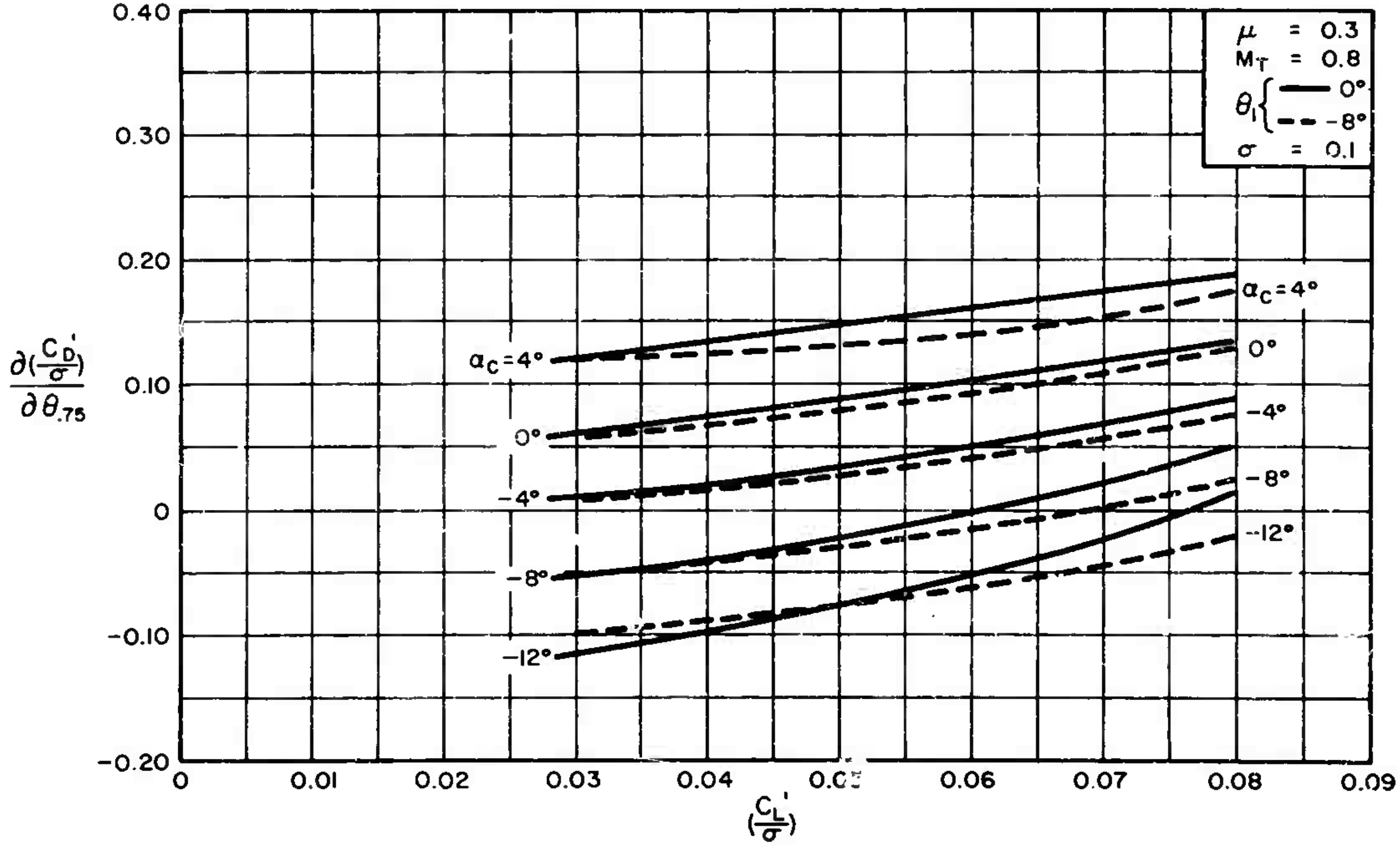

Figure 23. Continued

(b) $\partial(\frac{C_D'}{\sigma})/\partial\theta_{.75}$

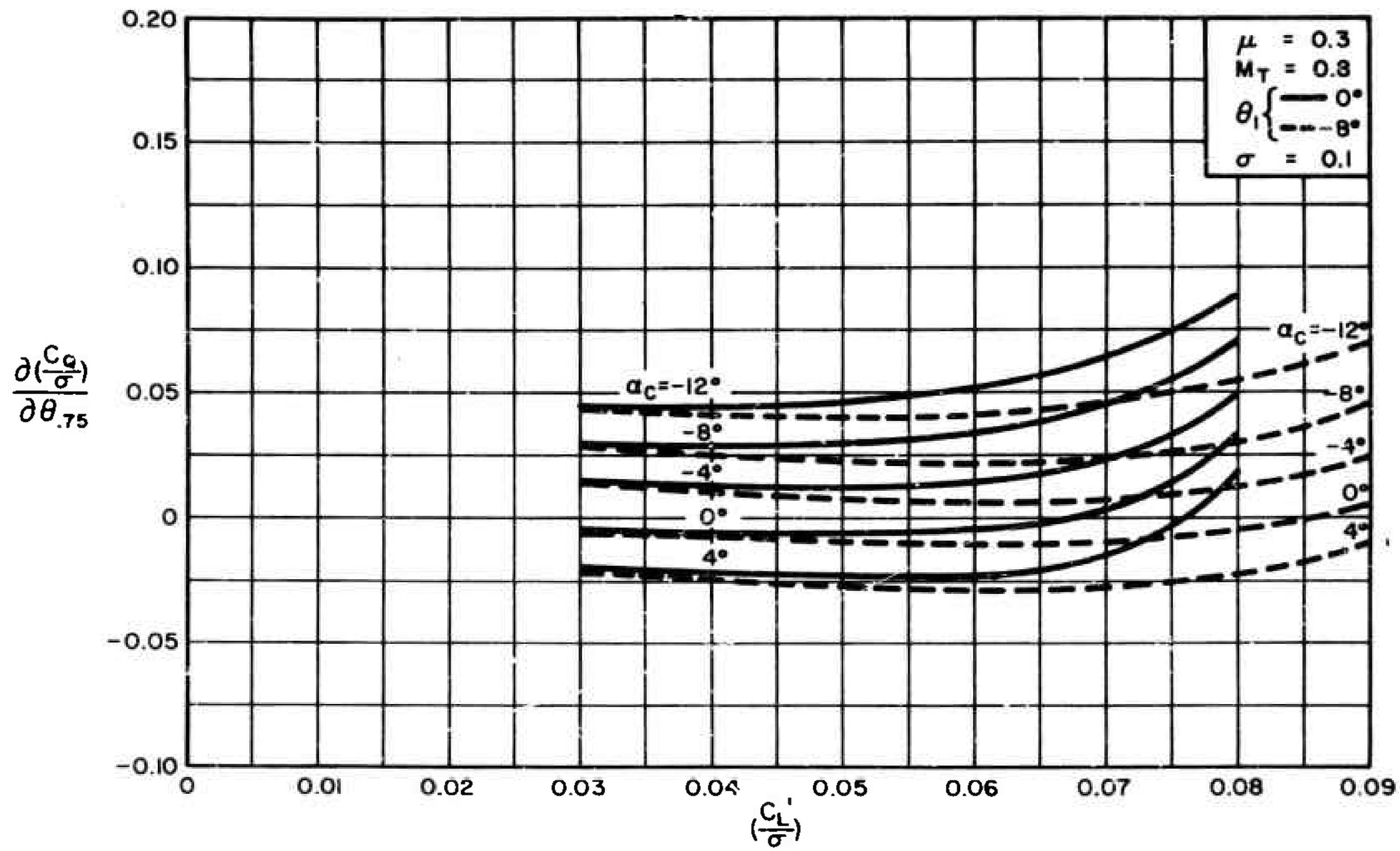

Figure 23. Continued

(c) $\partial(\frac{C_Q}{\sigma})/\partial\theta_{.75}$

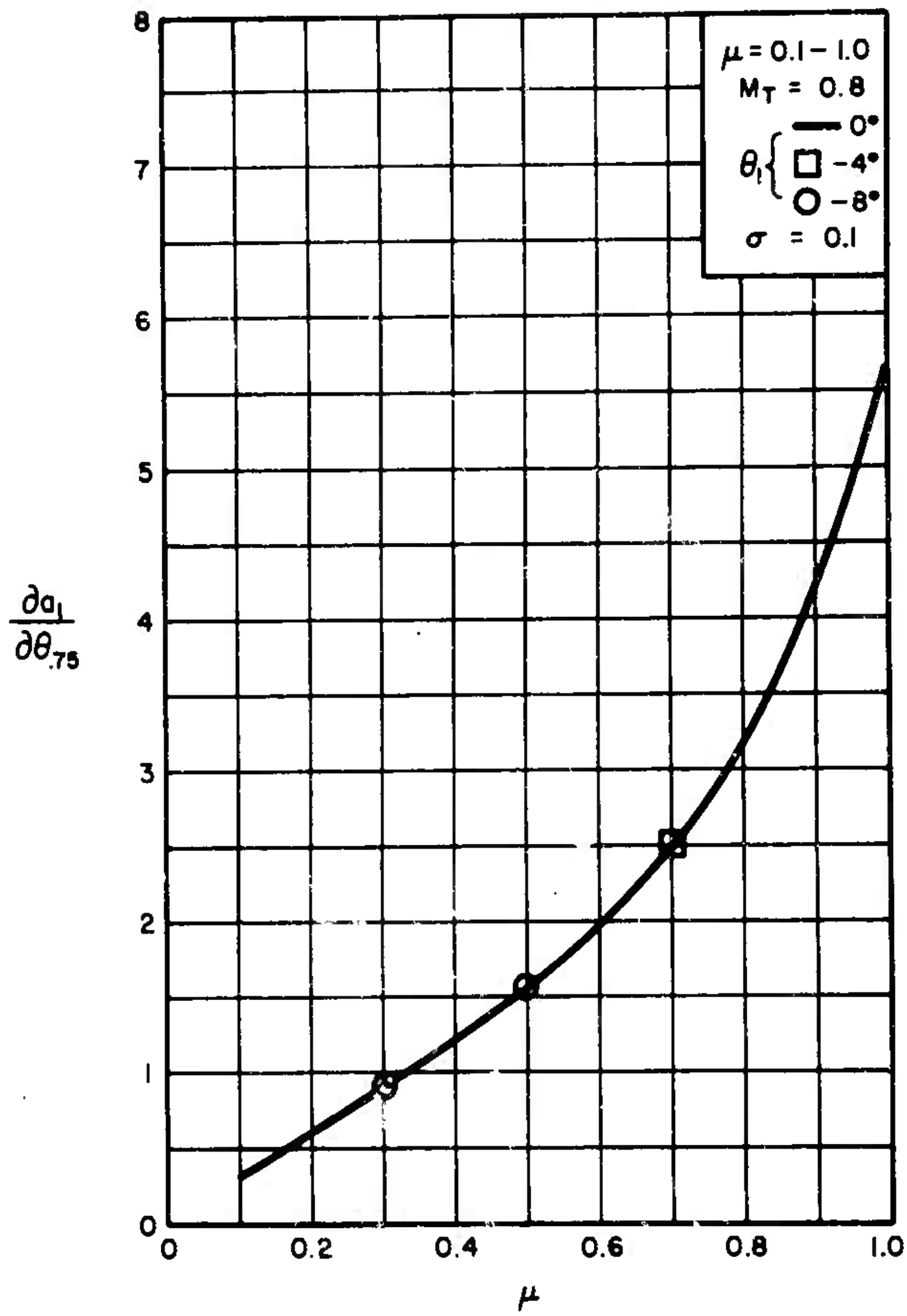

Figure 23. Continued

(d) $\partial a_1/\partial \theta_{.75}$

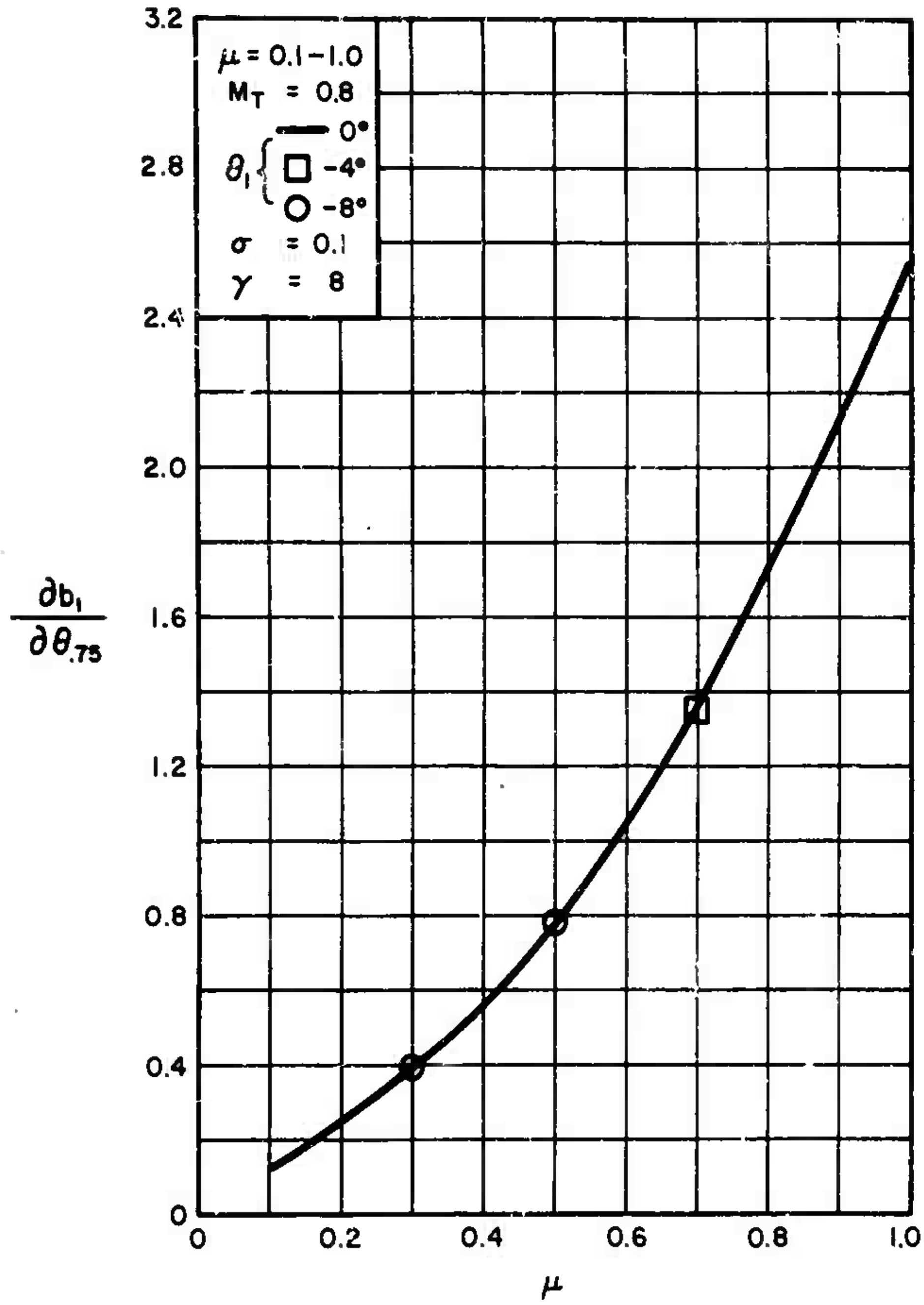

Figure 23. Continued

(e) $\partial b_1 / \partial \theta_{.75}$

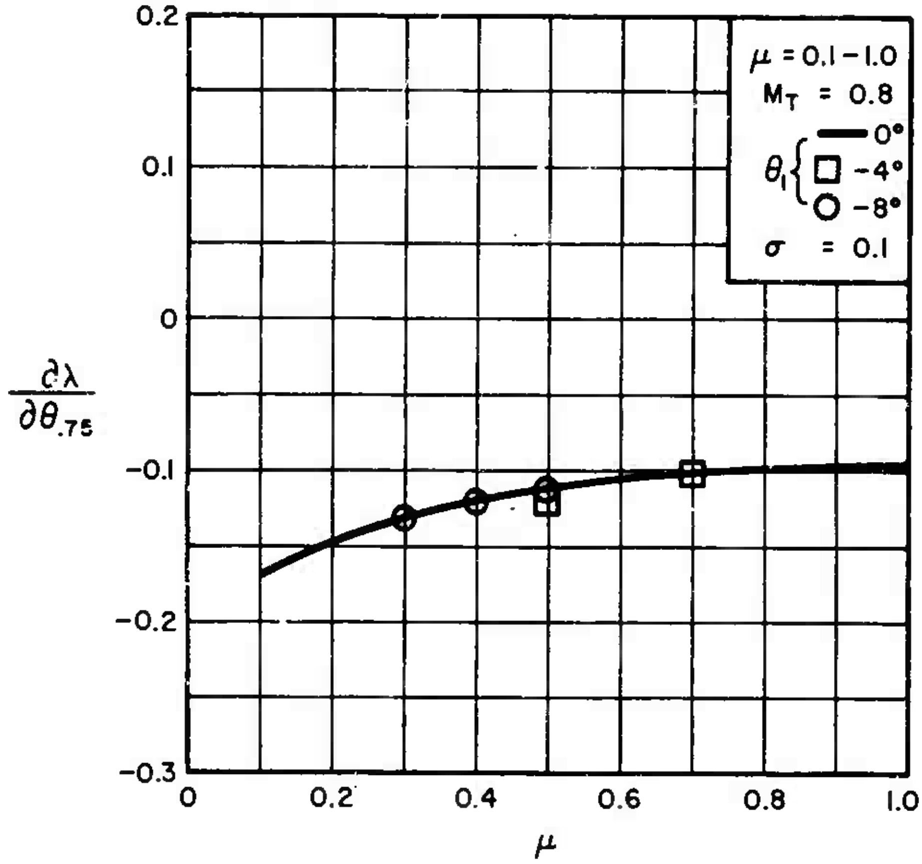

Figure 23. Concluded

(f) $\partial\lambda/\partial\theta_{.75}$

7.5.5 Effect of Compressibility on the Isolated Rotor Derivatives

This section presents the effect of advancing blade tip Mach number on various isolated rotor derivatives with respect to the basic aerodynamic variables μ , α_C , and $\theta_{.75}$.

The effect of compressibility on each rotor derivative for a selected range of pertinent rotor parameters is shown on the comparison plots of Figures 24 through 26.

The plots present the derivatives for the basic case of $M_T = 0.8$ and $\theta_1 = 0$, together with the derivatives obtained for $M_T = 0.9$ and $\theta_1 = 0$.

These plots are based on the theoretical data of Reference 1.

7.5.5.1 Effect of Compressibility on the Isolated Rotor Derivatives With Respect to μ

Figures 24(a) through 24(f) present an indication of the effect of Mach number variation on the isolated rotor derivatives $\partial(C_L'/\sigma)/\partial\mu$, $\partial(C_D'/\sigma)/\partial\mu$, $\partial(C_Q/\sigma)/\partial\mu$, $\partial a_1/\partial\mu$, $\partial b_1/\partial\mu$, and $\partial\lambda/\partial\mu$, respectively. The results are presented for tip speed ratio of $\mu = 0.4$ and a range of $\theta_{.75}$ from $\theta_{.75} = -4°$ through $\theta_{.75} = 12°$.

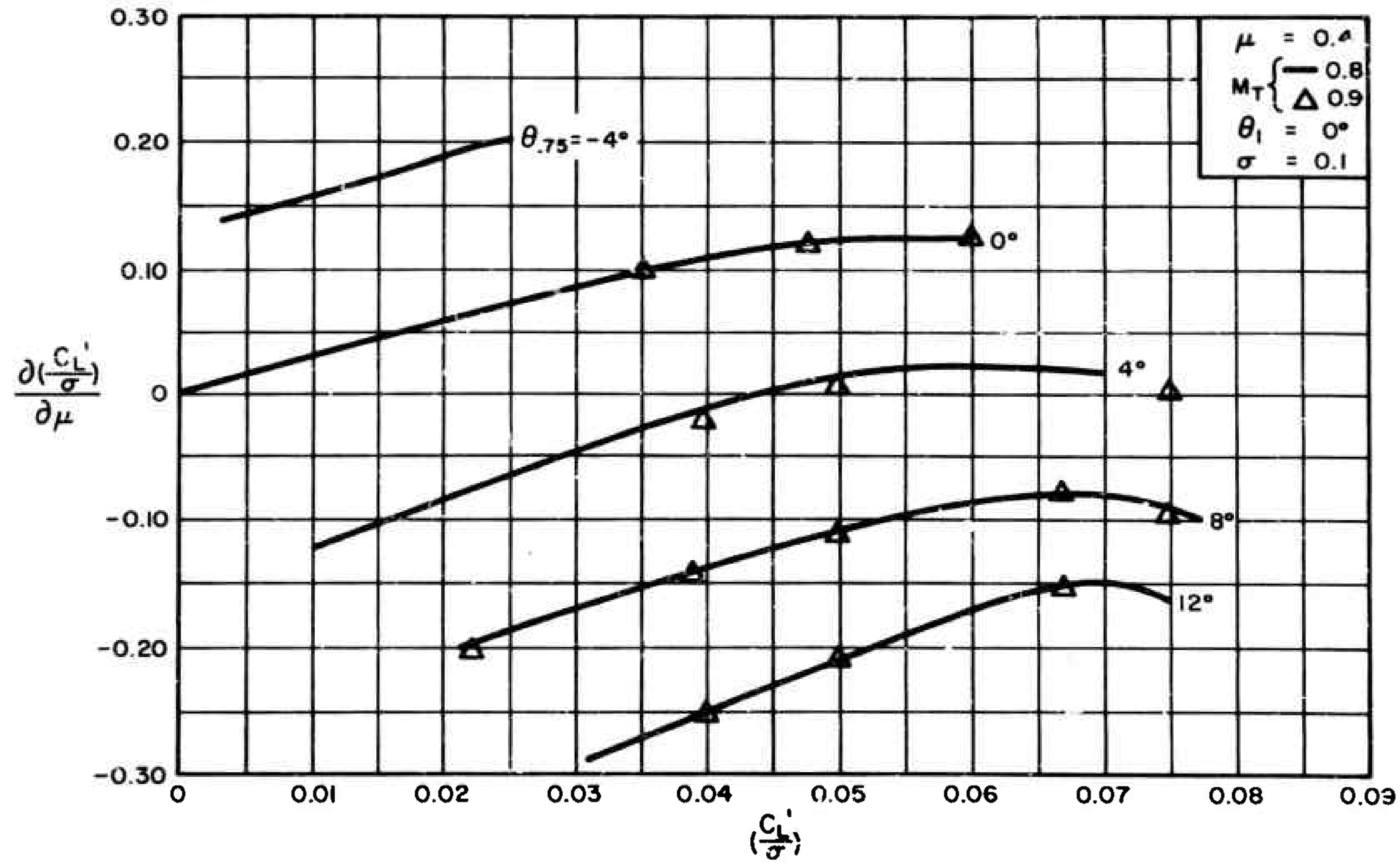

Figure 24. Effect of Compressibility on μ Derivatives

(a) $\partial(\frac{C_L'}{\sigma})/\partial\mu$

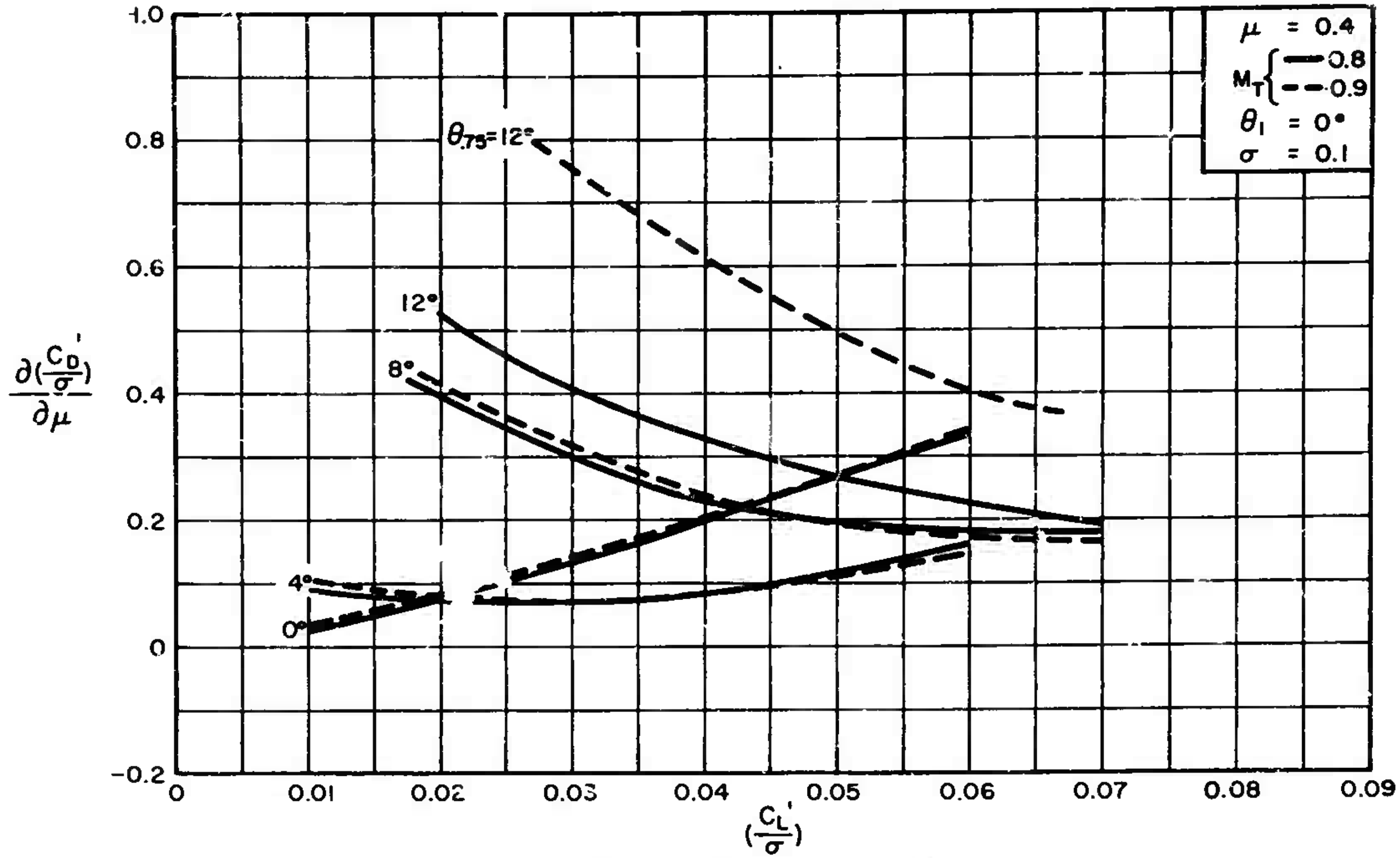

Figure 24. Continued

(b) $\partial(\frac{C_D'}{\sigma})/\partial\mu$

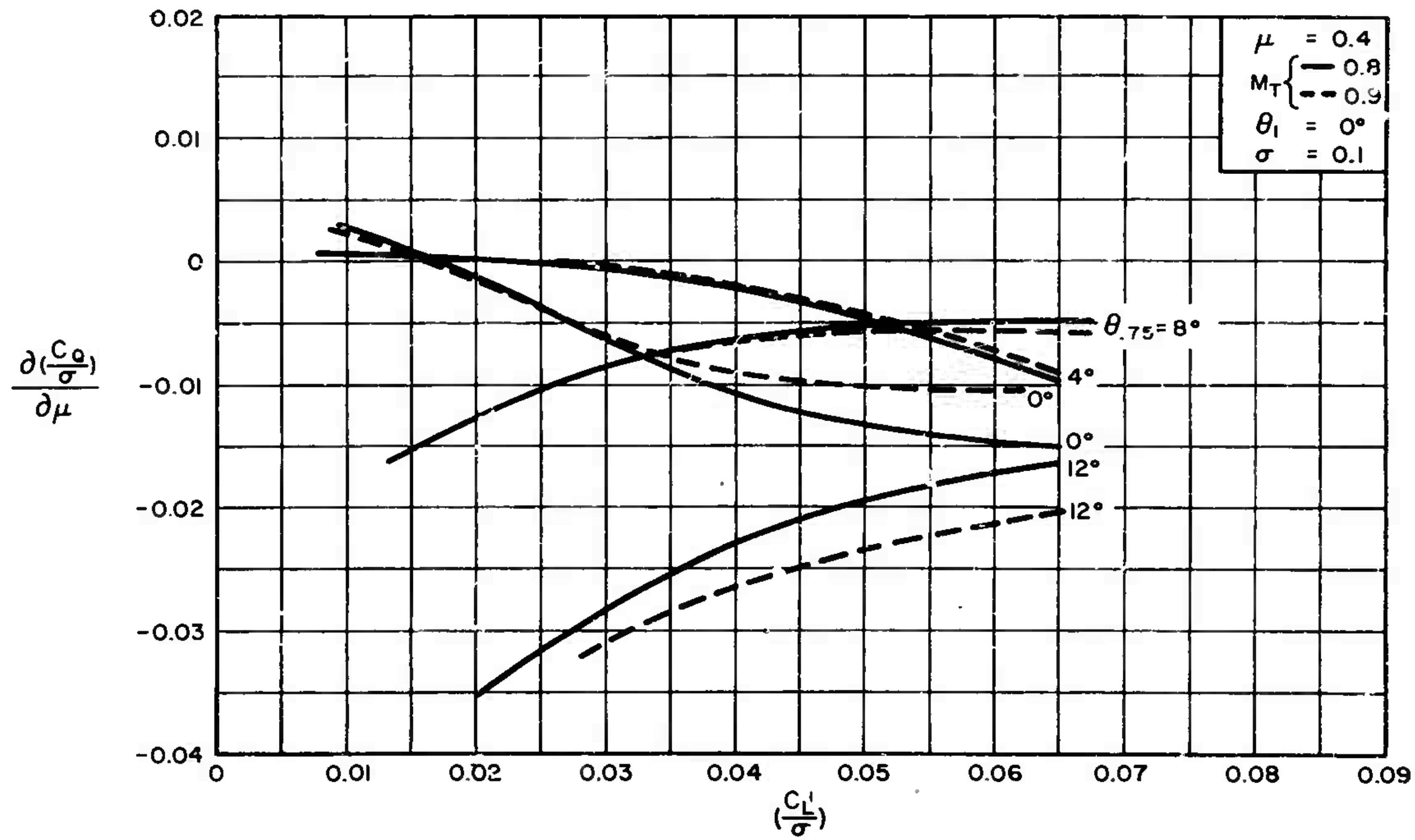

Figure 24. Continued

(c) $\partial(\frac{C_Q}{\sigma})/\partial\mu$

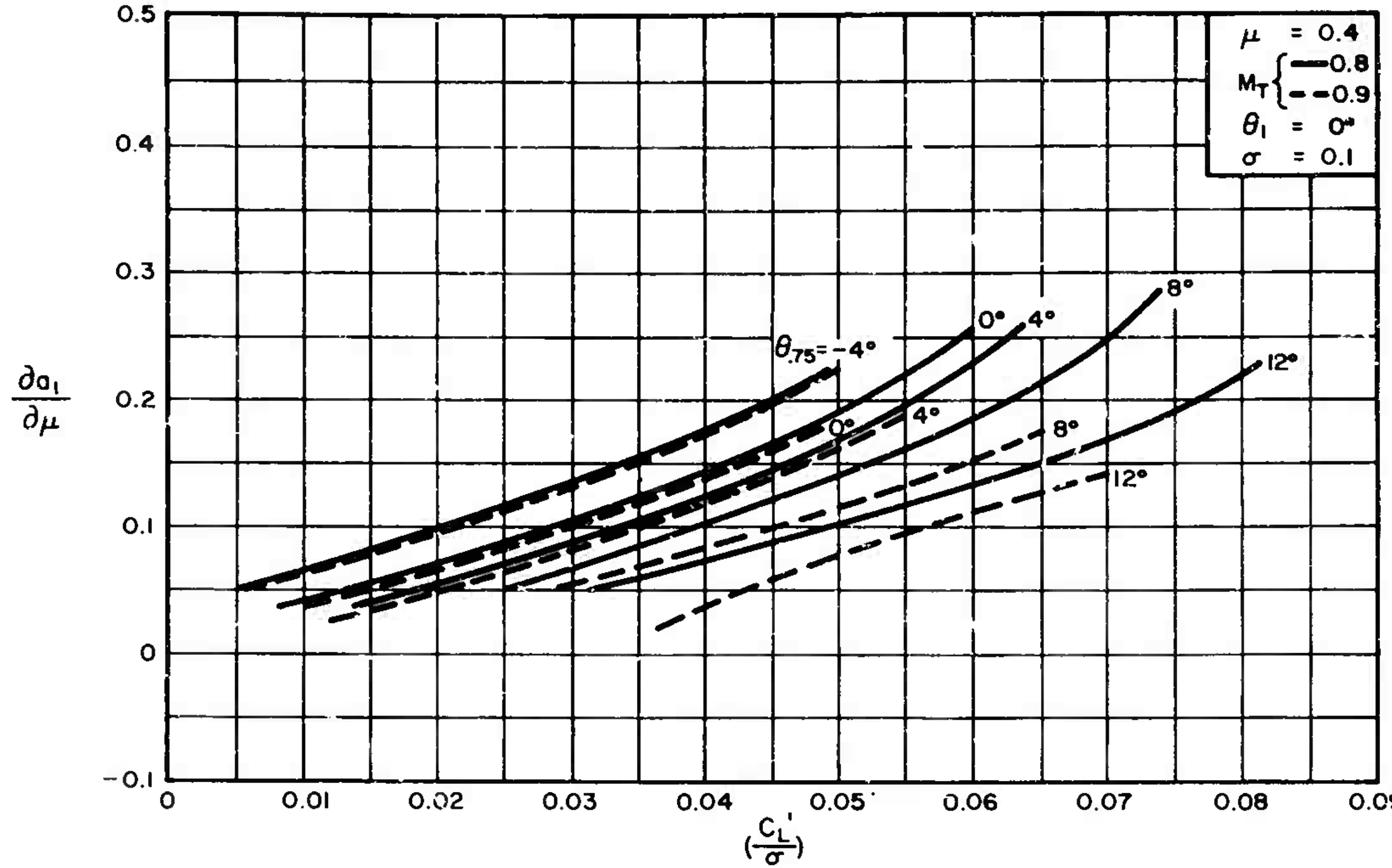

Figure 24. Continued

(d) $\partial a_1/\partial \mu$

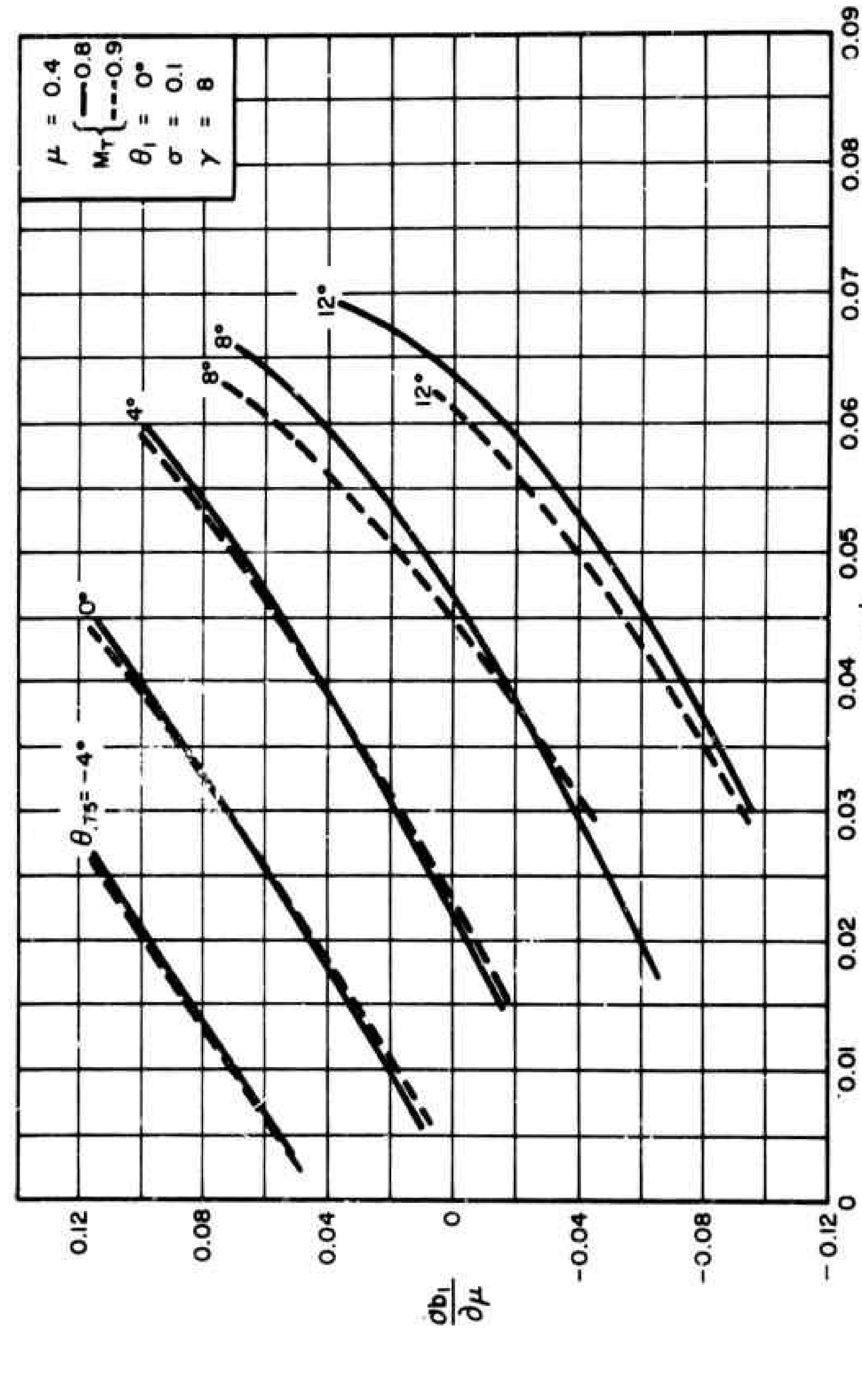

Figure 24. Continued

(e) $\partial b_1/\partial \mu$

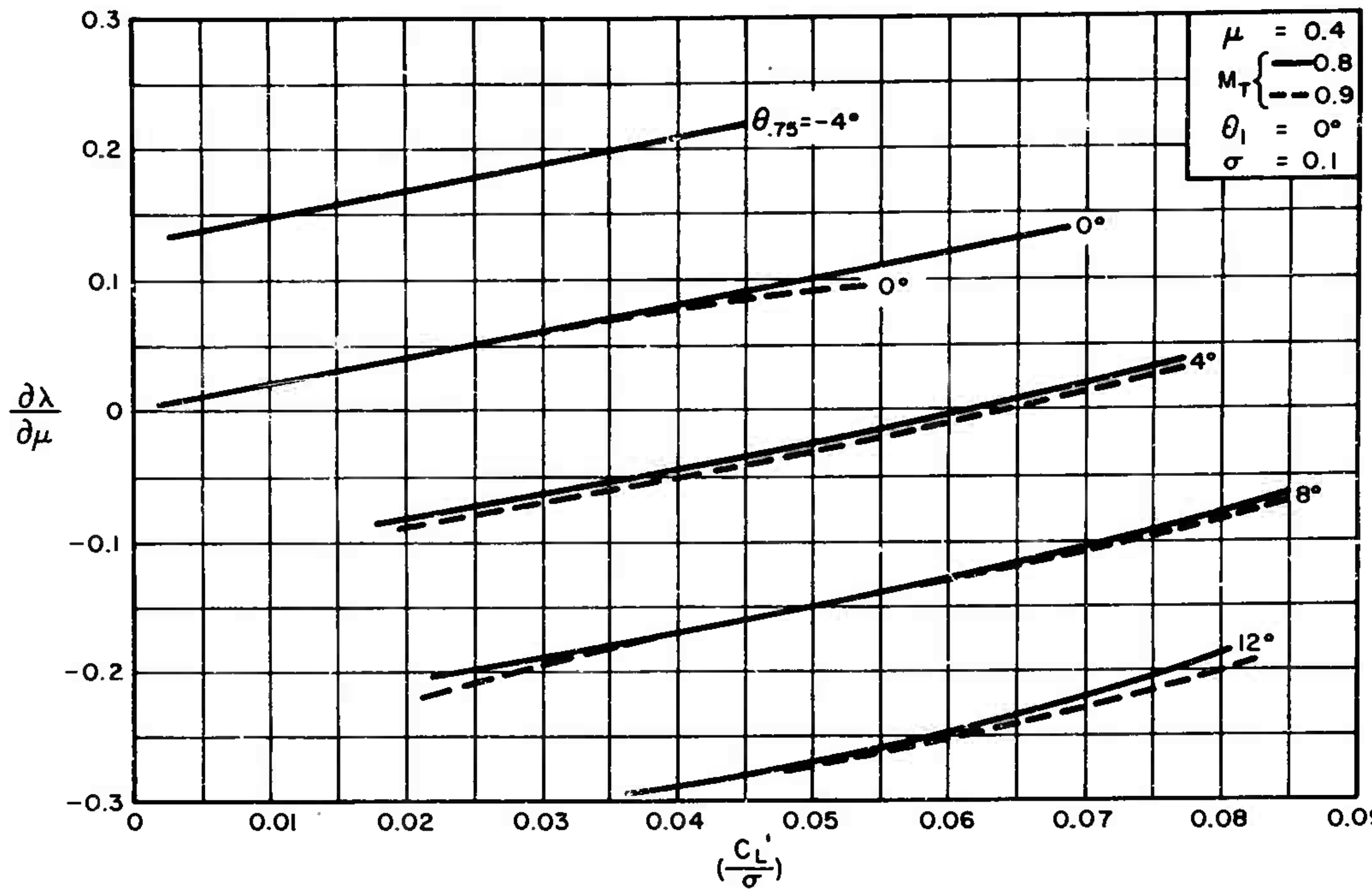

Figure 24. Concluded

(f) $\partial\lambda/\partial\mu$

7.5.5.2 Effect of Compressibility on the Isolated Rotor Derivatives With Respect to α_C

Figures 25(a) through 25(k) present an indication of the effect of Mach number variation on the isolated rotor derivatives $\partial(C_L'/\sigma)/\partial\alpha_C$, $\partial(C_D'/\sigma)/\partial\alpha_C$, $\partial(C_Q/\sigma)/\partial\alpha_C$, $\partial a_1/\partial\alpha_C$, $\partial b_1/\partial\alpha_C$, and $\partial\lambda/\partial\alpha_C$, respectively. The results are presented for two values of tip speed ratios, $\mu = 0.3$ and $\mu = 1.0$, and a representative range of collective pitch settings for each μ. The two different values of μ were purposely selected in order to show the compressibility effects in low and high speed regimes.

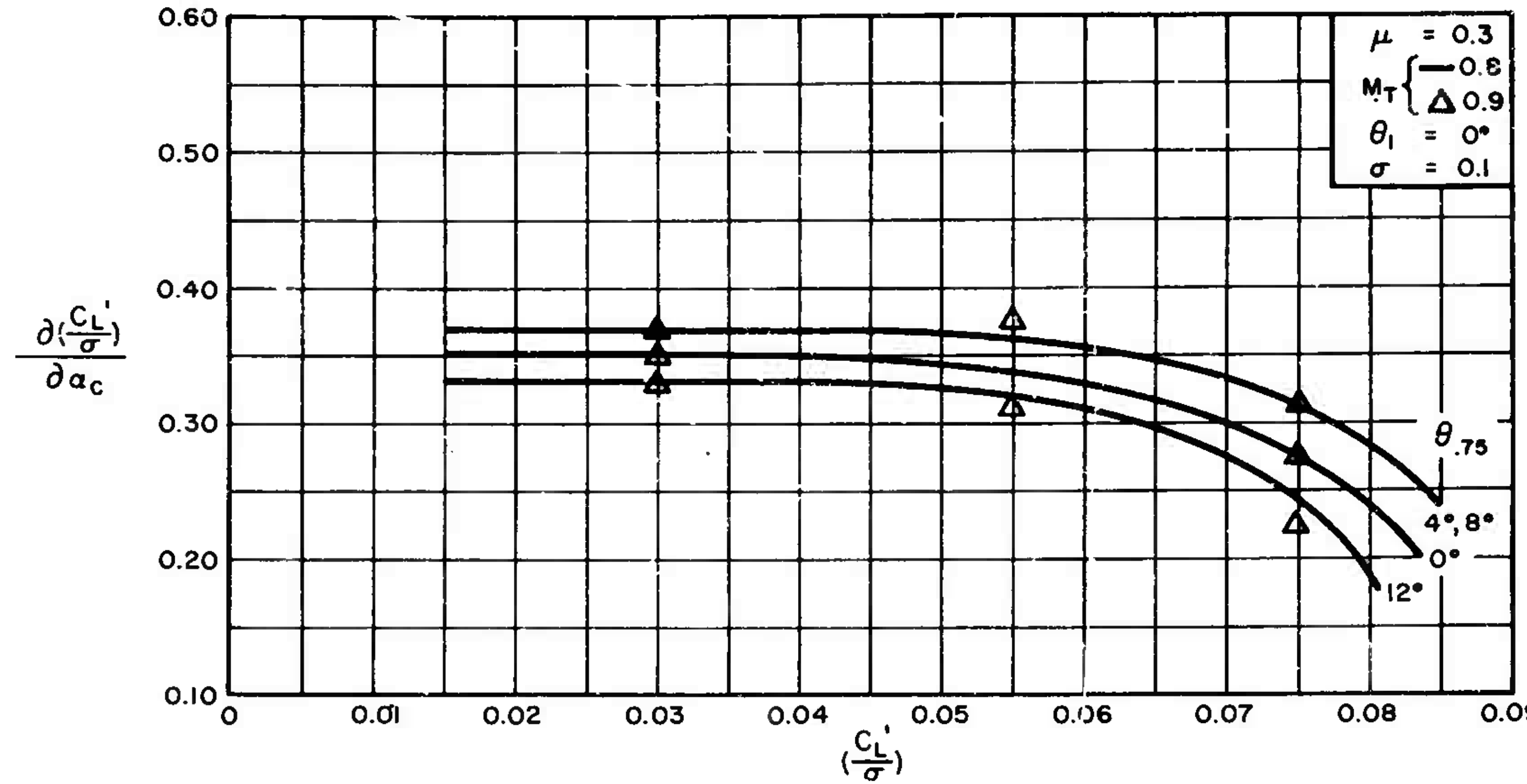

Figure 25. Effect of Compressibility on a_c Derivatives

(a) $\partial(\frac{C_L'}{\sigma})/\partial a_c$ for $\mu = 0.3$

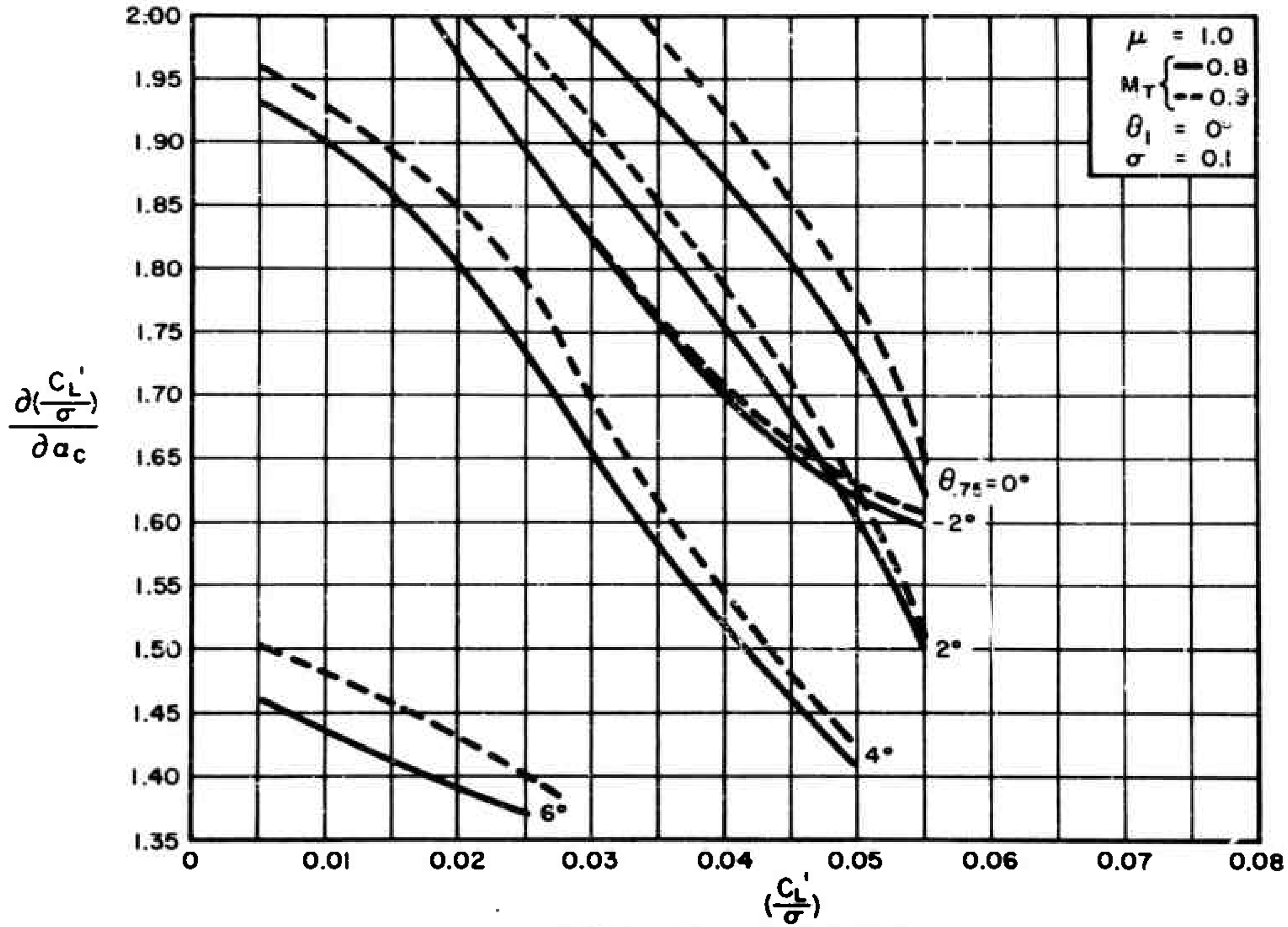

Figure 25. Continued.

(b) $\partial(\frac{C_L'}{\sigma})/\partial a_c$ for $\mu=1.0$

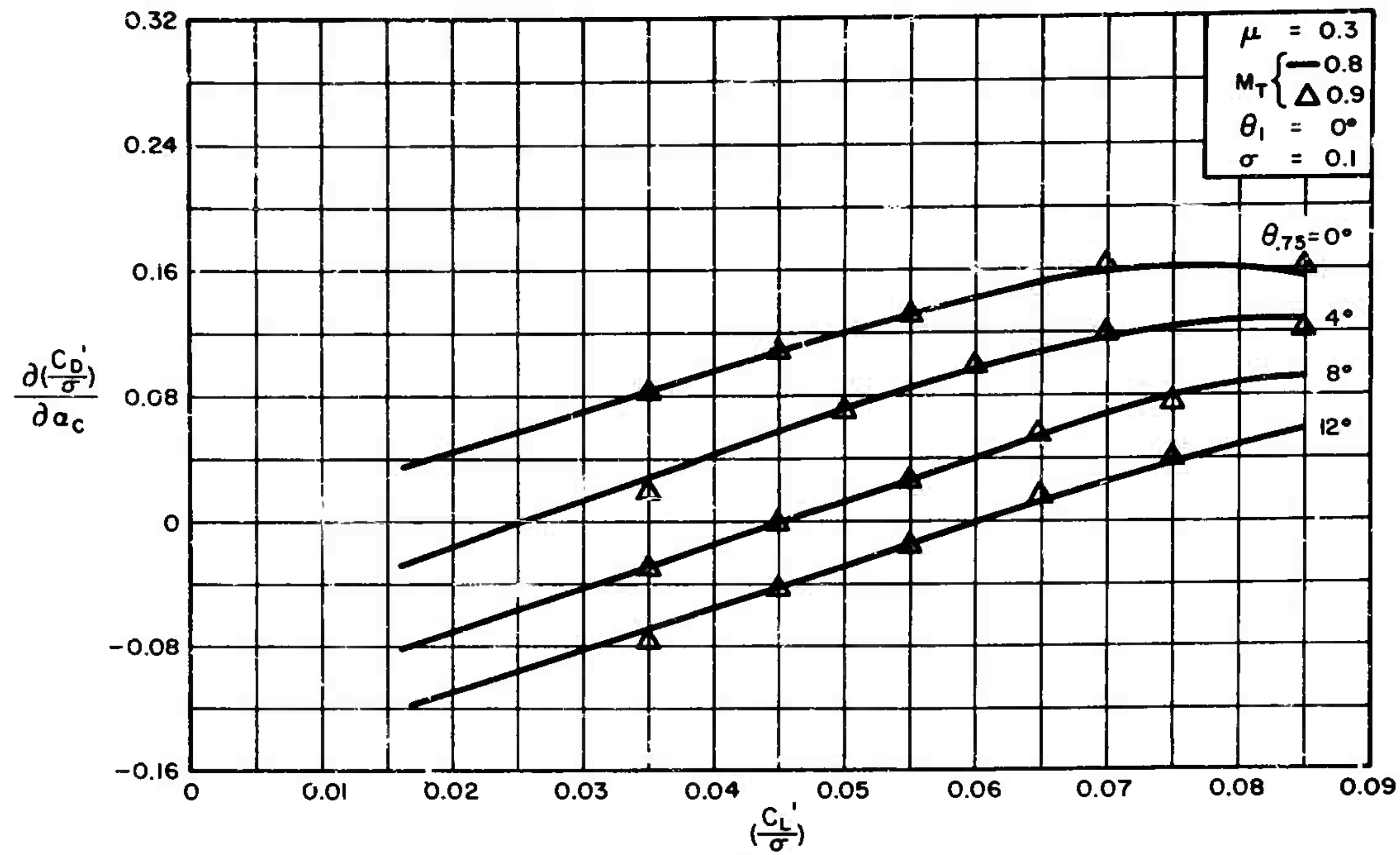

Figure 25. Continued

(c) $\partial(\frac{C_D'}{\sigma})/\partial a_c$ for $\mu = 0.3$

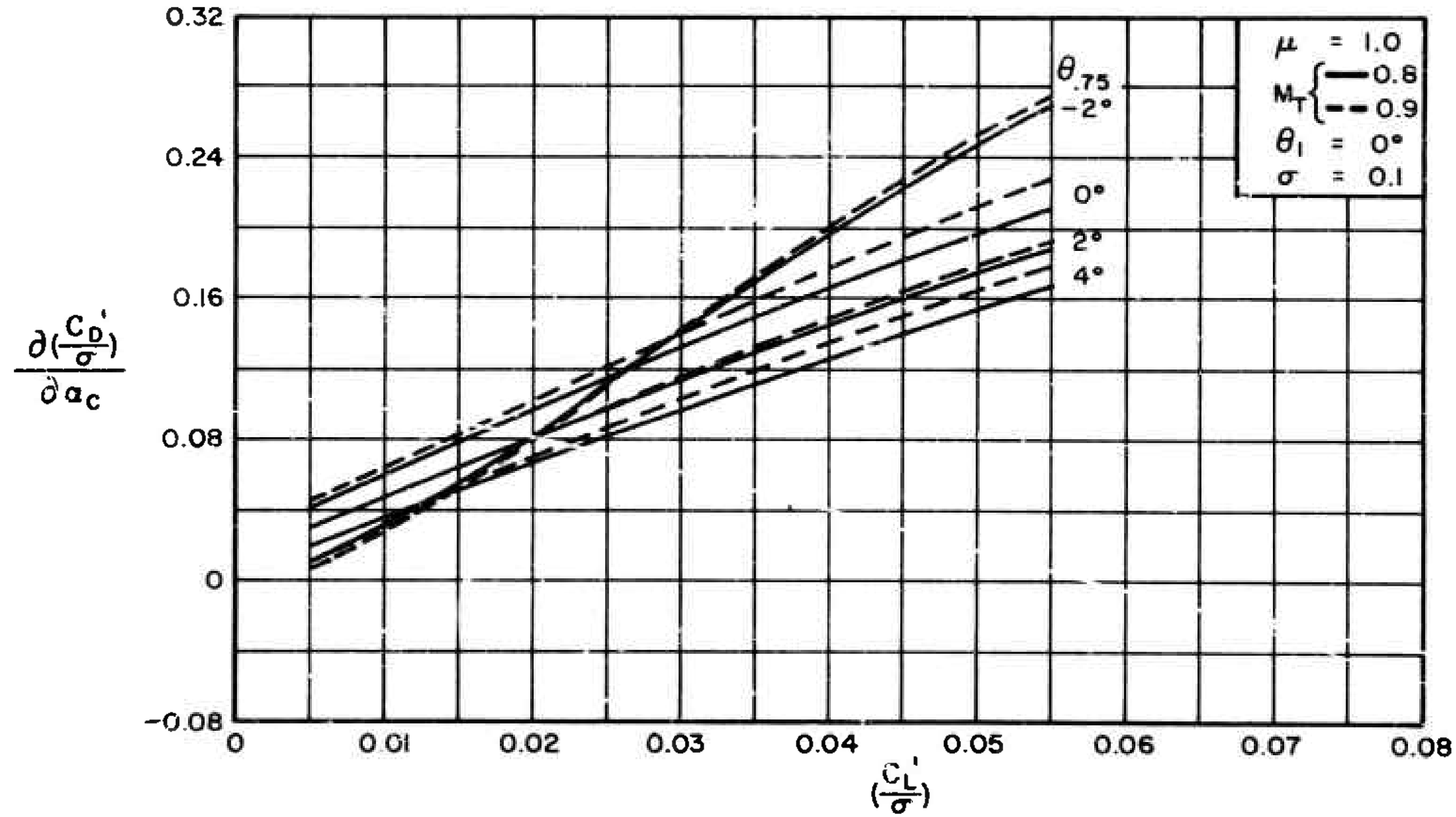

Figure 25. Continued

(d) $\partial(\frac{C_D'}{\sigma})/\partial a_c$ for $\mu = 1.0$

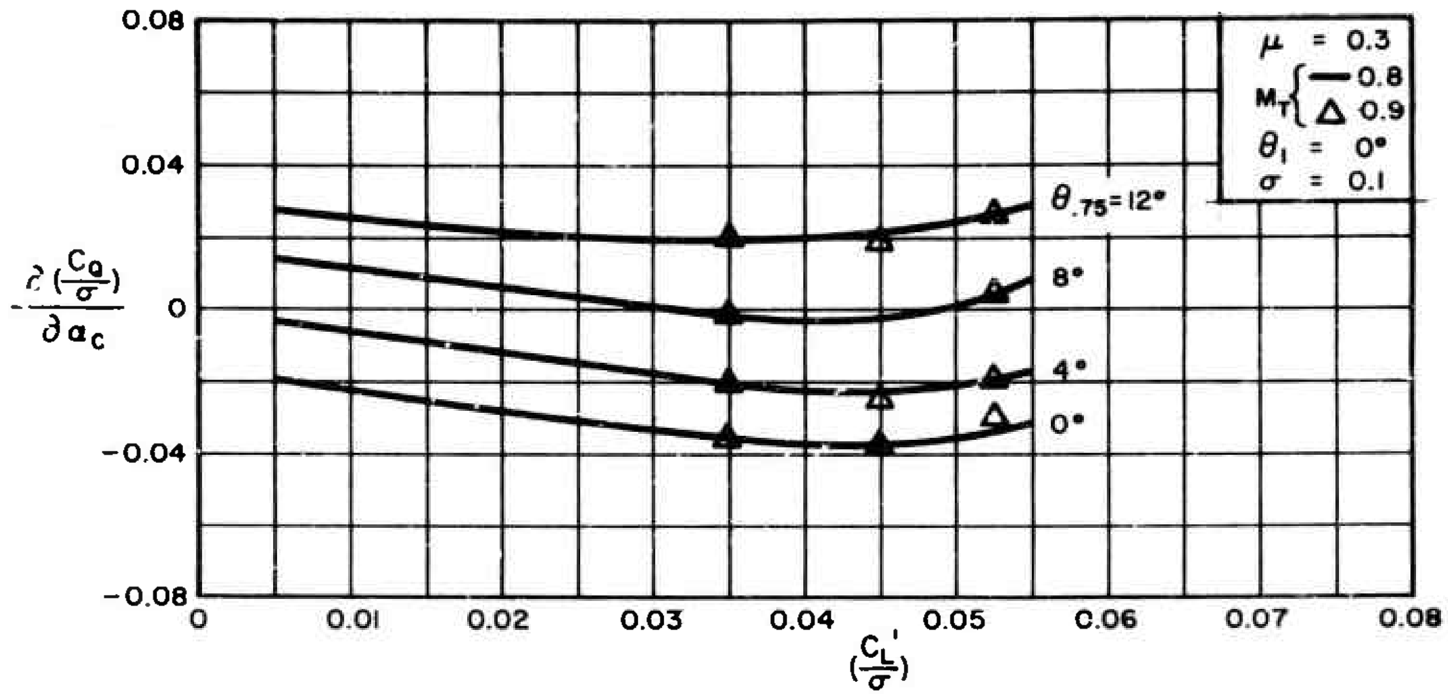

Figure 25. Continued

(e) $\partial(\frac{C_Q}{\sigma})/\partial a_c$ for $\mu = 0.3$

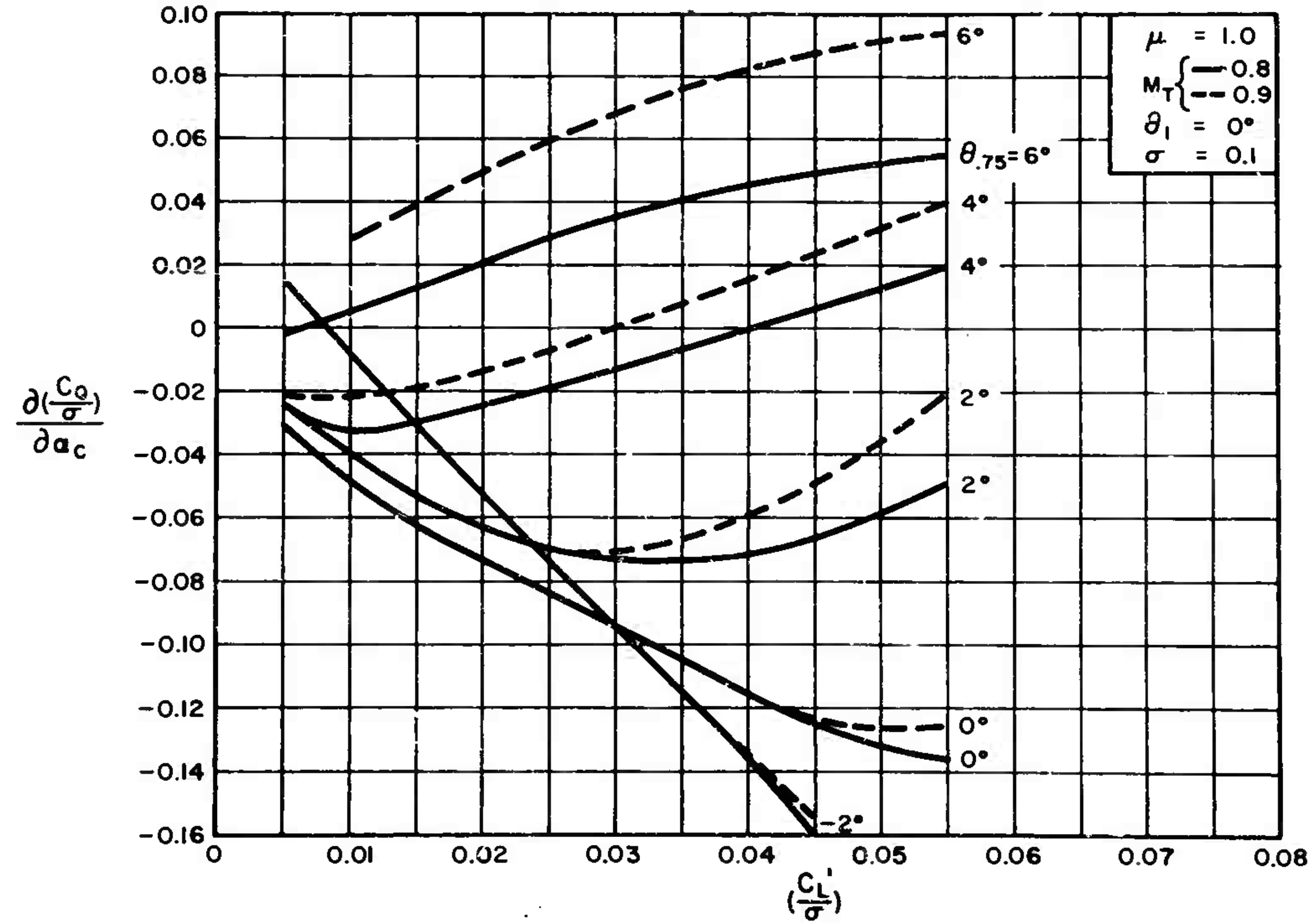

Figure 25. Continued

(f) $\partial(\frac{C_Q}{\sigma})/\partial a_c$ for μ=1.0

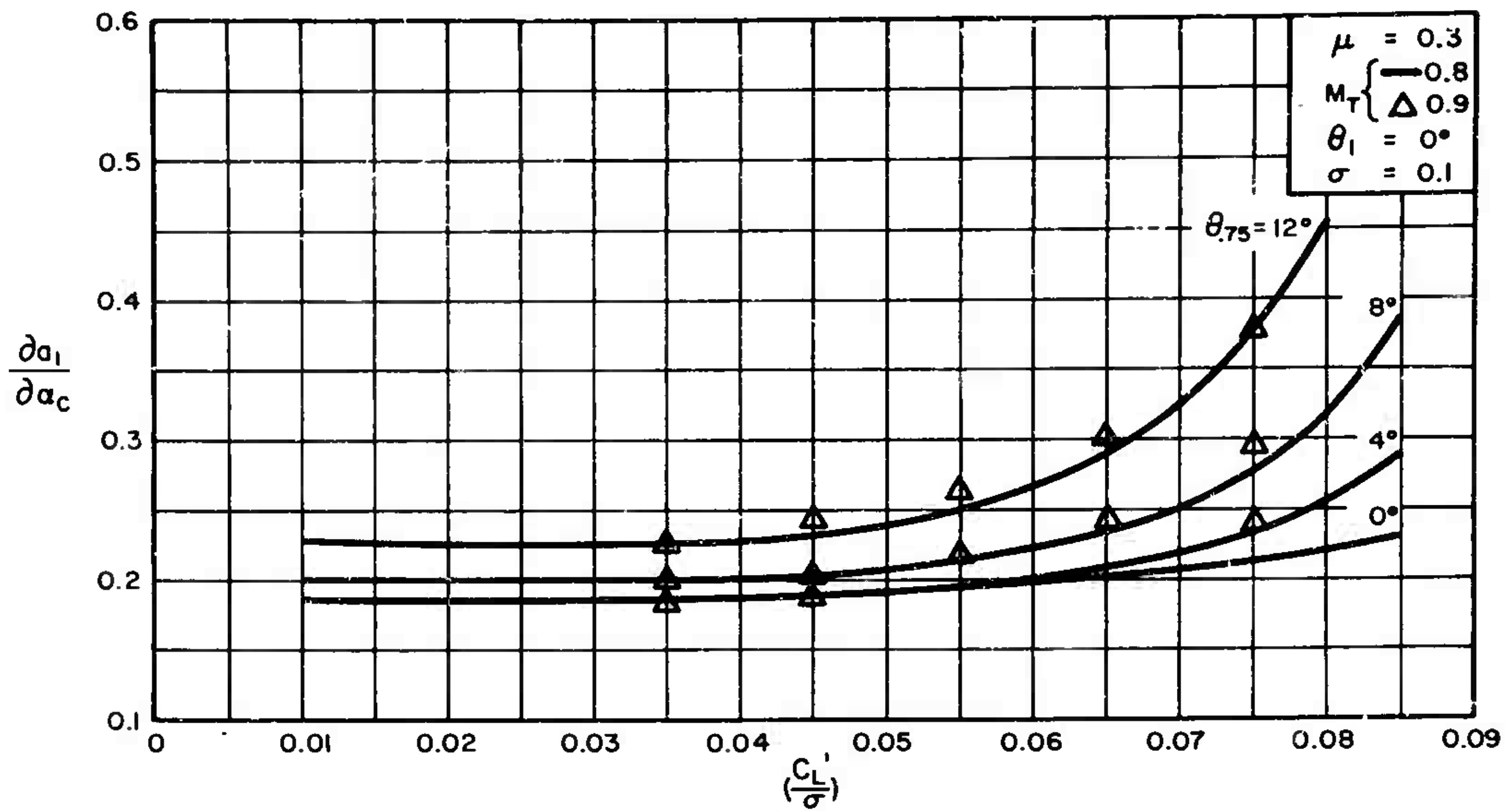

Figure 25. Continued

(g) $\partial a_1/\partial \alpha_c$ for $\mu = 0.3$

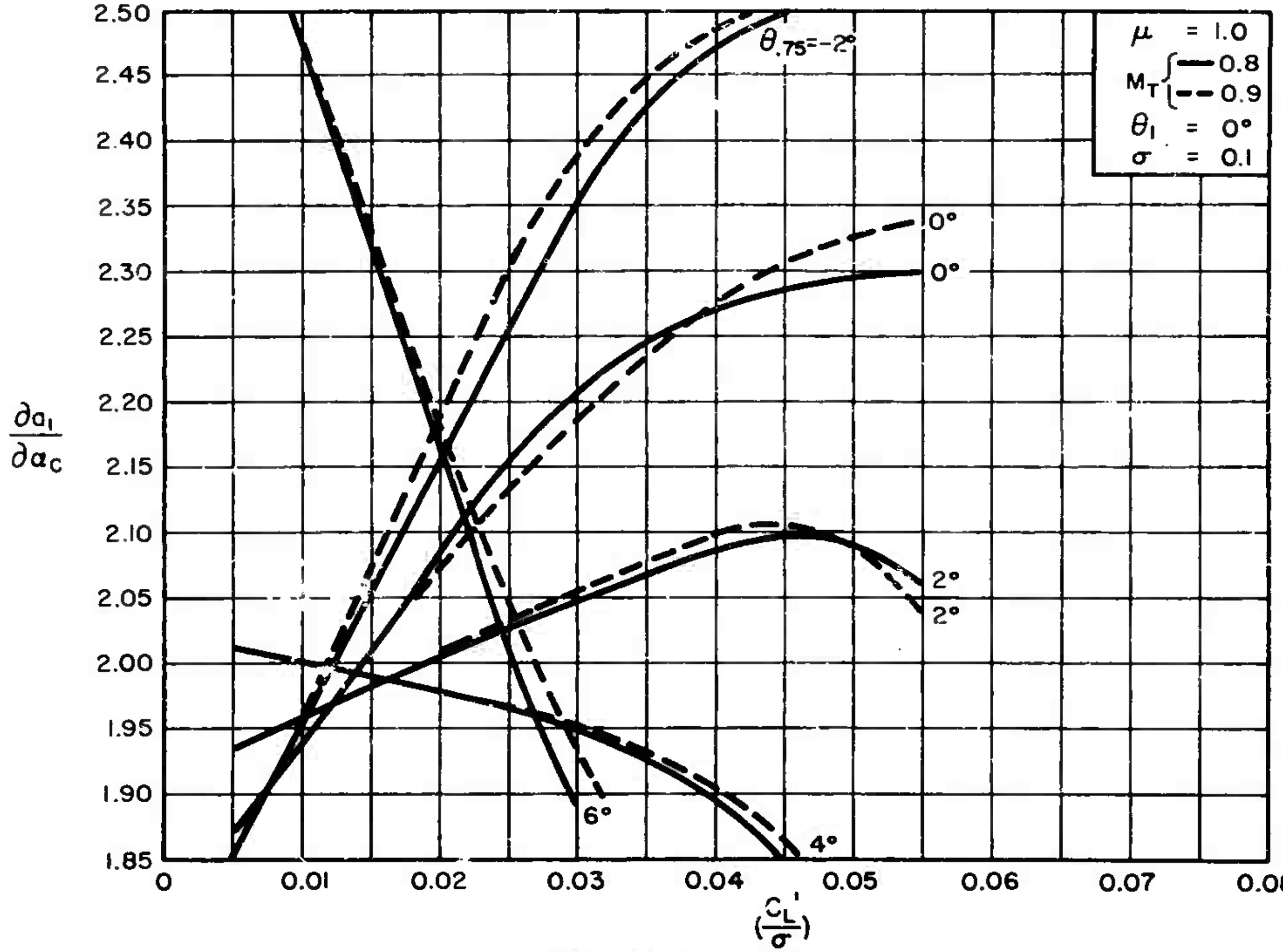

Figure 25. Continued

(h) $\partial a_1/\partial \alpha_c$ for $\mu = 1.0$

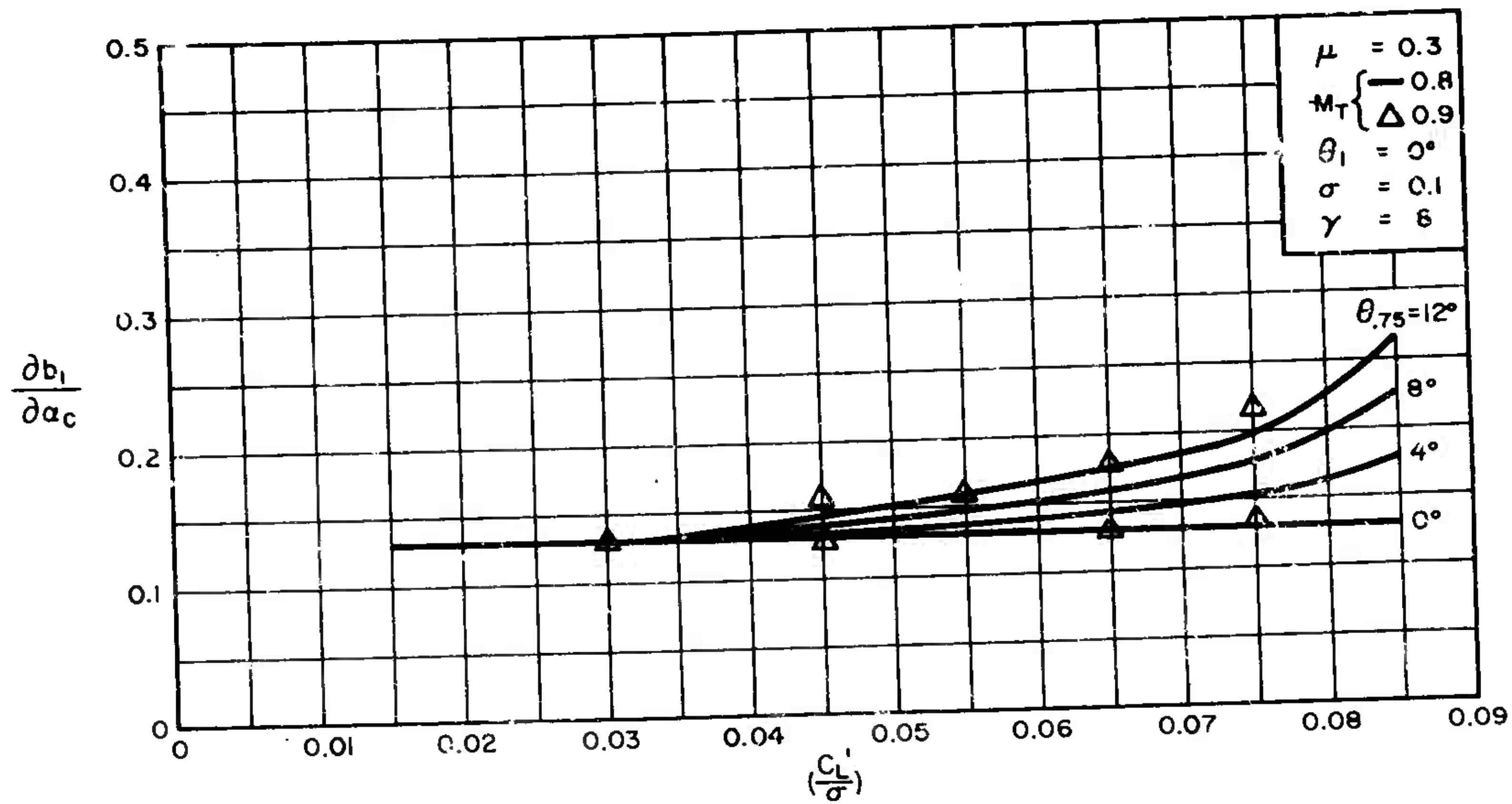

Figure 25. Continued

(i) $\partial b_1/\partial a_c$ for $\mu = 0.3$

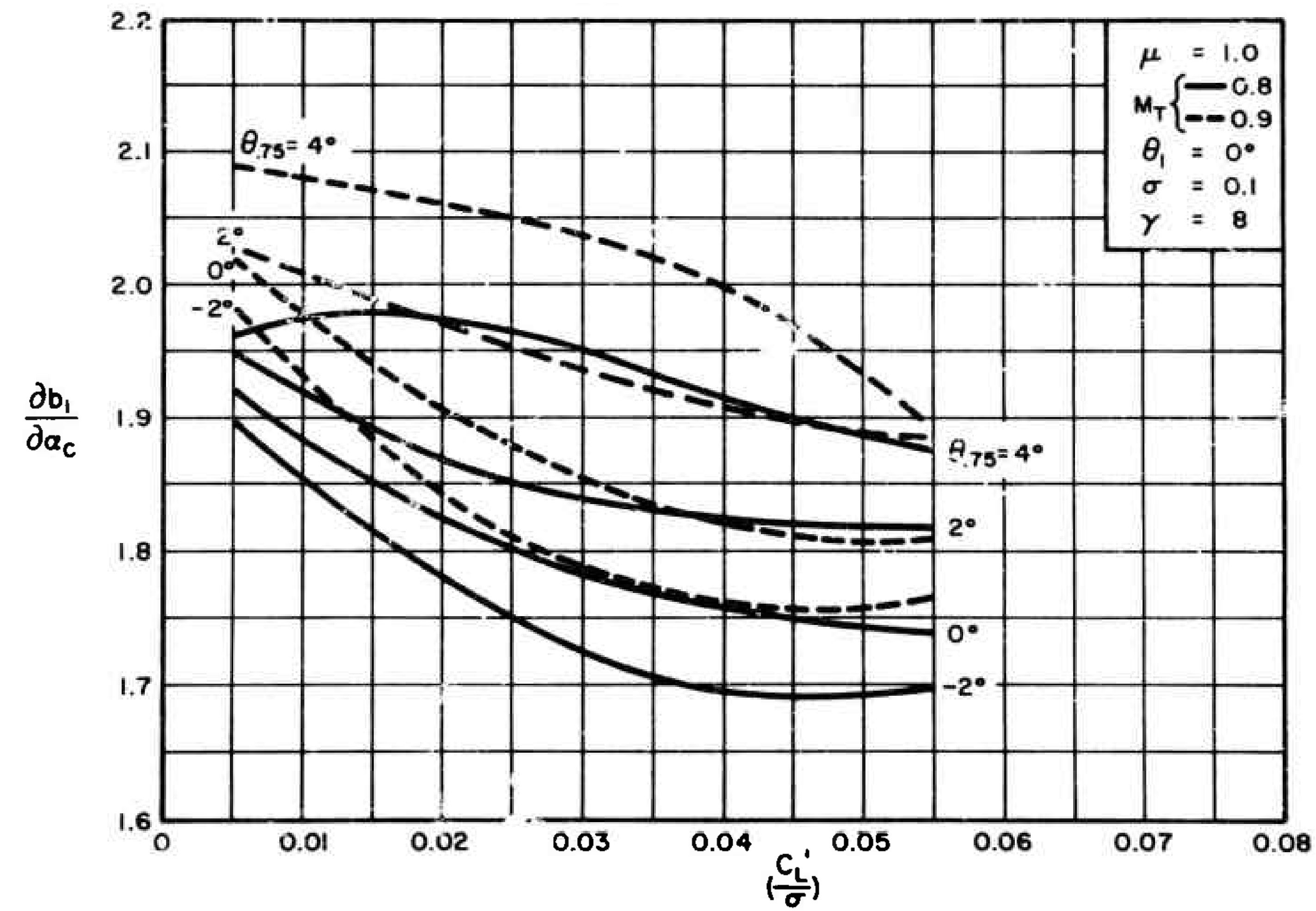

Figure 25. Continued

(j) $\partial b_1/\partial \alpha_c$ for $\mu = 1.0$

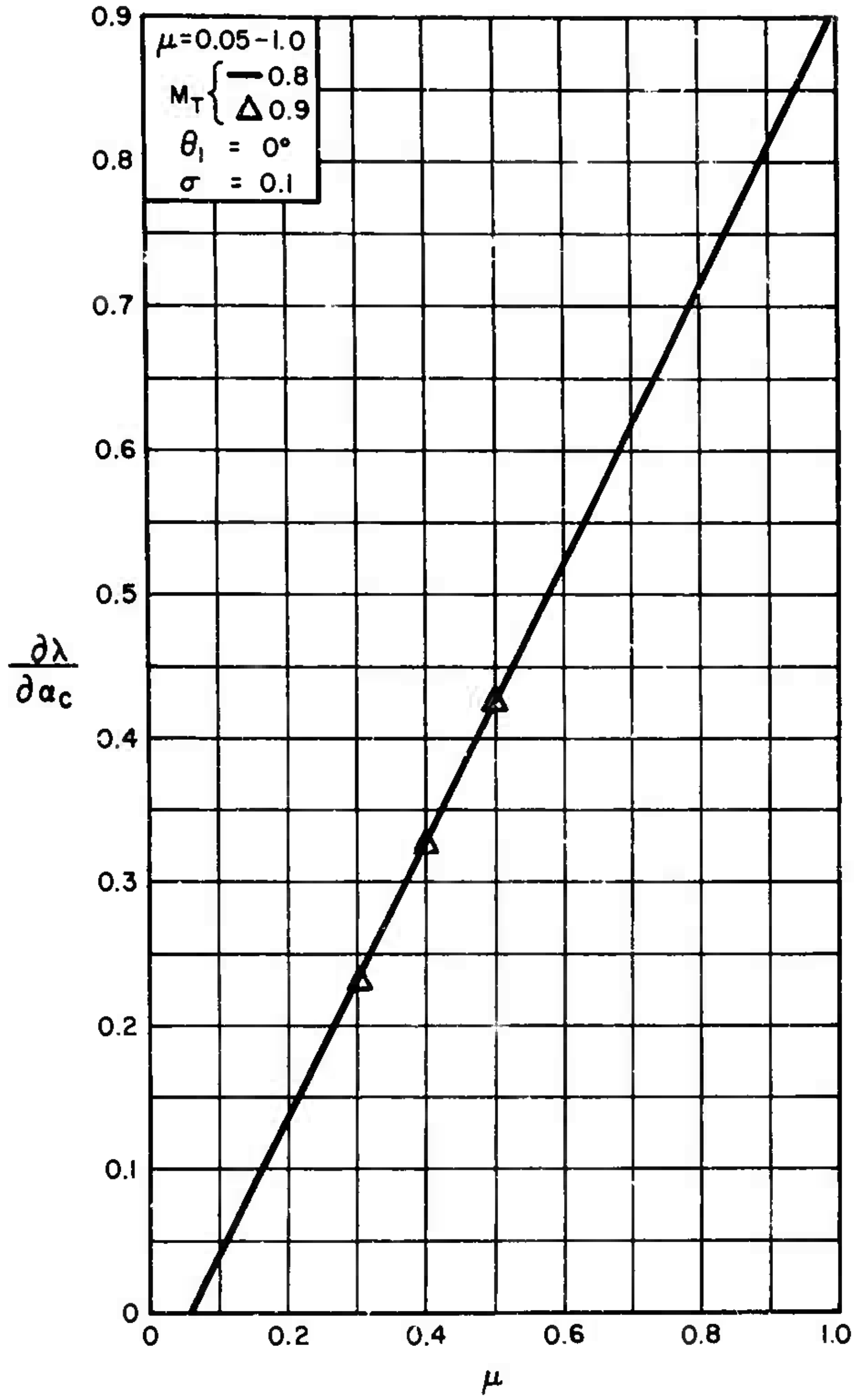

Figure 25. Concluded

(k) $\partial\lambda/\partial\alpha_c$ for all values of μ

7.5.5.3 Effect of Compressibility on the Isolated Rotor Derivatives With Respect to $\theta_{.75}$

Figures 26(a) through 26(i) present an indication of the effect of Mach number variation on the isolated rotor derivatives $\partial(C_L'/\sigma)\partial\theta_{.75}$, $\partial(C_D'/\sigma)\partial\theta_{.75}$, $\partial(C_Q/\sigma)/\partial\theta_{.75}$, $\partial a_1/\partial\theta_{.75}$, $\partial b_1/\partial\theta_{.75}$, and $\partial\lambda/\partial\theta_{.75}$, respectively. The results are presented for two values of tip speed ratios, $\mu = 0.4$ and $\mu = 1.0$, and a representative range of rotor angle of attack for each μ. The two different values of μ were purposely selected to show the compressibility effects in low and high speed regimes.

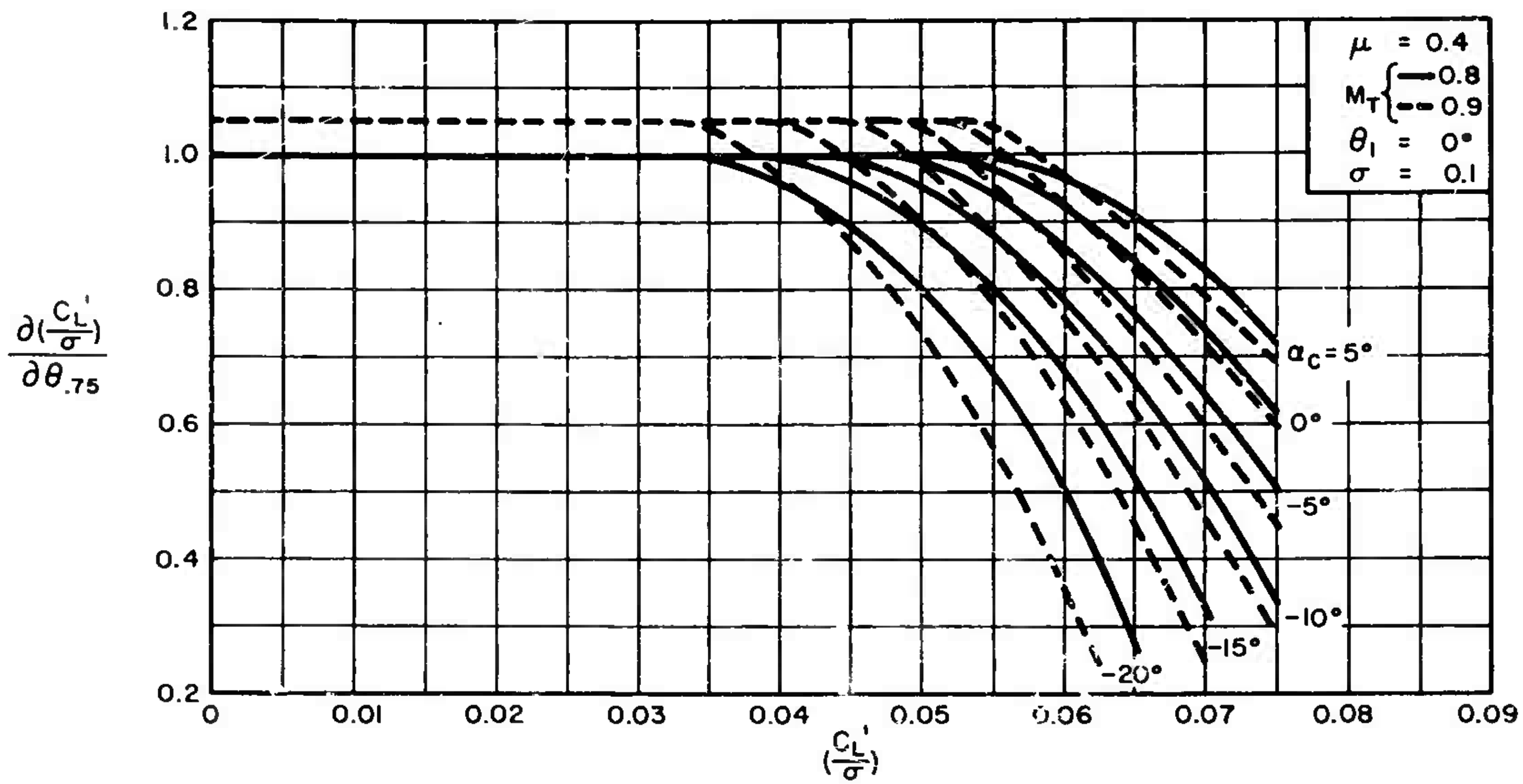

Figure 26. Effect of Compressibility on $\theta_{.75}$ Derivatives

(a) $\partial(\frac{C_L'}{\sigma})/\partial\theta_{.75}$ for $\mu = 0.4$

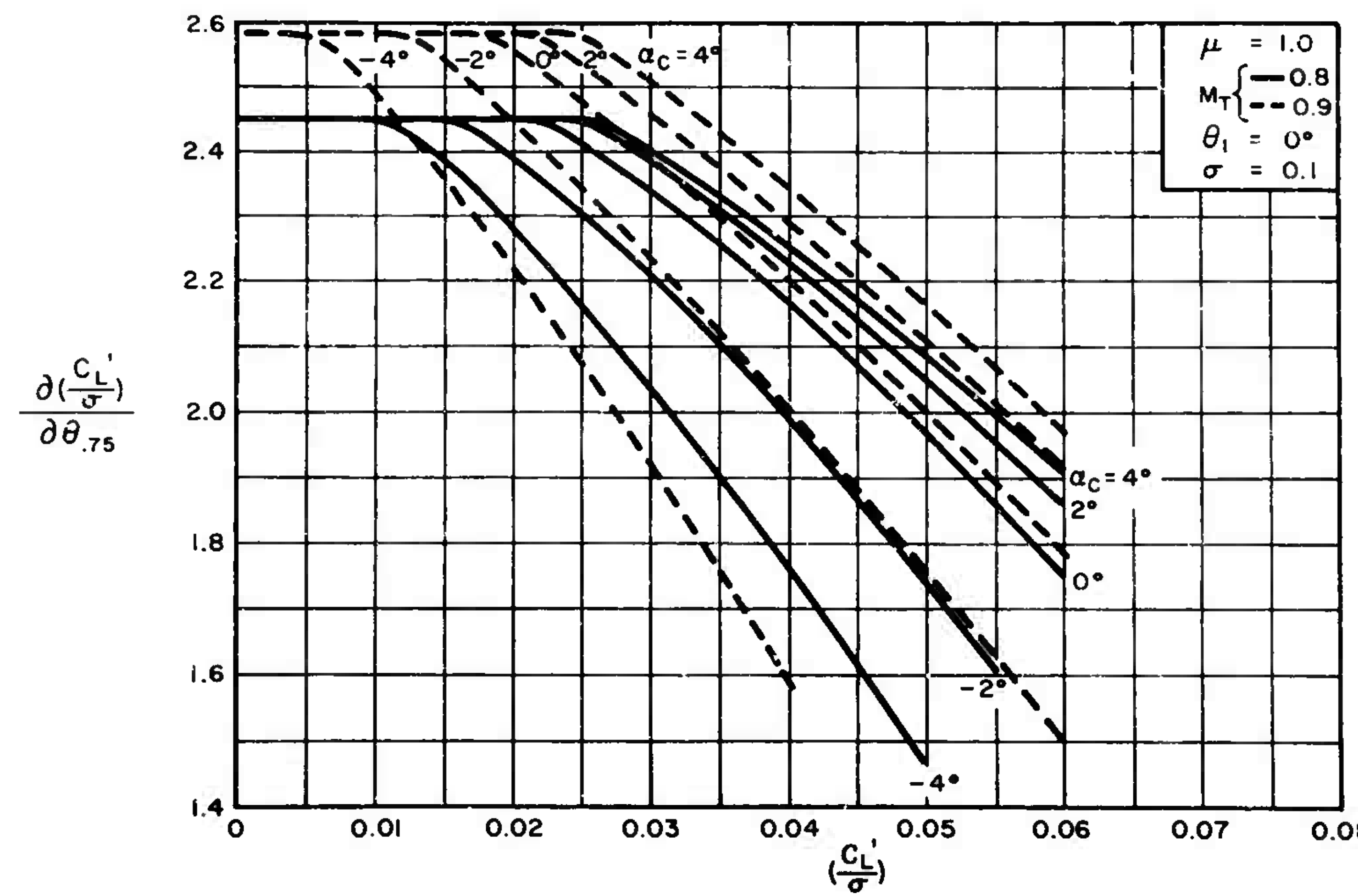

Figure 26. Continued

(b) $\partial(\frac{C_L'}{\sigma})/\partial\theta_{.75}$ for $\mu = 1.0$

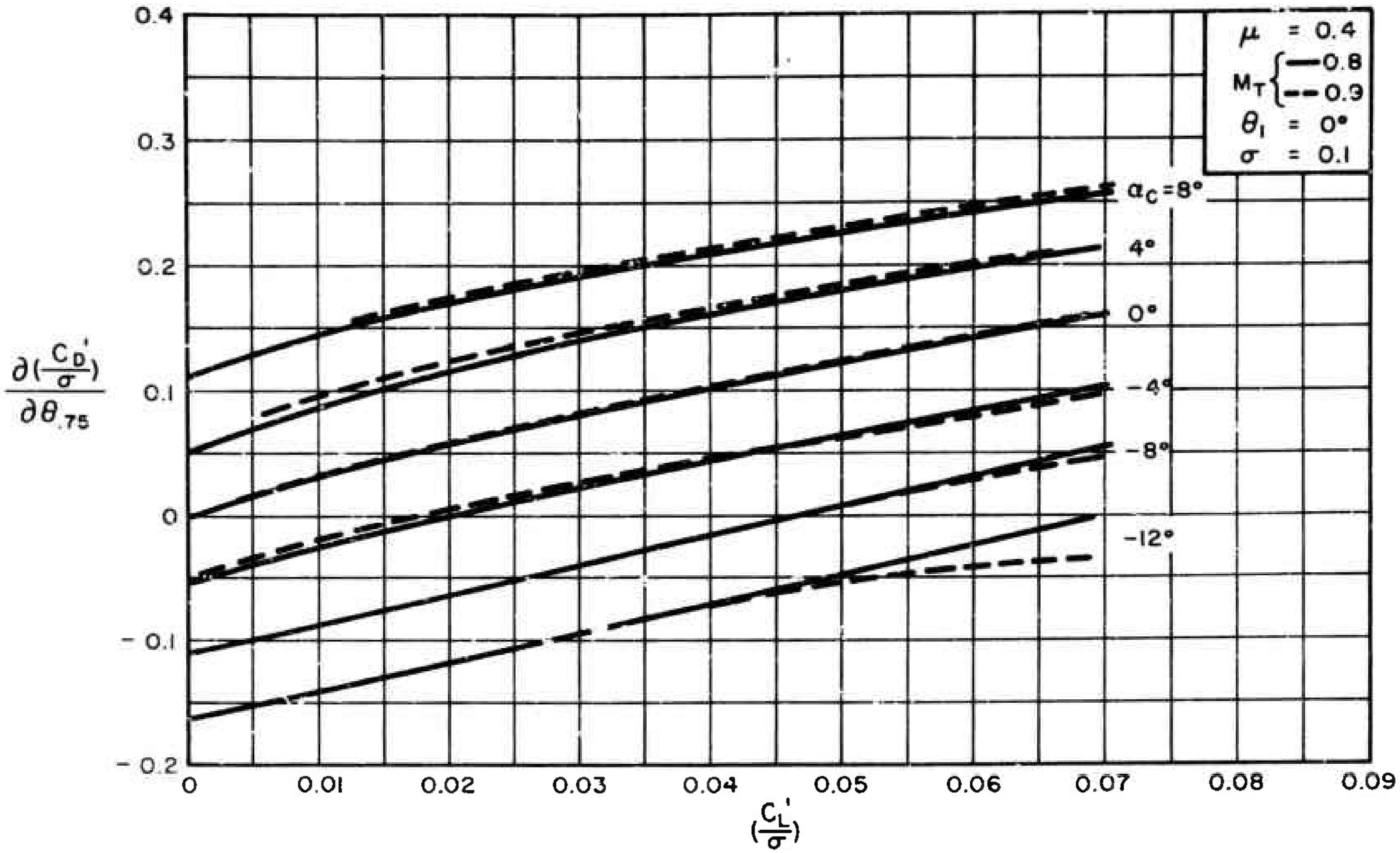

Figure 26. Continued

(c) $\partial(\frac{C_D'}{\sigma})/\partial\theta_{.75}$ for $\mu = 0.4$

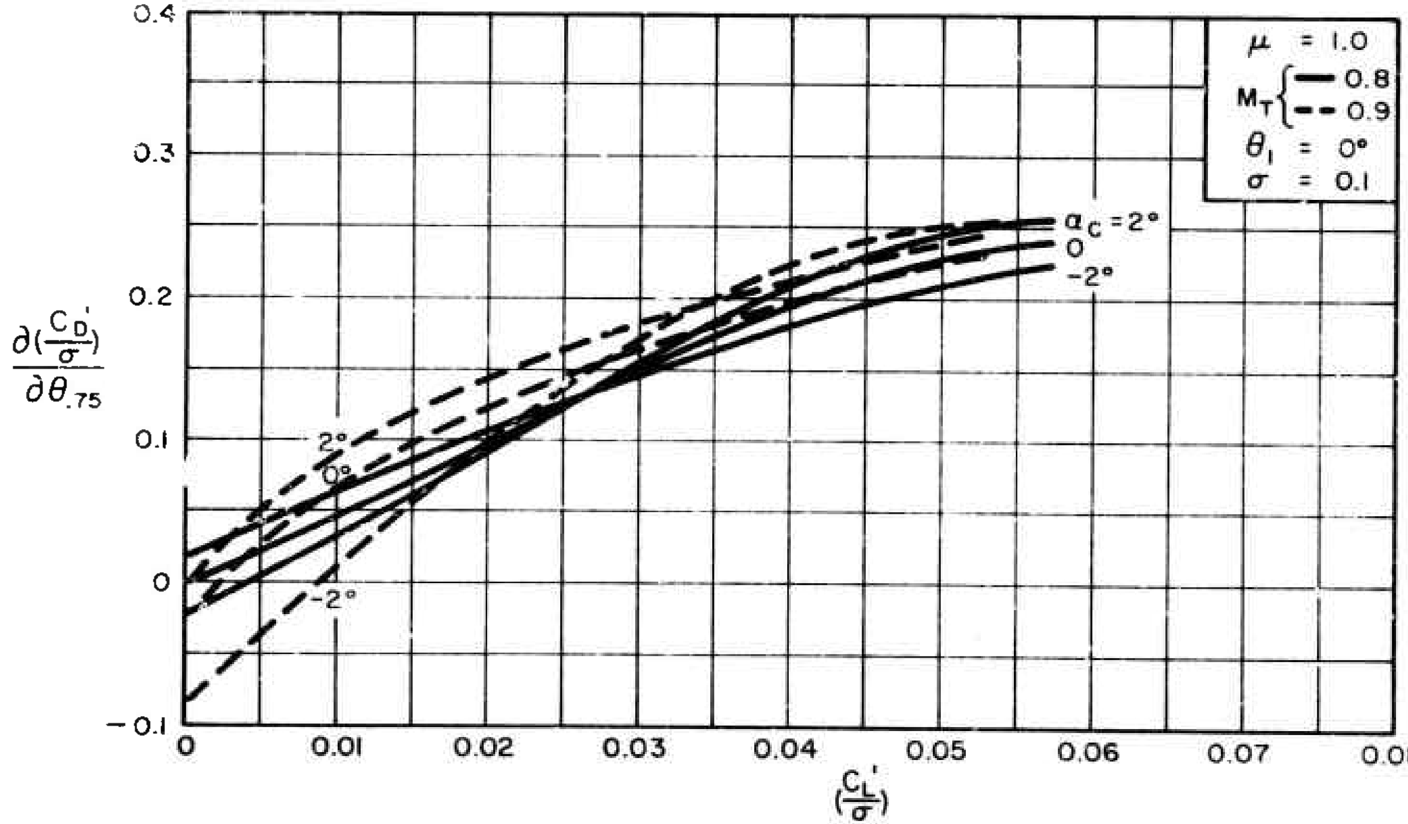

Figure 26. Continued

(d) $\partial(\frac{C_D'}{\sigma})/\partial\theta_{.75}$ for $\mu = 1.0$

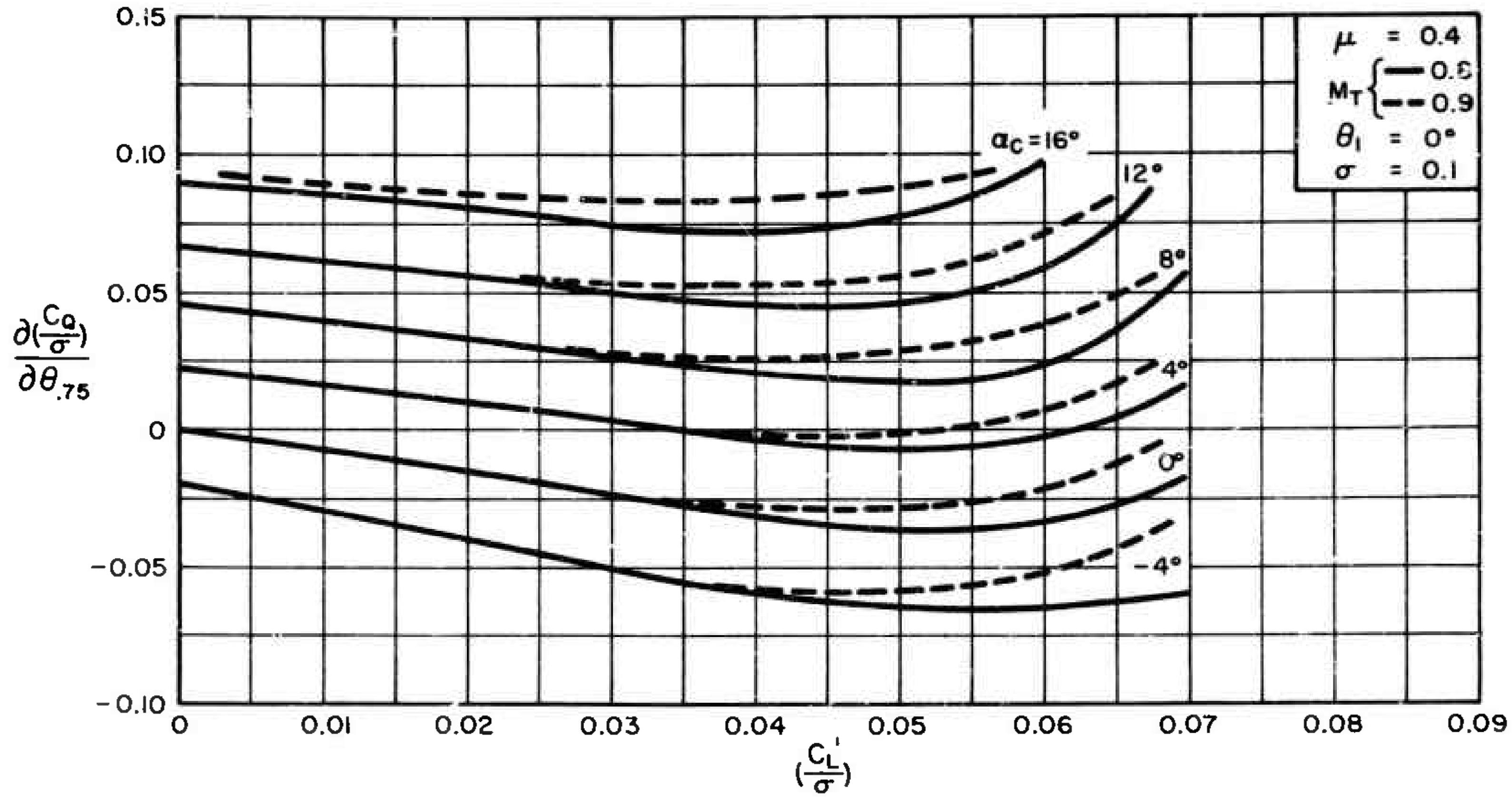

Figure 26. Continued

(e) $\partial(\frac{C_Q}{\sigma})/\partial\theta_{.75}$ for μ=0.4

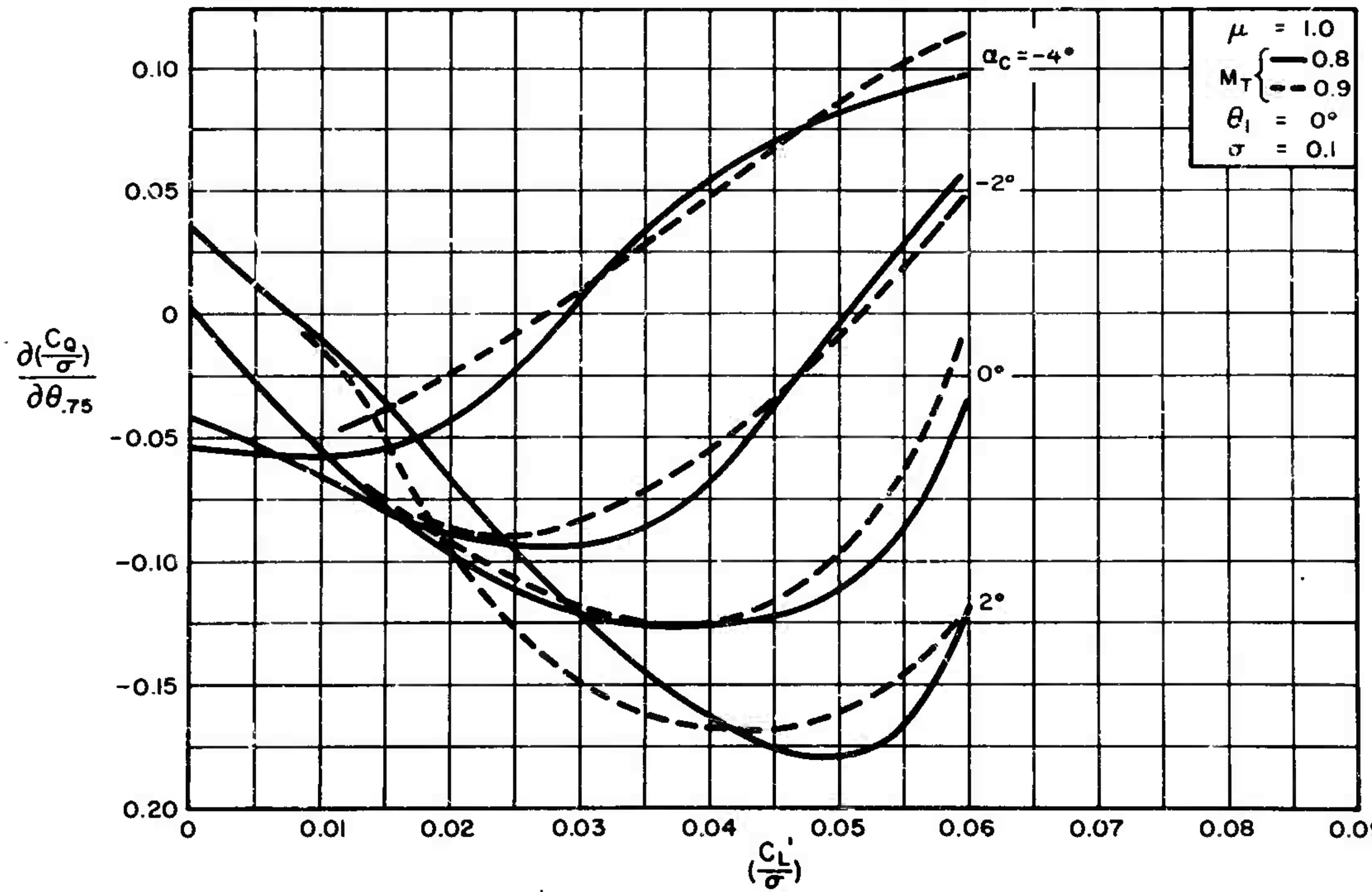

Figure 26. Continued

(f) $\partial(\frac{C_Q}{\sigma})/\partial\theta_{.75}$ for $\mu = 1.0$

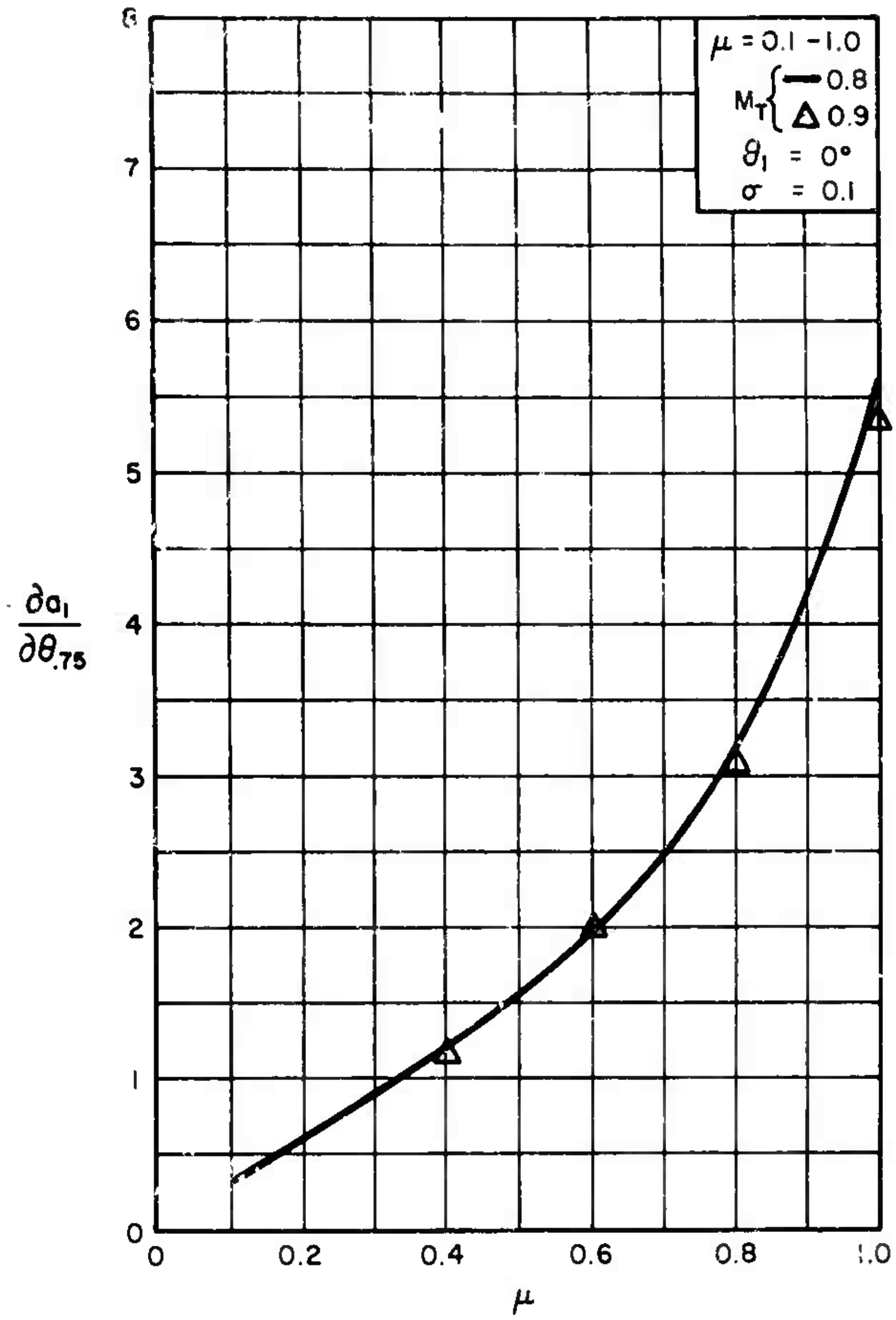

Figure 26. Continued

(g) $\partial a_1/\partial \theta_{.75}$ for all values of $\frac{C_L'}{\sigma}$

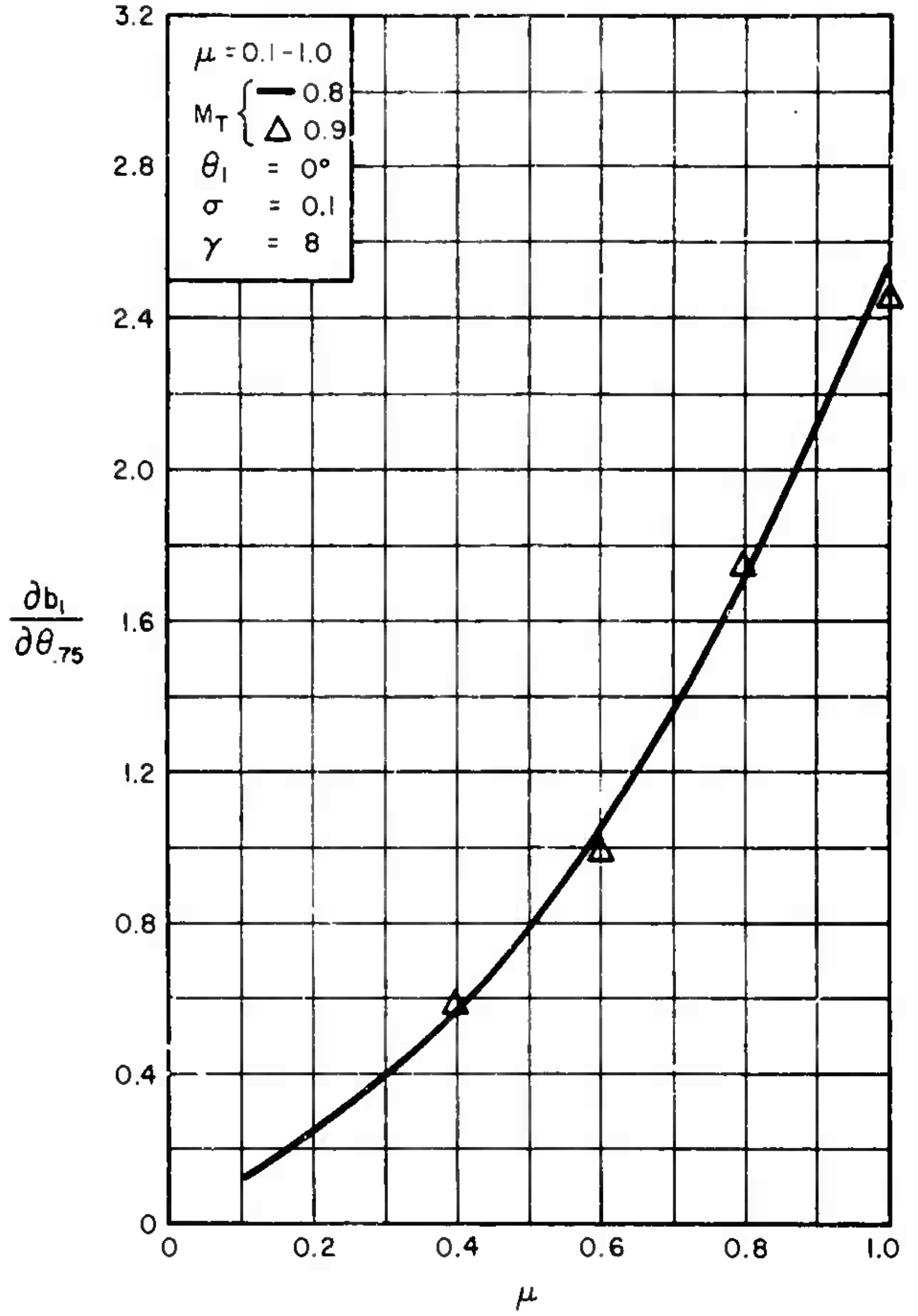

Figure 26. Continued

(h) $\partial b_1/\partial \theta_{.75}$ for all values of $\frac{C_L'}{\sigma}$

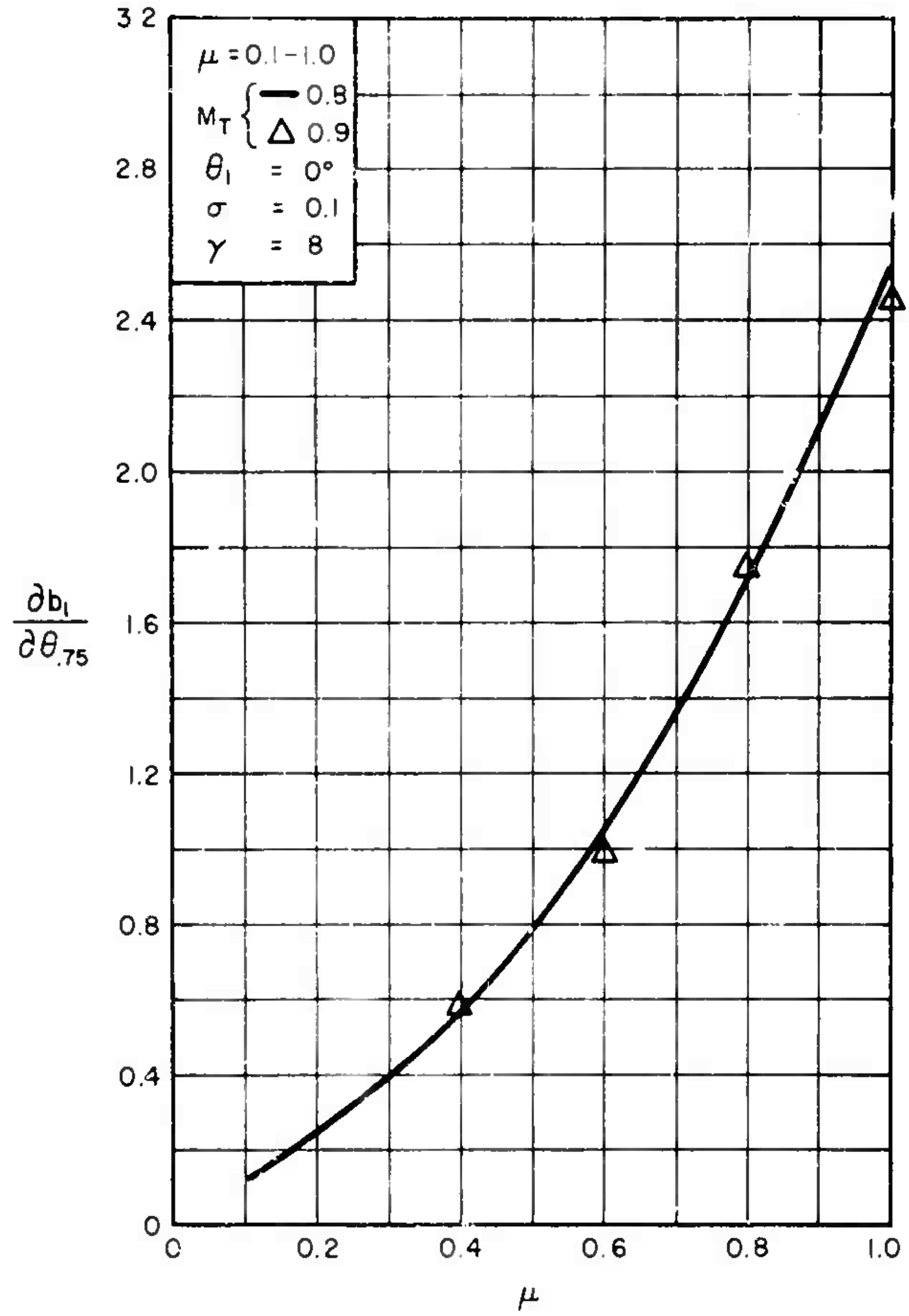

Figure 26. Continued

(h) $\partial b_1/\partial\theta_{.75}$ for all values of $\frac{C_L'}{\sigma}$

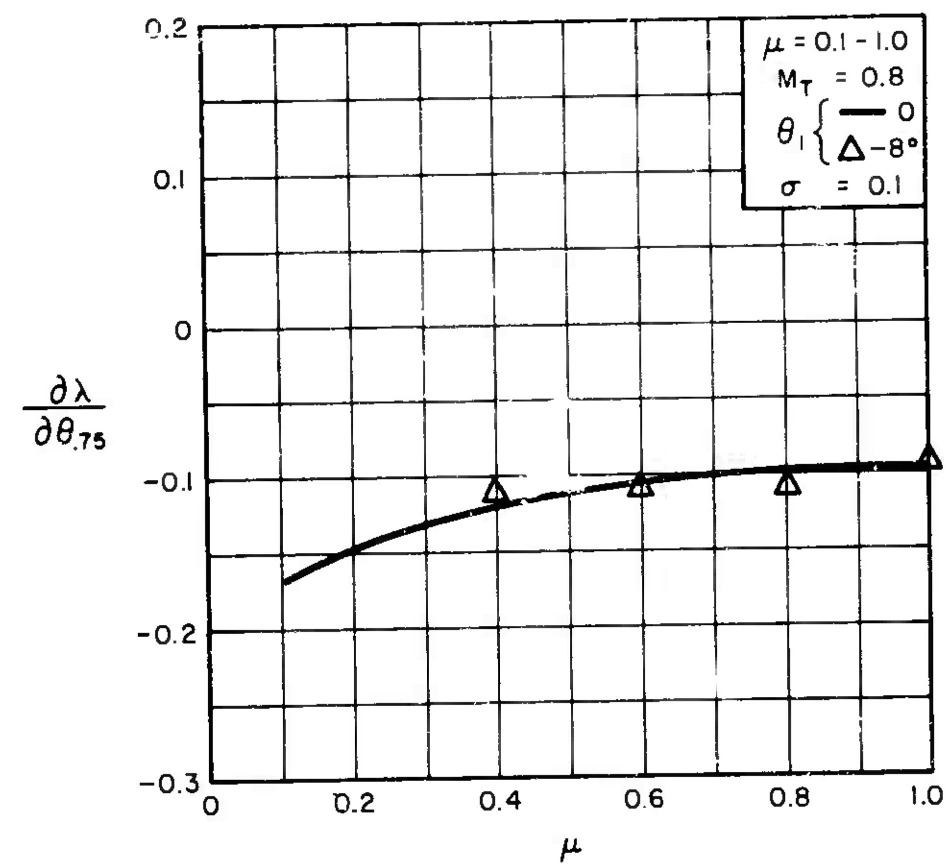

Figure 26. Concluded

(i) $\partial\lambda/\partial\theta_{.75}$ for all values of $\frac{C_L'}{\sigma}$

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1. Tanner, W. H., Charts for Estimating Rotary Wing Performance in Hover and at High Forward Speeds, NASA Contractor Report CR-114, National Aeronautics and Space Administration, Washington, D.C., November 1964.

2. Stability and Control Handbook for Helicopters, TRECOM Report 60-43, U. S. Army Transportation Research Command (presently, U. S. Army Aviation Materiel Laboratories), Fort Eustis, Virginia, August 1960.

3. Gessow, A., and Myers, G. C., Jr., Aerodynamics of the Helicopter, The MacMillan Company, New York, 1962.

7.6 DOWNWASH INTERFERENCE EFFECTS

Interference effects between aerodynamic components can be expressed in terms of changes of local velocity and local angle of attack. Changes in the effective velocity due to aerodynamic interference are generally small and are herein neglected. Changes in local angle of attack, however, can be appreciable. In general, the local angle of attack of an aerodynamic component can be expressed in terms of the remote stream angle of attack and interference angles as follows:

$$\alpha_{LOCAL} = \alpha + i - \epsilon$$

where

α = remote wind angle of attack relative to the X-axis

i = the geometric inclination of the specific aerodynamic component considered with respect to the X-axis

ϵ = aerodynamic interference angle.

For the helicopter configurations considered here, the aerodynamic interference is generated mainly by the downwash velocities of the rotors. Hence, each rotor can affect any other rotor, the fuselage, and any lifting surface attached to the fuselage. The downwash velocity of a rotor varies with time as well as with location. The determination of the effect of such a varying velocity distribution on the lift and drag of a rotor, fuselage, or lifting surface is an exceedingly complicated task; in fact, to be consistent with other assumptions made, it is not required in the stability and control analysis. Indeed, it is adequate to assume that the effective change of angle of attack of an aerodynamic component due to rotor downwash is equal to the average downwash velocity of the rotor, divided by the free stream velocity, and multiplied by an

appropriate downwash interference factor. Hence, the angle due to downwash interference of the front rotor on the rear rotor of a tandem helicopter is given by

$$\epsilon_R = K_{FR}\left(\frac{V_{i_F}}{V_a}\right)$$

where K_{FR} is the interference factor of the front rotor on the rear rotor, as identified by the subscripts FR; V_{i_F} is the average induced velocity at the front rotor plane. The term V_{i_F} is obtained by use of the momentum equation as follows:

$$\frac{V_{i_F}}{V_a} = \tan \alpha_{C_F} - \left(\frac{\lambda}{\mu}\right)_F$$

The downwash interference angles of the front rotor on the rear of a tandem rotor helicopter or front and rear rotors on other aerodynamic components can therefore be written as follows:

(a) Front Rotor on Rear Rotor

$$\epsilon_R = K_{FR}\left[\tan \alpha_C - \left(\frac{\lambda}{\mu}\right)\right]_F$$

(b) Fuselage

$$\epsilon_{FUS} = K_{FFUS}\left[\tan \alpha_C - \left(\frac{\lambda}{\mu}\right)\right]_F + K_{RFUS}\left[\tan \alpha_C - \left(\frac{\lambda}{\mu}\right)\right]_R$$

(c) Wing

$$\epsilon_W = K_{FW}\left[\tan \alpha_{C_F} - \left(\frac{\lambda}{\mu}\right)_F\right] + K_{RW}\left[\tan \alpha_{C_R} - \left(\frac{\lambda}{\mu}\right)_R\right]$$

(d) Horizontal Tail Surface

$$\epsilon_T = K_{FT}\left[\tan \alpha_{C_F} - \left(\frac{\lambda}{\mu}\right)_F\right] + K_{RT}\left[\tan \alpha_{C_R} - \left(\frac{\lambda}{\mu}\right)_R\right]$$

(e) Vertical Tail Surface

$$\epsilon_{VT} = K_{FVT}\left[\tan \alpha_{C_F} - \left(\frac{\lambda}{\mu}\right)_F\right] + K_{RVT}\left[\tan \alpha_{C_R} - \left(\frac{\lambda}{\mu}\right)_R\right]$$

(f) Rear Rotor on Front Rotor

$$\epsilon_F = K_{RF}\left[\tan \alpha_{C_R} - \left(\frac{\lambda}{\mu}\right)_R\right]$$

(g) Tail Rotor

$$\epsilon_{TR} = K_{FTR}\left[\tan \alpha_{C_F} - \left(\frac{\lambda}{\mu}\right)_F\right] + K_{RTR}\left[\tan \alpha_{C_R} - \left(\frac{\lambda}{\mu}\right)_R\right]$$

On the basis of data on the downwash behind a single rotor, such as Reference 1, it has been concluded by other investigators that a presentation of the downwash factor as a function of wake angle will yield more accurate results. The wake angle is defined by

$$\chi = a_1 + \tan^{-1}\left[\frac{1}{\left(-\frac{\lambda}{\mu}\right)}\right]$$

The variation of the interference factor K_{FR} as a function of χ, neglecting rotor overlap and differential rotor height, may be taken as that suggested in Reference 2, and is presented in Figure 1.

This factor is obtained from the theory of Reference 3 and represents the value of the ratio of the downwash at the location of the center of the rear rotor to the downwash at the center of the front rotor. The theory is based on the downwash due to one isolated rotor and, hence, neglects the effect of the presence of the rear rotor on the resultant flow of the front rotor. Correlation with test data, similar to those obtained in Reference 4, is required to check the validity of this assumption. It is recommended that until better information becomes available, the value of K_{FR} as presented in Figure 1 should be used. It is also recommended that K_{FT} be taken equal to K_{FR}. Very little information is available on the effects of the rear rotor on the front rotor. Some investigators recommend the use of $K_{RF} = -0.08$ to indicate the existence of a slight upwash. Until more reliable data become available, it is recommended that $K_{RF} = 0$ be utilized.

Measurements of fuselage lift and drag reported in Reference 5 indicate that for a single rotor helicopter K_{FFUS} is approximately 1.0. Also, test data presented in Reference 5 on the horizontal tail interference factor are reproduced here as Figure 2. These data were obtained for the horizontal tail, located approximately one rotor radius behind the rotor center. Based on these data, it is recommended that $K_{FT} = 1.0$ be utilized.

In summary, the following values for the downwash interference factors are recommended:

K_{FR} – see Figure 1

$K_{FT} = K_{FVT} = K_{FTR} = K_{FR}$

$K_{FFUS} = K_{FW} = 1.0$

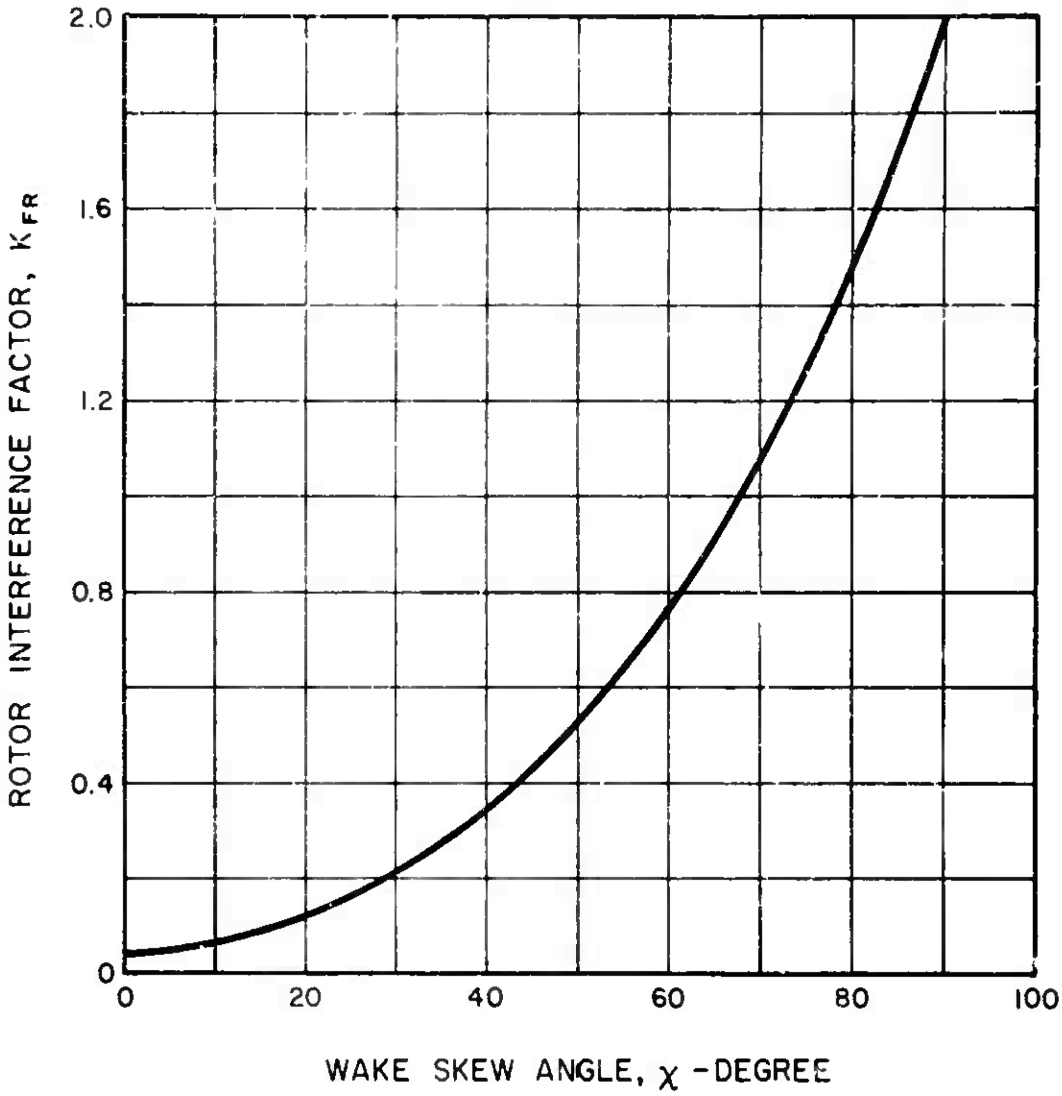

Figure 1. Variation of K_{FR} vs. χ

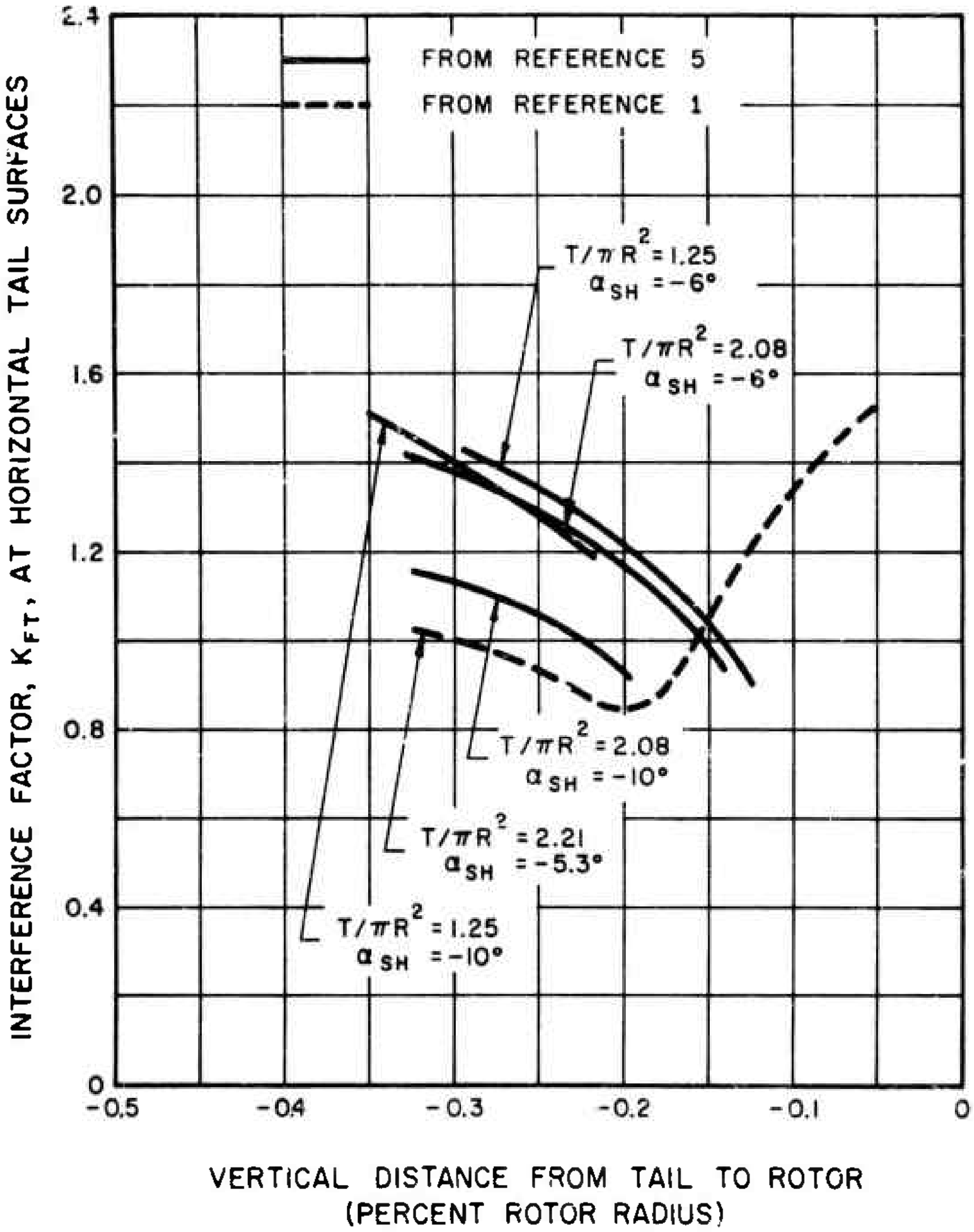

Figure 2. Interference Factor at a "Half Tee" Tail

$K_{RT} = K_{RVT} = K_{RTR} = K_{RFUS} = K_{RW} = 1.0$

$K_{RF} = 1.0$

NOTE: The values of K_{FW} and K_{RW} are given for a wing located at a distance of less than one rotor radius aft of front or rear rotor shaft, respectively.

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7.7 LIFTING SURFACE CHARACTERISTICS

The aerodynamic characteristics of lifting surfaces are documented in numerous NACA and NASA publications. As described in Reference 1, the aerodynamic coefficients and their derivatives with respect to pertinent stability parameters depend on the specific geometric configuration of the lifting surface. In general, it is adequate to use the following expressions for a lifting surface such as a wing:

$$C_{L_W} = \frac{L_W}{\frac{1}{2}\rho V_0^2 S_W}$$

$$= a_W \alpha_W$$

where

$$a_W = \frac{a_{0_W}}{1 + \frac{a_{0_W}}{\pi \mathcal{R}}}$$

a_{0_W} = wing section lift curve slope

$\mathcal{R}$ = wing aspect ratio

and

$$\alpha_W = \alpha + i_W - \epsilon_W$$

Also, wing drag coefficient is given by:

$$C_{D_W} = \frac{D_W}{\frac{1}{2}\rho V_0^2 S_W} = C_{D_{0_W}} + \frac{C_{L_W}^{\ 2}}{\pi \mathcal{R}}$$

where $C_{D_{O_W}}$ is the wing section profile drag coefficient. Values for the stability derivatives of lifting surfaces can be obtained from Reference 1.

REFERENCE

1. USAF Stability and Control Handbook (DATCOM), Flight Control Division, Air Force Flight Dynamics Laboratory, Wright-Patterson Air Force Base, Ohio, October 1960, Revised July 1963.

SECTION 8. STABILITY CHARACTERISTIC EQUATIONS

The linearized equations of motion given in Section 6 can be represented as a set of homogeneous algebraic equations containing the unknowns $\bar{u}, \bar{v}, \bar{w}, \bar{\theta}, \phi, \psi$, etc., and the operator Λ. This operator is defined as the time rate of change of the unknowns, thus: $\Lambda \equiv d(\ )/dt$ and $\Lambda^2 \equiv d^2(\ )/dt^2$, etc.

The simultaneous solution for the unknowns can be obtained by employing the usual determinant methods which yield

$$\bar{u} = \frac{f_1(\Lambda)}{F(\Lambda)}, \quad \bar{v} = \frac{f_2(\Lambda)}{F(\Lambda)}, \quad \bar{w} = \frac{f_3(\Lambda)}{F(\Lambda)}, \quad \bar{\theta} = \frac{f_4(\Lambda)}{F(\Lambda)}$$

The numerator determinants $f_1(\Lambda)$, $f_2(\Lambda)$ ... $f_4(\Lambda)$ ... are formed by replacing the coefficients of the appropriate unknown variables by the column of constants which pertain to the control inputs. The denominator determinant $F(\Lambda)$ consists of the coefficients of the homogeneous algebraic equations with control inputs fixed at zero. The determinant $F(\Lambda)$ is known as the stability determinant. Expansion of the determinant $F(\Lambda)$ leads to the stability characteristic equation. The property of this type of equation is that there can be nonzero values of the unknowns ($\bar{u}$, $\bar{v}$, $\bar{w}$) if, and only if, the determinant $F(\Lambda) = 0$. Setting the determinant equal to zero provides the condition for finding the roots of the characteristic equation $\Lambda_1, \Lambda_2, \ldots \Lambda_n$

In order to obtain the actual response solution of the unknown variables ($\bar{u}, \bar{v}, \bar{w}$) due to a given forcing function or control input, the Heaviside expansion theorem can be used. The Heaviside expansion method is developed in Reference 1, pages 436 to 438, and will not be duplicated in this section; however, the final response equations are given below.

If it is assumed that the stability characteristic equation $F(\Lambda)=0$ yields n real roots, $\Lambda_1, \Lambda_2, \ldots \Lambda_n$ and m pairs of complex roots, $\Lambda_m = a_n \pm b_m i$, the time history response

of any variable (e.g., variable $\bar{\theta}$) can be expressed as follows:

$$\bar{\theta} = \frac{f_4(\Lambda)}{F(\Lambda)} = \frac{f_4(0)}{F'(0)} + \sum_{\Lambda=\Lambda_1}^{\Lambda_n} \frac{f_4(\Lambda)}{\Lambda F'(\Lambda)} e^{\Lambda t} + \sum_{\Lambda=\Lambda_1}^{\Lambda_m} A e^{a_m t} \sin(b_m t + \Phi)$$

where the constants A and Φ can be obtained by using appropriate boundary conditions.

In the case that one of the real roots of the characteristic equation is zero (i.e., $\Lambda_1 = 0$), the response equation becomes

$$\bar{\theta} = \frac{f_4(\Lambda)}{F(\Lambda)} = \frac{f_4(0)}{F'(0)} t + \sum_{\Lambda=\Lambda_2}^{\Lambda_n} \frac{f_4(\Lambda)}{\Lambda F'(\Lambda)} e^{\Lambda t} + \sum_{\Lambda=\Lambda_1}^{\Lambda_m} A e^{a_m t} \sin(b_m t + \Phi)$$

REFERENCE

1. Perkins, C. D., and Hage, R. E., Airplane Performance Stability and Control, Third Edition, John Wiley and Sons, Inc., New York, Chapman and Hall Ltd., London, England, February 1953.

8.1 COUPLED LONGITUDINAL AND LATERAL MODES INCLUDING STABILITY AUGMENTATION SYSTEM

In this section, the generalized case of aircraft perturbation motion consisting of 6 degrees of freedom of aircraft motion and 2 degrees of freedom of the motion of the stability augmentation system is considered. The analysis presented herein is suitable for either analog or digital computer work.

The linearized equations of motion presented in Section 6 can be expressed as follows:

(a) The X-Force Equation

$$a_{11}\bar{u} + a_{12}\bar{v} + a_{13}\bar{w} + a_{14}\bar{\theta} + a_{15}\bar{\phi} + a_{16}\bar{\psi} + a_{17}\bar{B}_{1_S} + a_{18}\bar{A}_{1_S} = K_1$$

(b) The Y-Force Equation

$$a_{21}\bar{u} + a_{22}\bar{v} + a_{23}\bar{w} + a_{24}\bar{\theta} + a_{25}\bar{\phi} + a_{26}\bar{\psi} + a_{27}\bar{B}_{1_S} + a_{28}\bar{A}_{1_S} = K_2$$

(c) The Z-Force Equation

$$a_{31}\bar{u} + a_{32}\bar{v} + a_{33}\bar{w} + a_{34}\bar{\theta} + a_{35}\bar{\phi} + a_{36}\bar{\psi} + a_{37}\bar{B}_{1_S} + a_{38}\bar{A}_{1_S} = K_3$$

(d) The Rolling Moment($\mathcal{L}$) Equation

$$a_{41}\bar{u} + a_{42}\bar{v} + a_{43}\bar{w} + a_{44}\bar{\theta} + a_{45}\bar{\phi} + a_{46}\bar{\psi} + a_{47}\bar{B}_{1_S} + a_{48}\bar{A}_{1_S} = K_4$$

(e) The Pitching Moment (M) Equation

$$a_{51}\bar{u}+a_{52}\bar{v}+a_{53}\bar{w}+a_{54}\bar{\theta}+a_{55}\bar{\phi}+a_{56}\bar{\psi}+a_{57}\bar{B}_{1_S}+a_{58}\bar{A}_{1_S}=K_5$$

(f) The Yawing Moment (N) Equation

$$a_{61}\bar{u}+a_{62}\bar{v}+a_{63}\bar{w}+a_{64}\bar{\theta}+a_{65}\bar{\phi}+a_{66}\bar{\psi}+a_{67}\bar{B}_{1_S}+a_{68}\bar{A}_{1_S}=K_6$$

(g) Stability Augmentation System Equations

(i) Longitudinal Control (B_{1_S}) Equation

$$a_{71}\bar{u}+a_{72}\bar{v}+a_{73}\bar{w}+a_{74}\bar{\theta}+a_{75}\bar{\phi}+a_{76}\bar{\psi}+a_{77}\bar{B}_{1_S}+a_{78}\bar{A}_{1_S}=K_7$$

(ii) Lateral Control (A_{1_S}) Equation

$$a_{81}\bar{u}+a_{82}\bar{v}+a_{83}\bar{w}+a_{84}\bar{\theta}+a_{85}\bar{\phi}+a_{86}\bar{\psi}+a_{87}\bar{B}_{1_S}+a_{88}\bar{A}_{1_S}=K_8$$

The coefficients a_{mn} and the control parameters K_n are given in Table I.

The numerator determinant $f_4(\Lambda)$ required to determine the response of variable (θ) is given by

$$f_4(\Lambda)=\begin{vmatrix} a_{11} & a_{12} & a_{13} & K_1 & \cdot & \cdot & \cdot & a_{18} \\ a_{21} & a_{22} & a_{23} & K_2 & \cdot & \cdot & \cdot & a_{28} \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ a_{81} & a_{82} & a_{83} & K_8 & \cdot & \cdot & \cdot & a_{88} \end{vmatrix}$$

THE COEFFICIENTS OF THE

EQUAT.	a	1	2	3
X	1	$X_u + X_{\dot{u}} \Lambda$	X_v	$X_w + X_{\dot{w}} \Lambda$
Y	2	Y_u	$Y_v + Y_{\dot{v}} \Lambda$	Y_w
Z	3	Z_u	Z_v	$Z_w + Z_{\dot{w}} \Lambda$
$\mathcal{L}$	4	$\mathcal{L}_u$	$\mathcal{L}_v$	$\mathcal{L}_w + \mathcal{L}_w \Lambda$
M	5	M_u	M_v	$M_w + M_{\dot{w}} \Lambda$
N	6	N_u	N_v	$N_w + N_{\dot{w}} \Lambda$
B_{1_s}	7	$-k_3$	0	$-k_4$
A_{1_s}	8	0	k_7	0

TABLE I

DETERMINANT FOR AIRCRAFT RESPONSE ANALYSIS

4	5	6	7	8	K
$X_\theta + X_{\dot\theta}\Lambda + X_{\ddot\theta}\Lambda^2$	$X_{\dot\phi}\Lambda$	$X_{\dot\psi}\Lambda$	$X_{B_{1_S}} + X_{\dot B_{1_S}}\Lambda$	$X_{A_{1_S}} + X_{\dot A_{1_S}}\Lambda$	$-J_1 X_{B_{1_C}}\overline{B_{1_C}} - J_2 X_{A_{1_C}}\overline{A_{1_C}} - J_3 X_{\delta_{r_C}}\overline{\delta_{r_C}} - J_4 X_{\theta_C}\overline{\theta_C}$
$Y_{\dot\theta}\Lambda$	$Y_\phi + Y_{\dot\phi}\Lambda$	$Y_\psi + Y_{\dot\psi}\Lambda$	$Y_{B_{1_S}} + Y_{\dot B_{1_S}}\Lambda$	$Y_{A_{1_S}} + Y_{\dot A_{1_S}}\Lambda$	$-J_1 Y_{B_{1_C}}\overline{B_{1_C}} - J_2 Y_{A_{1_C}}\overline{A_{1_C}} - J_3 Y_{\delta_{r_C}}\overline{\delta_{r_C}} - J_4 Y_{\theta_C}\overline{\theta_C}$
$Z_\theta + Z_{\dot\theta}\Lambda + Z_{\ddot\theta}\Lambda^2$	$Z_{\dot\phi}\Lambda$	$Z_{\dot\psi}\Lambda$	0	0	$-J_1 Z_{B_{1_C}}\overline{B_{1_C}} - J_2 Z_{A_{1_C}}\overline{A_{1_C}} - J_3 Z_{\delta_{r_C}}\overline{\delta_{r_C}} - J_4 Z_{\theta_C}\overline{\theta_C}$
$\mathcal{L}_\theta + \mathcal{L}_{\dot\theta}\Lambda + \mathcal{L}_{\ddot\theta}\Lambda^2$	$\mathcal{L}_{\dot\phi}\Lambda + \mathcal{L}_{\ddot\phi}\Lambda^2$	$\mathcal{L}_{\dot\psi}\Lambda + \mathcal{L}_{\ddot\psi}\Lambda^2$	0	0	$-J_1 \mathcal{L}_{B_{1_C}}\overline{B_{1_C}} - J_2 \mathcal{L}_{A_{1_C}}\overline{A_{1_C}} - J_3 \mathcal{L}_{\delta_{r_C}}\overline{\delta_{r_C}} - J_4 \mathcal{L}_{\theta_C}\overline{\theta_C}$
$M_{\dot\theta}\Lambda + M_{\ddot\theta}\Lambda^2$	$M_{\dot\phi}\Lambda + M_{\ddot\phi}\Lambda^2$	$M_{\dot\psi}\Lambda + M_{\ddot\psi}\Lambda^2$	0	0	$-J_1 M_{B_{1_C}}\overline{B_{1_C}} - J_2 M_{A_{1_C}}\overline{A_{1_C}} - J_3 M_{\delta_{r_C}}\overline{\delta_{r_C}} - J_4 M_{\theta_C}\overline{\theta_C}$
$N_{\dot\theta}\Lambda + N_{\ddot\theta}\Lambda^2$	$N_{\dot\phi}\Lambda + N_{\ddot\phi}\Lambda^2$	$N_{\dot\psi}\Lambda + N_{\ddot\psi}\Lambda^2$	$N_{B_{1_S}} + N_{\dot B_{1_S}}\Lambda$	0	$-J_1 N_{B_{1_C}}\overline{B_{1_C}} - J_2 N_{A_{1_C}}\overline{A_{1_C}} - J_3 N_{\delta_{r_C}}\overline{\delta_{r_C}} - J_4 N_{\theta_C}\overline{\theta_C}$
$-(k_1\Lambda + k_2\Lambda^2)$	0	0	$\Lambda + D_1 + D_2\overline{B_{1_S}}$	0	0
0	$k_5\Lambda + k_6\Lambda^2$	0	0	$\Lambda + D_1 + D_2\overline{A_{1_S}}$	0

The numerator determinants $f_1(\Lambda)$, $f_2(\Lambda)$, $f_3(\Lambda)$, etc. required for response calculations of the perturbation variables $\bar{u}, \bar{v}, \bar{w}$, etc., are obtained by replacing the coefficients of columns 2 and 3 by the set of control coefficients $K_1, K_2, \ldots\ldots\ldots K_8$, respectively.

The stability determinant $F(\Lambda)$ is given by:

$$F(\Lambda) = \begin{vmatrix} a_{11} & a_{12} & a_{13} & \cdot & \cdot & \cdot & a_{18} \\ a_{21} & a_{22} & a_{23} & \cdot & \cdot & \cdot & a_{28} \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ a_{81} & a_{82} & a_{83} & \cdot & \cdot & \cdot & a_{88} \end{vmatrix}$$

Expanding the stability determinant $F(\Lambda) = 0$ yields the generalized characteristic equation as follows:

$$A\Lambda^{n} + B\Lambda^{n-1} + C\Lambda^{n-2} + \cdot \quad \cdot \quad \cdot + E = 0$$

where n is an integer denoting the highest order of the stability characteristic equation and A, B, C, ... E are coefficients of the characteristic equation in terms of total stability derivatives.

8.2 UNCOUPLED LONGITUDINAL MODE (Three Degrees of Freedom)

Considering decoupled longitudinal motion as affected by changes in the stability variables u, w, and θ, the corresponding stability determinant $F(\Lambda)$ is obtained by deleting the remaining stability variables v, ϕ, ψ, etc., in the equations for X, Z, and M, thus:

$$F(\Lambda) = \begin{vmatrix} a_{11} & a_{13} & a_{14} \\ a_{31} & a_{33} & a_{34} \\ a_{51} & a_{53} & a_{54} \end{vmatrix} = 0$$

Expanding the stability determinant $F(\Lambda)$ yields

$$F(\Lambda) = a_{11}(a_{33}a_{54} - a_{53}a_{34}) - a_{13}(a_{31}a_{54} - a_{51}a_{34})$$
$$+ a_{14}(a_{31}a_{53} - a_{51}a_{33})$$

Substituting the values for the coefficients from Table I of Section 8.1 yields

$$F(\Lambda) = (X_u + X_{\dot{u}}\Lambda)\Big[(Z_w + Z_{\dot{w}}\Lambda)(M_{\dot{\theta}}\Lambda + M_{\ddot{\theta}}\Lambda^2)$$
$$-(M_w + M_{\dot{w}}\Lambda)(Z_\theta + Z_{\dot{\theta}}\Lambda + Z_{\ddot{\theta}}\Lambda^2)\Big]$$
$$-(X_w + X_{\dot{w}}\Lambda)\Big[Z_u(M_{\dot{\theta}}\Lambda + M_{\ddot{\theta}}\Lambda^2) - M_u(Z_\theta + Z_{\dot{\theta}}\Lambda + Z_{\ddot{\theta}}\Lambda^2)\Big]$$
$$+(X_\theta + X_{\dot{\theta}}\Lambda + X_{\ddot{\theta}}\Lambda^2)\Big[Z_u(M_w + M_{\dot{w}}\Lambda) - M_u(Z_w + Z_{\dot{w}}\Lambda)\Big]$$

Thus:

$$F(\Lambda) = A\Lambda^4 + B\Lambda^3 + C\Lambda^2 + D\Lambda + E = 0$$

where

$$A = G_1 X_{\dot{u}}$$

$$B = G_1 X_u + G_2 X_{\dot{u}} + G_3 X_{\dot{w}} + G_4 X_{\ddot{\theta}}$$

$$C = G_2 X_u + G_3 X_w + G_4 X_{\dot{\theta}} + G_5 X_{\dot{u}} + G_6 X_{\dot{w}} + G_7 X_{\ddot{\theta}}$$

$$D = G_4 X_\theta + G_5 X_u + G_6 X_w + G_7 X_{\dot{\theta}} + G_8 Z_\theta$$

$$E = G_7 X_\theta + G_9 Z_\theta$$

and

$$G_1 = Z_{\dot{w}} M_{\ddot{\theta}} - M_{\dot{w}} Z_{\ddot{\theta}}$$

$$G_2 = Z_w M_{\ddot{\theta}} + Z_{\dot{w}} M_{\dot{\theta}} - M_w Z_{\ddot{\theta}} - M_{\dot{w}} Z_{\dot{\theta}}$$

$$G_3 = Z_{\ddot{\theta}} M_u - Z_u M_{\ddot{\theta}}$$

$$G_4 = Z_u M_{\dot{w}} - M_u Z_{\dot{w}}$$

$$G_5 = Z_w M_{\dot{\theta}} - M_w Z_\theta \cdot Z_\theta M_{\dot{w}}$$

$$G_6 = M_u Z_{\dot\theta} - Z_u M_{\dot\theta}$$

$$G_7 = Z_u M_w - M_u Z_w$$

$$G_8 = M_u X_{\dot w} - M_w X_{\dot u}$$

$$G_9 = X_w M_u - X_u M_w$$

In order to determine the aircraft response, say, in pitch $[\theta = f_4(\Lambda)/F(\Lambda)]$ due to a step input of the longitudinal control $(\overline{B_{1c}})$, it is necessary to evaluate the numerator determinant $f_4(\Lambda)$. The determinant $f_4(\Lambda)$ is formed by replacing the coefficients of (θ) (namely, a_{14}, a_{34}, and a_{54}) of the stability determinant $F(\Lambda)$ given above by the control input functions K_1, K_3, and K_5.

The function $f_4(\Lambda)$ can be obtained as follows:

$$f_4(\Lambda) = \begin{vmatrix} a_{11} & a_{13} & K_1 \\ a_{31} & a_{33} & K_3 \\ a_{51} & a_{53} & K_5 \end{vmatrix}$$

$$= a_{11}(K_5 a_{33} - K_3 a_{53}) - a_{13}(K_5 a_{31} - K_3 a_{51}) + K_1(a_{31} a_{53} - a_{51} a_{33})$$

Since, in this case, the uncoupled longitudinal motion of 3 degrees of freedom with no stability augmentation system is considered, the stability authority ratios J_1, J_2, J_3 ...., etc., are taken as unity, and all control inputs other than $(\overline{B_{1_C}})$ are taken as zero.

Thus, when the values for the coefficients a_{13}, a_{15} ......, etc., and the appropriate control inputs K_1, K_3 and K_5 from Table I of Section 8.1 are substituted, the function $f_4(\Lambda)$ becomes

$$f_4(\Lambda) = (X_u + X_{\dot{u}}\Lambda)\left[-M_{B_{1_C}}\overline{B_{1_C}}(Z_w + Z_{\dot{w}}\Lambda) + Z_{B_{1_C}}\overline{B_{1_C}}(M_w + M_{\dot{w}}\Lambda)\right]$$

$$-(X_w + X_{\dot{w}}\Lambda)\left[-M_{B_{1_C}}\overline{B_{1_C}}(Z_u) + Z_{B_{1_C}}\overline{B_{1_C}}(M_u)\right]$$

$$-X_{B_{1_C}}\overline{B_{1_C}}\left[Z_u(M_w + M_{\dot{w}}\Lambda) - M_u(Z_w + Z_{\dot{w}}\Lambda)\right]$$

Thus:

$$f_4(\Lambda) = \overline{B_{1_C}}(A\Lambda^2 + B\Lambda + C)$$

where

$$A = X_{\dot{u}}(Z_{B_{1_C}} M_{\dot{w}} - M_{B_{1_C}} Z_{\dot{w}})$$

$$B = X_u(Z_{B_{1_C}} M_{\dot{w}} - M_{B_{1_C}} Z_{\dot{w}}) + X_{\dot{u}}(Z_{B_{1_C}} M_w - M_{B_{1_C}} Z_w)$$

$$+ X_{\dot{w}}(M_{B_{1_C}} Z_u - Z_{B_{1_C}} M_u) + X_{B_{1_C}}(M_u Z_{\dot{w}} - Z_u M_{\dot{w}})$$

$$C = X_u(Z_{B_{1_C}} M_w - M_{B_{1_C}} Z_w) + X_w(M_{B_{1_C}} Z_u - Z_{B_{1_C}} M_u)$$

$$+ X_{B_{1_C}}(M_u Z_w - M_w Z_u)$$

8.3 UNCOUPLED LATERAL MODE (Three Degrees of Freedom)

When the longitudinal stability variables u, v, and θ and longitudinal equations of motion X, Z, M are deleted, the stability determinant for the 3 lateral degrees of freedom becomes

$$F(\Lambda) = \begin{vmatrix} a_{22} & a_{25} & a_{26} \\ a_{42} & a_{45} & a_{46} \\ a_{62} & a_{65} & a_{66} \end{vmatrix} = 0$$

Expanding the above determinant yields

$$F(\Lambda) = a_{22}(a_{45}a_{66} - a_{65}a_{46}) - a_{25}(a_{42}a_{66} - a_{62}a_{46})$$

$$+ a_{26}(a_{42}a_{65} - a_{62}a_{45})$$

Substituting the values for the coefficients a_{mn} from Table I of Section 8.1 yields

$$F(\Lambda) = (Y_u + Y_{\dot u}\Lambda)\Big[(L_{\dot\phi}\Lambda + L_{\ddot\phi}\Lambda^2)(N_{\dot\psi}\Lambda + N_{\ddot\psi}\Lambda^2)$$

$$-(N_{\dot\phi}\Lambda + N_{\ddot\phi}\Lambda^2)(L_{\dot\psi}\Lambda + L_{\ddot\psi}\Lambda^2)\Big]$$

$$-(Y_\phi + Y_{\dot\phi}\Lambda)\Big[L_v(N_{\dot\psi}\Lambda + N_{\ddot\psi}\Lambda^2) - N_v(L_{\dot\psi}\Lambda + L_{\ddot\psi}\Lambda^2)\Big]$$

$$-(Y_\psi + Y_{\dot\psi}\Lambda)\Big[L_v(N_{\dot\phi}\Lambda + N_{\ddot\phi}\Lambda^2) - N_v(L_{\dot\phi}\Lambda + L_{\ddot\phi}\Lambda^2)\Big]$$

Thus:

$$F(\Lambda) = \Lambda\left[A\Lambda^4 + B\Lambda^3 + C\Lambda^2 + D\Lambda + E\right] = 0$$

where

$$A = H_1 Y_{\dot{v}}$$

$$B = H_1 Y_v + H_2 Y_{\dot{v}}$$

$$C = H_2 Y_v + H_3 Y_{\dot{v}} + H_4 Y_{\dot{\phi}} + H_5 Y_{\dot{\psi}}$$

$$D = H_3 Y_v + H_4 Y_{\phi} + H_5 Y_{\psi} + H_6 Y_{\dot{\phi}} + H_7 Y_{\dot{\psi}}$$

$$E = H_6 Y_{\phi} + H_7 Y_{\psi}$$

and

$$H_1 = L_{\ddot{\phi}} N_{\ddot{\psi}} - N_{\ddot{\phi}} L_{\ddot{\psi}}$$

$$H_2 = L_{\dot{\phi}} N_{\ddot{\psi}} + L_{\ddot{\phi}} N_{\dot{\psi}} - N_{\dot{\phi}} L_{\ddot{\psi}} - N_{\ddot{\phi}} L_{\dot{\psi}}$$

$$H_3 = L_{\dot{\phi}} N_{\dot{\psi}} - N_{\dot{\phi}} L_{\dot{\psi}}$$

$$H_4 = N_v L_{\ddot{\psi}} - L_v N_{\ddot{\psi}}$$

$$H_5 = L_v N_{\ddot{\phi}} - N_v L_{\ddot{\phi}}$$

$$H_6 = N_v L_{\dot{\psi}} - L_v N_{\dot{\psi}}$$

$$H_7 = L_v N_{\dot{\phi}} - N_v L_{\dot{\phi}}$$

8.4 CRITERIA FOR STABILITY

The requirement for positive stability is that there be no positive real root or positive real part of the complex roots of the characteristic equation.

If there are to be no unstable modes, certain conditions pertaining to the coefficients of the characteristic equation must be met. These conditions can be expressed in terms of Routh stability criteria which involve sign tests of the coefficients of the characteristic equation and the magnitude and sign of the Routh discriminant R^*. A more detailed information on the subject can be obtained from Reference 1.

The sections below present a summary of the Routh stability criteria for various types of the characteristic equations commonly encountered in stability work.

8.4.1 Routh Criteria for a Cubic

Let the cubic equation be

$$A\Lambda^3 + B\Lambda^2 + C\Lambda + D = 0 \qquad (A > 0)$$

The necessary and sufficient conditions for stability are

(a) The coefficients A, B, C, $D > 0$
(b) $R^* = BC - AD > 0$

8.4.2 Routh Criteria for a Quartic

Let the quartic equation be

$$A\Lambda^4 + B\Lambda^3 + C\Lambda^2 + D\Lambda + E = 0$$

The necessary and sufficient conditions for stability are

(a) The coefficients $A, B, C, D, E > 0$
(b) $R^* = D(BC-AD)-B^2E > 0$

8.4.3 Routh Criteria for a Quintic

Let the quintic equation be

$$A\Lambda^5 + B\Lambda^4 + C\Lambda^3 + D\Lambda^2 + E\Lambda + F = 0$$

The necessary and sufficient conditions for stability are

(a) The coefficients $A, B, C, D, E, F > 0$
(b) $BC-AD > 0$
(c) $R^* = D(BC-AD)(BE-AF)-B(BF-AF)^2-F(BC-AD)^2 > 0$

REFERENCE

1. Routh, E. J., Dynamics of a System of Rigid Bodies, 6th Edition, MacMillan and Company, London, England, 1905.

8.5 SOLUTION OF THE CHARACTERISTIC EQUATION

There are many methods in the literature for obtaining the roots of the characteristic equation. The method used will depend on the order of the characteristic equation and particularly on whether the roots are to be extracted by hand or by machine.

Reference 1, pages 2.1.1-167 to 2.1.1-190, gives a fairly detailed review of the most commonly used methods for extracting roots of the characterisitc equation ranging from 3rd to 6th order equations.

Since quartic equations occur most frequently in aircraft stability work, a method is herein given for solution of stability quartics. The method is known as an "Analytical Solution of Quartics" and is based on the Ferrari reducing cubic method. Some of the advantages of this analytical method are that it is independent of the relative magnitude of the coefficients, does not require initial knowledge of the order of magnitudes of the roots, and is particularly useful if no real roots exist. This method is equally as well applicable for hand calculation as it is for machine computation.

The calculation procedure of this method is as follows:

(a) Determine the coefficients B, C, D and E of a given quatric as follows:

$$\Lambda^4 + B\Lambda^3 + C\Lambda^2 + D\Lambda + E = 0$$

(b) Calculate

$$S^* = BD + C^2 - 4E$$

$$R^* = BCD - EB^2 - D^2$$

(Note that R^* is the usual Routh discriminant discussed in Section 8.4)

(c) Compute

$$h_1 = \frac{1}{3}(3S^* - 4C^2)$$

$$h_2 = \frac{1}{27}(18CS^* - 16C^3 - 27R^*)$$

(d) Evaluate discriminant (Δ)

$$\Delta = \frac{h_2^{\,2}}{4} + \frac{h_1^{\,3}}{27}$$

(e) Determine (Π_n) as follows:

(i) If $\Delta > 0$, then

$$\Pi_n = \sqrt[3]{-\frac{h_2}{2} + \sqrt{\Delta}} + \sqrt[3]{-\frac{h_2}{2} - \sqrt{\Delta}}$$

(ii) If $\Delta = 0$, then

$$\Pi_1 = 2\sqrt[3]{-\frac{h_2}{2}}$$

$$\Pi_2 = \Pi_3 = -\sqrt[3]{-\frac{h_2}{2}}$$

(iii) If $\Delta < 0$, then calculate

$$\cos\Phi = -\frac{h_2}{2} \Big/ \sqrt{-\frac{h_1^3}{27}}$$

and obtain

$$\Pi_1 = 2\left[\sqrt{-\frac{h_1}{3}} \cos\left(\frac{\Phi}{3}\right)\right]$$

$$\Pi_2 = 2\left[\sqrt{-\frac{h_1}{3}} \cos\left(\frac{\Phi}{3} + 120°\right)\right]$$

$$\Pi_3 = 2\left[\sqrt{-\frac{h_1}{3}} \cos\left(\frac{\Phi}{3} + 240°\right)\right]$$

(f) Select the algebraically smallest value of (Π_n) using step (i), (ii), or (iii), whichever applies, and compute

$$\zeta\eta \equiv \Pi_n + \frac{2C}{3} \leq \frac{B^2}{4}$$

(g) Calculate

$$\varsigma = \frac{C - \zeta\eta}{2} + \sqrt{\left(\frac{C - \zeta\eta}{2}\right)^2 - E}$$

$$\nu = \frac{E}{\varsigma}$$

$$\eta = \frac{D - B\varsigma}{\nu - \varsigma}$$

$$\zeta - B - \eta$$

(h) Finally, determine the four roots of the quartic thus:

$$\Lambda_{1,2} = -\frac{\zeta}{2} \pm \sqrt{\left(\frac{\zeta}{2}\right)^2 - \nu}$$

$$\Lambda_{3,4} = -\frac{\eta}{2} \pm \sqrt{\left(\frac{\eta}{2}\right)^2 - \varsigma}$$

REFERENCE

1. Stability and Control Handbook for Helicopters, TRECOM Report 60-43, U. S. Army Transportation Research Command (presently, U. S. Army Aviation Materiel Laboratories), Fort Eustis, Virginia, August 1960.

8.6 ROOTS OF THE CHARACTERISTIC EQUATION

The roots of the characteristic equation can be

(a) Real
(b) Complex
(c) Combination of real and complex

The real roots correspond to a periodic motion, converging in amplitude as time passes if negative, and diverging in amplitude if positive.

The complex roots ($\Lambda_m = a_m \pm b_m i$) always occur in pairs, and each pair corresponds to an oscillatory mode

$$Ae^{a_m t} \sin(b_m t + \Phi)$$

where A is the amplitude of the oscillation and Φ is the phase angle.

The real part a_m of the complex pair of roots determines the converging or diverging behavior of the mode.

If $a_m > 0$, the amplitude of the mode will increase with time (t), resulting in unstable, divergent oscillation.
If $a_m = 0$, the amplitude remains unchanged (neutral stability).
If $a_m < 0$, the amplitude will decrease with time, (t) resulting in a stable or damped oscillation.

The complex part b_m describes the frequency of the mode, in radians/second.

If the real root or real part of the complex root is negative, the time constant τ of the mode is defined as

$$\tau = \begin{cases} \dfrac{1}{-\Lambda} \text{ for real } \Lambda \\ \\ \dfrac{1}{-a_m} \text{ for complex } \Lambda \end{cases}$$

The time constant τ corresponds to the time required for the motion to reach 36.8% of its original value. If the real root, or real part of the complex root, is positive, it is more convenient to express the characteristics of the mode in terms of the time required to double its initial amplitude $\tau_{2/1}$ where

$$\tau_{2/1} = \begin{cases} \dfrac{0.693}{+\Lambda} \text{ for real } \Lambda \\ \\ \dfrac{0.693}{a_m} \text{ for complex } \Lambda \end{cases}$$

The converging characteristics of stable modes is sometimes also expressed in terms of the time required to reduce to half its initial amplitude $\tau_{1/2}$ where

$$\tau_{1/2} = \begin{cases} \dfrac{0.693}{-\Lambda} \text{ for real } \Lambda \\ \\ \dfrac{0.693}{-a_m} \text{ for complex } \Lambda \end{cases}$$

The period P of an oscillatory mode is given by

$$P = \frac{2\pi}{b_m} \text{ seconds}$$

SECTION 9. RAPID METHODS FOR ESTIMATING THE LONGITUDINAL STABILITY CHARACTERISTICS OF SINGLE AND TANDEM ROTOR HELICOPTERS

Sections 4 through 8 present complete methods for evaluating the dynamic stability and control response of generalized helicopter configurations including as many as 8 degrees of freedom of coupled motion.

The methods presented therein can be used for "formal solutions". In preliminary design work, there is often a need for quick and approximate estimates of the longitudinal stability characteristics of single and tandem rotor helicopters. Therefore, some simplified stability methods may often be required where time rather than accuracy is of prime consideration.

This section presents such methods, which can be used for rapid estimation of the longitudinal stability characteristics of conventional single and tandem rotor helicopters with only a slight loss in accuracy. The accuracy of the final results using the simplified methods is within 5% of the "formal" solutions contained in the preceding sections.

The applicability of the simplified stability methods presented in this section is limited to low and medium forward speed regimes, i.e., those covering the tip speed ratio range of $0.1 \leq \mu \leq 0.3$.

The simplified stability equations presented below have been verified by numerical calculations for typical single and tandem rotor helicopters. The numerical results are presented in the tables of Sections 9.1 and 9.2, respectively.

9.1 SIMPLIFIED STABILITY METHOD FOR A SINGLE ROTOR HELICOPTER

The following procedure can be used for rapid estimation of the longitudinal stability characteristics of a single rotor helicopter:

9.1.1 Trim Calculation

The required design parameters and the longitudinal trim conditions for a single rotor helicopter can be obtained by using the procedure given in Section 5.1.

The trim procedure as described in Section 5.1 cannot be appreciably simplified to save time and effort in this approach. However, it is apparent that the computation pertaining to lateral trim can be omitted, with the exception of tail rotor thrust T_{TR} . This parameter is required in the evaluation of the pitching moment derivatives.

The downwash interference effects of the front rotor on the fuselage ϵ_{FUS} and the front rotor on the horizontal tailplane ϵ_T are retained; however, the interference angle at the tail rotor ϵ_{TR} due to front rotor downwash can be neglected.

9.1.2 The Total Stability Derivatives

Using the equations of Section 7.1, the total stability derivatives for a single rotor helicopter can be simplified as shown below. These derivatives are expressed in terms of local stability derivatives which can be obtained from Section 7.3.

9.1.2.1 The X-Force Derivatives

(a) $X_u = (X_u)_F + (X_u)_{FUS} + (X_u)_T$

where

$$(X_u)_F = \frac{\partial L_F}{\partial u_F}\alpha - \frac{\partial D_F}{\partial u_F}$$

$$(X_u)_{FUS} = \frac{\partial L_{FUS}}{\partial u_{FUS}}(\alpha - \epsilon_{FUS}) - \frac{\partial D_{FUS}}{\partial u_{FUS}}$$

$$+ \frac{\partial \alpha_{FUS}}{\partial u}\left[\frac{\partial L_{FUS}}{\partial \alpha_{FUS}}(\alpha - \epsilon_{FUS}) - \frac{\partial D_{FUS}}{\partial \alpha_{FUS}}\right]$$

$$(X_u)_T = \frac{\partial L_T}{\partial u_T}(\alpha - \epsilon_T) - \frac{\partial D_T}{\partial u_T} + \frac{\partial \alpha_T}{\partial u}\left[\frac{\partial L_T}{\partial \alpha_T}(\alpha - \epsilon_T) - \frac{\partial D_T}{\partial \alpha_T}\right]$$

(b) $X_{\dot{u}} = -\frac{W}{g}$

(c) $X_w = (X_w)_F + (X_w)_{FUS} + (X_w)_T$

where

$$(X_w)_F = \frac{1}{V_0}\left(\frac{\partial L_F}{\partial \alpha_F}\alpha - \frac{\partial D_F}{\partial \alpha_F} + L_F\right)$$

$$(X_w)_{FUS} = \frac{1}{V_0}\left(\frac{\partial \alpha_{FUS}}{\partial \alpha}\right)\left[\left(\frac{\partial L_{FUS}}{\partial \alpha_{FUS}} + D_{FUS}\right)(\alpha - \epsilon_{FUS}) - \frac{\partial D_{FUS}}{\partial \alpha_{FUS}} + L_{FUS}\right]$$

$$(X_w)_T = \frac{1}{V_0}\left(\frac{\partial \alpha_T}{\partial \alpha}\right)\left[\frac{\partial L_T}{\partial \alpha_T}(\alpha - \epsilon_T) - \frac{\partial D_T}{\partial \alpha_T} + L_T\right]$$

(d) $X_\theta = -W$

(e) $X_{\dot{\theta}} = -L_F\left(\frac{\partial \alpha_{IF}}{\partial q}\right) - \frac{W}{g}V_0\alpha$

9.1.2.2 The Z-Force Derivatives

(a) $Z_u = (Z_u)_F + (Z_u)_{FUS} + (Z_u)_T$

where

$$(Z_u)_F = -\left(\frac{\partial L_F}{\partial u_F} + \frac{\partial D_F}{\partial u_F}\alpha\right)$$

$$(Z_u)_{FUS} = -\left[\frac{\partial L_{FUS}}{\partial u_{FUS}} + \frac{\partial D_{FUS}}{\partial u_{FUS}}(\alpha - \epsilon_{FUS})\right]$$

$$- \frac{\partial \alpha_{FUS}}{\partial u}\left[\frac{\partial L_{FUS}}{\partial \alpha_{FUS}} + \frac{\partial D_{FUS}}{\partial \alpha_{FUS}}(\alpha - \epsilon_{FUS}) + D_{FUS}\right]$$

$$(Z_u)_T = -\left[\frac{\partial L_T}{\partial u_T} + \frac{\partial L_T}{\partial \alpha_T}\left(\frac{\partial \alpha_T}{\partial u}\right)\right]$$

(b) $Z_w = (Z_w)_F + (Z_w)_{FUS} + (Z_w)_T$

where

$$(Z_w)_F = -\frac{1}{V_0}\left(\frac{\partial L_F}{\partial \alpha_F}\right)$$

$$(Z_w)_{FUS} = -\frac{1}{V_0}\left(\frac{\partial L_{FUS}}{\partial \alpha_{FUS}}\right)\left(\frac{\partial \alpha_{FUS}}{\partial \alpha}\right)$$

$$(Z_w)_T = -\frac{1}{V_0}\left(\frac{\partial L_T}{\partial \alpha_T}\right)\left(\frac{\partial \alpha_T}{\partial \alpha}\right)$$

(c) $Z_{\dot{w}} = -\dfrac{W}{g}$

(d) $Z_\theta = -W\theta$

(e) $Z_{\dot{\theta}} = \frac{W}{g} V_0$

9.1.2.3 The Pitching Moment (M) Derivatives

(a) $M_u = (X_u)_F \ell_{Z_F} - (Z_u)_F \ell_{X_F} - (Z_u)_T \ell_{X_T}$

$$+ (X_u)_{TR} \ell_{Z_{TR}} - (Z_u)_{TR} \ell_{X_{TR}} + \frac{\partial M_{FUS}}{\partial u} + \frac{\partial M_{HUB_F}}{\partial u}$$

where

$$(X_u)_{TR} = -\left[\frac{\partial D_{TR}}{\partial u_{TR}} + \frac{\partial D_{TR}}{\partial \alpha_{TR}}\left(\frac{\partial \alpha_{TR}}{\partial u}\right)\right]$$

$$(Z_u)_{TR} = -\left[\frac{\partial D_{TR}}{\partial u_{TR}} - \frac{\partial D_{TR}}{\partial \alpha_{TR}}\left(\frac{\partial \alpha_{TR}}{\partial u}\right)\right](\alpha - \epsilon_{TR})$$

(b) $M_w = (X_w)_F \ell_{Z_F} - (Z_w)_F \ell_{X_F} - (Z_w)_T \ell_{X_T}$

$$-(Z_w)_{TR} \ell_{X_{TR}} + \frac{1}{V_0}\left(\frac{\partial M_{FUS}}{\partial \alpha} + \frac{\partial M_{HUB_F}}{\partial \alpha}\right)$$

where

$$(Z_w)_{TR} = -\frac{1}{V_0} D_{TR}$$

(c) $M_{\dot{\theta}} = Z_{w_T} \ell_{X_T}^{\,2} - \ell_{Z_F}\left(\frac{\partial o_{1F}}{\partial q}\right) L_F + \frac{\partial M_{HUB_F}}{\partial q}\Big)$

where

$$Z_{w_T} = -\frac{1}{V_0}\left(\frac{\partial L_T}{\partial \alpha_T}\right)$$

(d) $M_{\ddot{\theta}} = -I_{yy}$

9.1.3 Coefficient of the Characteristic Equation

Using the values for the total derivatives presented above, the coefficients of the characteristic equations can be calculated from Section 8.2.

9.1.4 Roots of the Characteristic Equation

The stability roots of the characteristic equation can be extracted by using the numerical procedures given in Section 8.5.

9.1.5 Numerical Comparison of the "Rapid Stability Method" With the "Formal Solution"

Tables I through III show a comparison of the results calculated using the "rapid stability method", as described above, versus the results obtained by the "formal method" presented in Sections 7 and 8.

The sample helicopter under consideration is a single rotor helicopter having the same design parameters as used in the sample trim calculation presented in Section 10.1.

In order to examine most critical variations of the pertinent stability parameters, a range of tip speed ratios of $0.1 \leq \mu \leq 0.3$ was selected for this comparison.

The tables presented in this section show that the results obtained using the "rapid stability method" are well within the tolerable limits of those obtained by the "formal method".

TABLE I

NUMERICAL COMPARISON OF THE SIMPLIFIED VERSUS FORMAL STABILITY METHOD FOR SINGLE ROTOR HELICOPTERS

$\mu = 0.1$

Stability Variables	Formal Method	Simplified Method
X_u	-5.99	-5.56
$X_{\dot{u}}$	-236.00	-236.00
X_w	7.86	7.98
$X_{\dot{w}}$	0	0
X_θ	-7600.00	-7600.00
$X_{\dot{\theta}}$	-904.28	-899.38
$X_{\ddot{\theta}}$	0	0
Z_u	-21.76	-21.82
Z_w	-149.47	-149.30
$Z_{\dot{w}}$	-236.00	-236.00
Z_θ	-660.00	-660.00
$Z_{\dot{\theta}}$	17640.00	17465.84
$Z_{\ddot{\theta}}$	0	0
M_u	44.07	44.27
M_w	43.22	43.17
$M_{\dot{w}}$	0	0
$M_{\dot{\theta}}$	-4992.00	-4749.29
$M_{\ddot{\theta}}$	-9100.00	-9100.00
A	1	1
B	1.20561	1.17809
C	0.04321	0.02794
D	0.16875	0.16895
E	0.08538	0.08576
Λ_1	-0.33740	-0.33639
Λ_2	-1.23600	-1.22171
$\Lambda_{3,4}$	0.18390 $\pm 0.41342i$	0.19001 $\pm 0.41541i$

TABLE II

NUMERICAL COMPARISON OF THE SIMPLIFIED VERSUS FORMAL STABILITY METHOD FOR SINGLE ROTOR HELICOPTERS

$\mu = 0.2$

Stability Variables	Formal Method	Simplified Method
X_u	-10.68	-10.59
$X_{\dot{u}}$	-236.00	-236.00
X_w	14.67	14.90
$X_{\dot{w}}$	0	0
X_θ	-7600.00	-7600.00
$X_{\dot{\theta}}$	-2210.17	-2233.02
$X_{\ddot{\theta}}$	0	0
Z_u	-0.15	-0.14
Z_w	-188.50	-188.92
$Z_{\dot{w}}$	-236.00	-236.00
Z_θ	-623.00	-623.00
$Z_{\dot{\theta}}$	34896.00	34931.68
$Z_{\ddot{\theta}}$	0	0
M_u	50.61	50.66
M_w	6.39	7.64
$M_{\dot{w}}$	0	0
$M_{\dot{\theta}}$	-6929.90	-6718.64
$M_{\ddot{\theta}}$	-9100.00	-9100.00
A	1	1
B	1.60551	1.58369
C	0.62716	0.58852
D	0.19429	0.19261
E	0.14404	0.14452
Λ_1	-0.71582	-0.68464
Λ_2	-1.07335	-1.09856
$\Lambda_{3,4}$	0.09183 $\pm$0.42312i	0.09975 $\pm$0.42686i

TABLE III

NUMERICAL COMPARISON OF THE SIMPLIFIED VERSUS FORMAL STABILITY METHOD FOR SINGLE ROTOR HELICOPTERS

$\mu = 0.3$

Stability Variables	Formal Method	Simplified Method
X_u	-6.51	-6.30
$X_{\dot{u}}$	-164.60	-164.60
X_w	-2.59	-2.67
$X_{\dot{w}}$	0	0
X_θ	-5300.00	-5300.00
$X_{\dot{\theta}}$	1906.86	1911.11
$X_{\ddot{\theta}}$	0	0
Z_u	18.53	18.49
Z_w	-179.47	-179.50
$Z_{\dot{w}}$	-164.60	-164.60
Z_θ	231.24	231.24
$Z_{\dot{\theta}}$	33709.35	34071.43
$Z_{\ddot{\theta}}$	0	0
M_u	13.67	13.57
M_w	-150.88	-150.54
$M_{\dot{w}}$	0	0
$M_{\dot{\theta}}$	-7098.36	-6944.56
$M_{\ddot{\theta}}$	-8400.00	-8400.00
A	1	1
B	1.97472	1.95553
C	4.65925	4.66766
D	0.26817	0.26354
E	-0.00761	-0.00710
Λ_1	0.02080	0.01987
Λ_2	-0.08049	-0.07833
$\Lambda_{3,4}$	-0.95751 $\pm$1.90515i	-0.94853 $\pm$1.91275i

9.2 SIMPLIFIED STABILITY METHOD FOR A TANDEM ROTOR HELICOPTER

The following procedure can be used for rapid estimation of the longitudinal stability characteristics of a tandem rotor configuration.

9.2.1 Trim Calculation

The required design parameters and the longitudinal trim conditions for a tandem rotor helicopter can be obtained from Section 5.2.

The trim procedure as described in that section can be simplified by omitting all the computation pertaining to the lateral trim parameters.

9.2.2 Total Stability Derivatives

Using the equations of Section 7.1, the total stability derivatives for a tandem rotor helicopter can be simplified as shown below. These derivatives are expressed in terms of the local stability derivatives which can be obtained from Section 7.3.

9.2.2.1 The X-Force Derivatives

(a) $X_u = (X_u)_F + (X_u)_R + (X_u)_{FUS}$

where

$$(X_u)_F = \frac{\partial L_F}{\partial u_F}\alpha - \frac{\partial D_F}{\partial u_F}$$

$$(X_u)_R = \frac{\partial L_R}{\partial u_R}(\alpha - \epsilon_R) - \frac{\partial D_R}{\partial u_R} + \frac{\partial \alpha_R}{\partial u}\left[\frac{\partial L_R}{\partial \alpha_R}(\alpha - \epsilon_R) - \frac{\partial D_R}{\partial \alpha_R} + L_R\right]$$

$$(X_u)_{FUS} = -\left[\frac{\partial D_{FUS}}{\partial u_{FUS}} + \frac{\partial D_{FUS}}{\partial \alpha_{FUS}}\left(\frac{\partial \alpha_{FUS}}{\partial u}\right)\right]$$

(b) $X_{\dot{u}} = -\dfrac{W}{g}$

(c) $X_{w} = (X_{w})_F + (X_{w})_R + (X_{w})_{FUS}$

where

$$(X_{w})_F = \frac{1}{V_0}\left(\frac{\partial L_F}{\partial \alpha_F}\alpha - \frac{\partial D_F}{\partial \alpha_F} + L_F\right)$$

$$(X_{w})_R = \frac{1}{V_0}\left(\frac{\partial \alpha_R}{\partial \alpha}\right)\left[\frac{\partial L_R}{\partial \alpha_R}(\alpha - \epsilon_R) - \frac{\partial D_R}{\partial \alpha_R} + L_R\right]$$

$$(X_{w})_{FUS} = \frac{1}{V_0}\left(\frac{\partial \alpha_{FUS}}{\partial \alpha}\right)\left[\left(\frac{\partial L_{FUS}}{\partial \alpha_{FUS}} + D_{FUS}\right)(\alpha - \epsilon_{FUS}) + L_{FUS}\right]$$

(d) $X_{\theta} = -W$

(e) $$X_{\dot{\theta}} = X_{u_F}\ell_{z_F} - X_{w_F}\ell_{x_F} - \left(\frac{\partial a_{1F}}{\partial q}\right)L_F$$

$$+ X_{u_R}\ell_{z_R} - X_{w_R}\ell_{x_R} - \left(\frac{\partial a_{1R}}{\partial q}\right)L_R$$

where

$$X_{u_F} = (X_u)_F$$

$$X_{w_F} = (X_{w})_F$$

$$X_{u_R} = \frac{\partial L_R}{\partial u_R}(\alpha - \epsilon_R) - \frac{\partial D_R}{\partial u_R}$$

$$X_{w_R} = \frac{1}{V_0}\left[\frac{\partial L_R}{\partial \alpha_R}(\alpha - \epsilon_R) - \frac{\partial D_R}{\partial \alpha_R} + L_R\right]$$

9.2.2.2 The Z-Force Derivatives

(a) $Z_u = (Z_u)_F + (Z_u)_R + (Z_u)_{FUS}$

where

$$(Z_u)_F = -\frac{\partial L_F}{\partial u_F}$$

$$(Z_u)_R = -\left[\frac{\partial L_R}{\partial u_R} + \frac{\partial L_R}{\partial \alpha_R}\left(\frac{\partial \alpha_R}{\partial u}\right)\right]$$

$$(Z_u)_{FUS} = -\left[\frac{\partial L_{FUS}}{\partial u_{FUS}} + \frac{\partial L_{FUS}}{\partial \alpha_{FUS}}\left(\frac{\partial \alpha_{FUS}}{\partial u}\right)\right]$$

(b) $Z_w = (Z_w)_F + (Z_w)_R + (Z_w)_{FUS}$

where

$$(Z_w)_F = -\frac{1}{V_0}\left(\frac{\partial L_F}{\partial \alpha_F}\right)$$

$$(Z_w)_F = -\frac{1}{V_0}\left(\frac{\partial L_R}{\partial \alpha_R}\right)\left(\frac{\partial \alpha_\partial}{\partial \alpha}\right)$$

$$(Z_w)_{FUS} = -\frac{1}{V_0}\left(\frac{\partial L_{FUS}}{\partial \alpha_{FUS}}\right)\left(\frac{\partial \alpha_{FUS}}{\partial \alpha}\right)$$

(c) $Z_{\dot{w}} = -\frac{W}{g}$

(d) $Z_\theta = -W\theta$

(e) $Z_{\dot{\theta}} = \frac{W}{g} V_0$

9.2.2.3 The Pitching Moment (M) Derivatives

(a) $M_u = (X_u)_F \ell_{z_F} - (Z_u)_F \ell_{x_F} + (X_u)_R \ell_{z_R} - (Z_u)_R \ell_{x_R}$

$$+ \frac{\partial M_{FUS}}{\partial u} + \frac{\partial M_{HUB_F}}{\partial u} + \frac{\partial M_{HUB_R}}{\partial u}$$

(b) $M_w = (X_w)_F \ell_{z_F} - (Z_w)_F \ell_{x_F} + (X_w)_R \ell_{z_R} - (Z_w)_R \ell_{x_R}$

$$+ \frac{1}{V} \left(\frac{\partial M_{FUS}}{\partial \alpha} + \frac{\partial M_{HUB_F}}{\partial \alpha} + \frac{\partial M_{HUB_R}}{\partial \alpha} \right)$$

(c) $M_{\dot{\theta}} = Z_{w_F} \ell_{x_F}{}^2 + Z_{w_R} \ell_{x_R}{}^2 - \ell_{z_F} \left(\frac{\partial a_{1_F}}{\partial q} \right) L_F$

$$- \ell_{z_R} \left(\frac{\partial a_{1_R}}{\partial q} \right) L_R + \frac{\partial M_{HUB_F}}{\partial q} + \frac{\partial M_{HUB_R}}{\partial q}$$

where

$Z_{w_F} = (Z_w)_F$

$Z_{w_R} = -\frac{1}{V_0} \left(\frac{\partial L_R}{\partial \alpha_R} \right)$

(d) $M_{\ddot{\theta}} = -I_{yy}$

9.2.3 Coefficient of the Characteristic Equation

In the case of tandem rotor helicopters, the coefficients of the characteristic equation can be further simplified as compared to these presented in Section 8.2, thus:

$$A = X_{\dot{u}} Z_{\dot{w}} M_{\ddot{\theta}}$$

$$B = X_u Z_{\dot{w}} M_{\ddot{\theta}} + X_{\dot{u}} (Z_{\dot{w}} M_{\dot{\theta}} + Z_w M_{\ddot{\theta}})$$

$$C = X_u (Z_w M_{\ddot{\theta}} + Z_{\dot{w}} M_{\dot{\theta}}) + X_{\dot{u}} (Z_w M_{\dot{\theta}} - M_w Z_{\dot{\theta}})$$

$$D = X_u (Z_w M_{\dot{\theta}} - M_w Z_{\dot{\theta}}) + X_{\dot{\theta}} (Z_u M_w - M_u Z_w)$$

$$+ X_w M_u Z_{\dot{\theta}} - X_{\dot{u}} M_w Z_{\theta} - X_{\theta} M_u Z_{\dot{w}}$$

$$E = X_{\theta} (Z_u M_w - M_u Z_w) + Z_{\theta} (X_w M_u - X_u M_w)$$

9.2.4 Roots of the Characteristic Equation

The stability roots of the above characteristic equation can be extracted by using the numerical procedures given in Section 8.5.

9.2.5 Numerical Comparison of the "Rapid Stability Method" With the "Formal Solution"

Tables I through III show a comparison of the results calculated using the "rapid stability method", as described above, versus the results obtained by the "formal method" presented in Sections 7 and 8.

TABLE I

NUMERICAL COMPARISON OF THE SIMPLIFIED VERSUS FORMAL STABILITY METHOD FOR TANDEM ROTOR HELICOPTERS, FORWARD C.G. POSITION (13.3% OF ROTOR RADIUS)

Stability Variables	Formal Method	Simplified Method
X_u	-40.44	-39.98
$X_{\dot{u}}$	-885.92	-885.92
X_w	62.27	62.46
$X_{\dot{w}}$	0	0
X_θ	-28500.00	-28500.00
$X_{\dot{\theta}}$	-2911.71	-2908.07
$X_{\ddot{\theta}}$	0	0
Z_u	36.90	37.75
Z_w	-931.05	-936.61
$Z_{\dot{w}}$	-885.92	-885.92
Z_θ	-876.09	-876.09
$Z_{\dot{\theta}}$	183591.15	182603.60
$Z_{\ddot{\theta}}$	0	0
M_u	-844.97	-865.26
M_w	1201.53	1093.75
$M_{\dot{w}}$	0	0
$M_{\dot{\theta}}$	-355707.54	-366075.77
$M_{\ddot{\theta}}$	-158041.00	-158041.00
A	1	1
B	3.34666	3.37389
C	0.91761	1.17531
D	-0.07531	-0.06211
E	-0.17053	-0.17671
Λ_1	0.30793	0.29411
Λ_2	-3.04303	-2.97905
$\Lambda_{3,4}$	-0.30578 $\pm 0.29747i$	-0.34447 $\pm 0.28814i$

TABLE II

NUMERICAL COMPARISON OF THE SIMPLIFIED VERSUS FORMAL STABILITY METHOD FOR TANDEM ROTOR HELICOPTERS, MID C.G. POSITION (1.4% OF ROTOR RADIUS)

Stability Variables	Formal Method	Simplified Method
X_u	-43.06	-42.47
$X_{\dot{u}}$	-885.92	-885.92
X_w	73.00	73.16
$X_{\dot{w}}$	0	0
X_θ	-28500.00	-28500.00
$X_{\dot{\theta}}$	-2621.65	-2615.32
$X_{\ddot{\theta}}$	0	0
Z_u	50.87	51.64
Z_w	-931.34	-937.38
$Z_{\dot{w}}$	-885.92	-885.92
Z_θ	-756.08	-756.08
$Z_{\dot{\theta}}$	187093.24	182603.60
$Z_{\ddot{\theta}}$	0	0
M_u	-1002.71	-1020.18
M_w	4360.70	4481.17
$M_{\dot{w}}$	0	0
$M_{\dot{\theta}}$	-371145.70	-384178.22
$M_{\ddot{\theta}}$	-158041.00	-158041.00
A	1	1
B	3.44800	3.48843
C	-3.21600	-3.10744
D	-0.25950	-0.24420
E	-0.16290	-0.16533
Λ_1	0.88759	0.83944
Λ_2	-4.22781	-4.21420
$\Lambda_{3,4}$	-0.05389 $\pm$0.20500i	-0.05691 $\pm$0.20859i

TABLE III

NUMERICAL COMPARISON OF THE SIMPLIFIED VERSUS FORMAL STABILITY METHOD FOR TANDEM ROTOR HELICOPTERS, AFT C.G. POSITION (6.9% OF ROTOR RADIUS)

Stability Variables	Formal Method	Simplified Method
X_u	-88.27	-88.14
$X_{\dot{u}}$	-885.92	-885.92
X_w	83.91	84.11
$X_{\dot{w}}$	0	0
X_θ	-28500.00	-28500.00
$X_{\dot{\theta}}$	-2198.48	-2197.29
$X_{\ddot{\theta}}$	0	0
Z_u	53.70	53.67
Z_w	-933.18	-940.29
$Z_{\dot{w}}$	-885.92	-885.92
Z_θ	-706.28	-706.28
$Z_{\dot{\theta}}$	189190.60	182603.60
$Z_{\ddot{\theta}}$	0	0
M_u	-895.19	-923.53
M_w	6596.49	6696.55
$M_{\dot{w}}$	0	0
$M_{\dot{\theta}}$	-398042.50	-414506.48
$M_{\ddot{\theta}}$	-158041.00	-158041.00
A	1	1
B	3.67096	3.68372
C	-5.92492	-5.58366
D	-0.67904	-0.63996
E	-0.10712	-0.11355
Λ_1	1.30714	1.25017
Λ_2	-4.86168	-4.81644
$\Lambda_{3,4}$	-0.05814 $\pm$0.11608i	-0.05873 $\pm$0.12413i

The sample helicopter under consideration is a tandem rotor configuration operating at a tip speed ratio of $\mu = 0.3$ and $M_T = 0.8$. The design parameters selected for the sample helicopter are the same as those used in the sample trim calculation presented in Section 10.2.

In order to examine the most critical variations of the pertinent stability parameters for tandem rotor helicopters, this comparison is based upon a wide range of C.G. travel.

The tables presented in this section show that the results obtained using the "rapid stability method" are well within the tolerable limits of those obtained by the "formal method".

SECTION 10. SAMPLE CALCULATIONS

In order to better illustrate the stability methods presented in the previous sections, sample calculations are herein performed for both single and tandem rotor helicopters.

For each helicopter configuration, the computations are performed according to the sequence of operations outlined in the previous sections. Specifically, the sample calculations are initiated with the aircraft trim computations followed by local and total stability derivatives and are ended with the solution of the stability characteristic equation and analog computations of aircraft response.

10.1 SINGLE ROTOR HELICOPTER

The sample single rotor helicopter considered is a medium utility type aircraft as illustrated in Figure 1 of Section 3.3. It consists of a preconed, two-bladed teetering main rotor and a rigid tail rotor. The horizontal tailplane has no end plates and is of NACA 0015 symmetric series. The fuselage shape, shown here in Figure 1, resembles that of the configuration D of Reference 1. Hence, the model test data corresponding to this configuration is utilized to obtain the required fuselage characteristics for the sample helicopter. These data have been appropriately nondimensionalized and are presented in Figure 2.

The aircraft operating conditions assumed in this sample calculation correspond to a forward speed of $V_o = 207$ ft/sec, a rotor tip speed of $\Omega R = 690$ ft/sec, and a pressure altitude corresponding to sea level standard day.

10.1.1 Trim Calculation for a Single Rotor Helicopter

The sample trim calculation for a single rotor helicopter is performed utilizing the analytical procedure outlined in Subsection 5.1.2 as follows:

(a) Determine the required helicopter design parameter as shown in Table I.

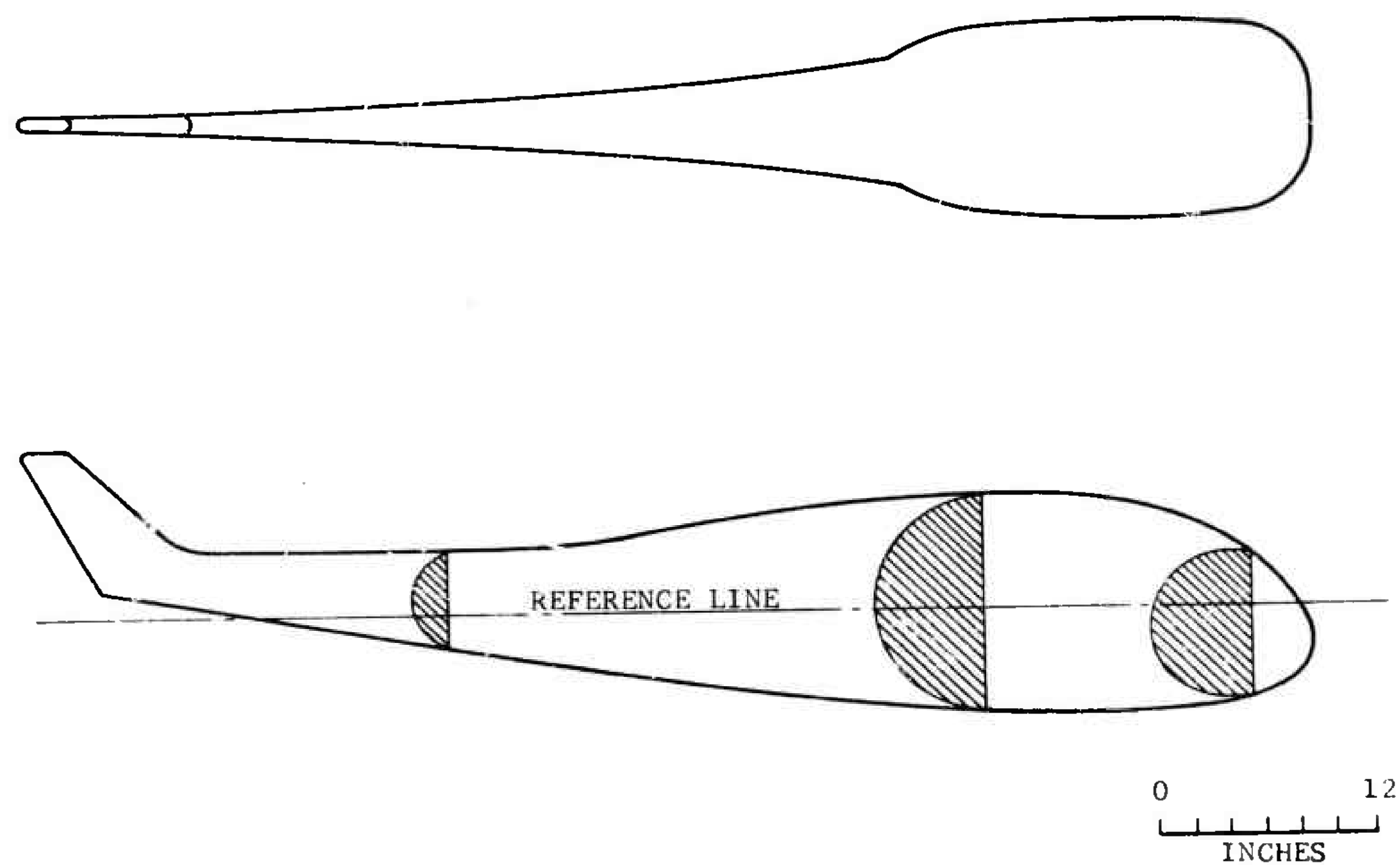


Figure 1. Fuselage Shape of the Sample Single Rotor Helicopter.

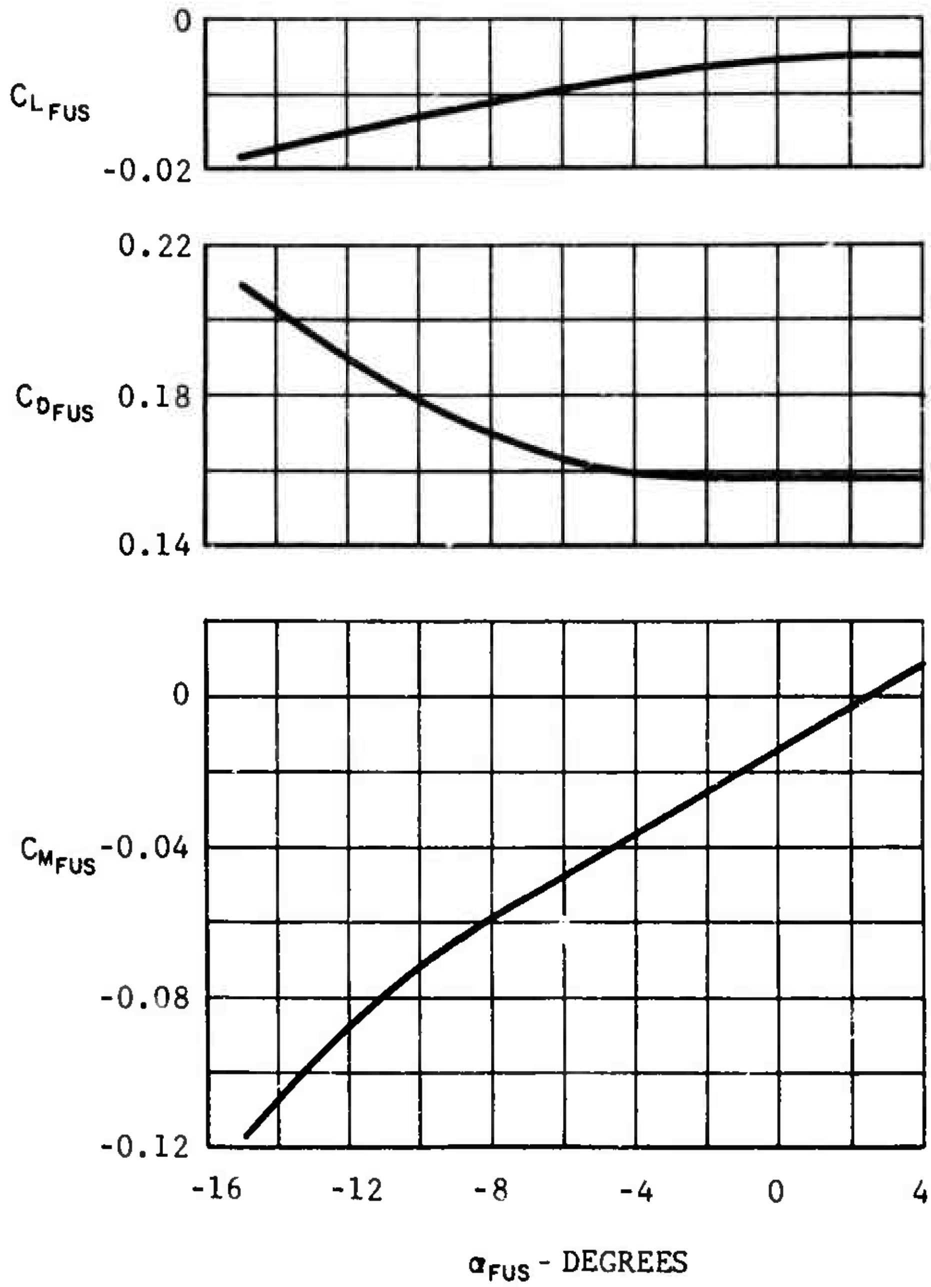


Figure 2. Fuselage Characteristics for the Sample Single Rotor Helicopter ($\beta_s = 0$).

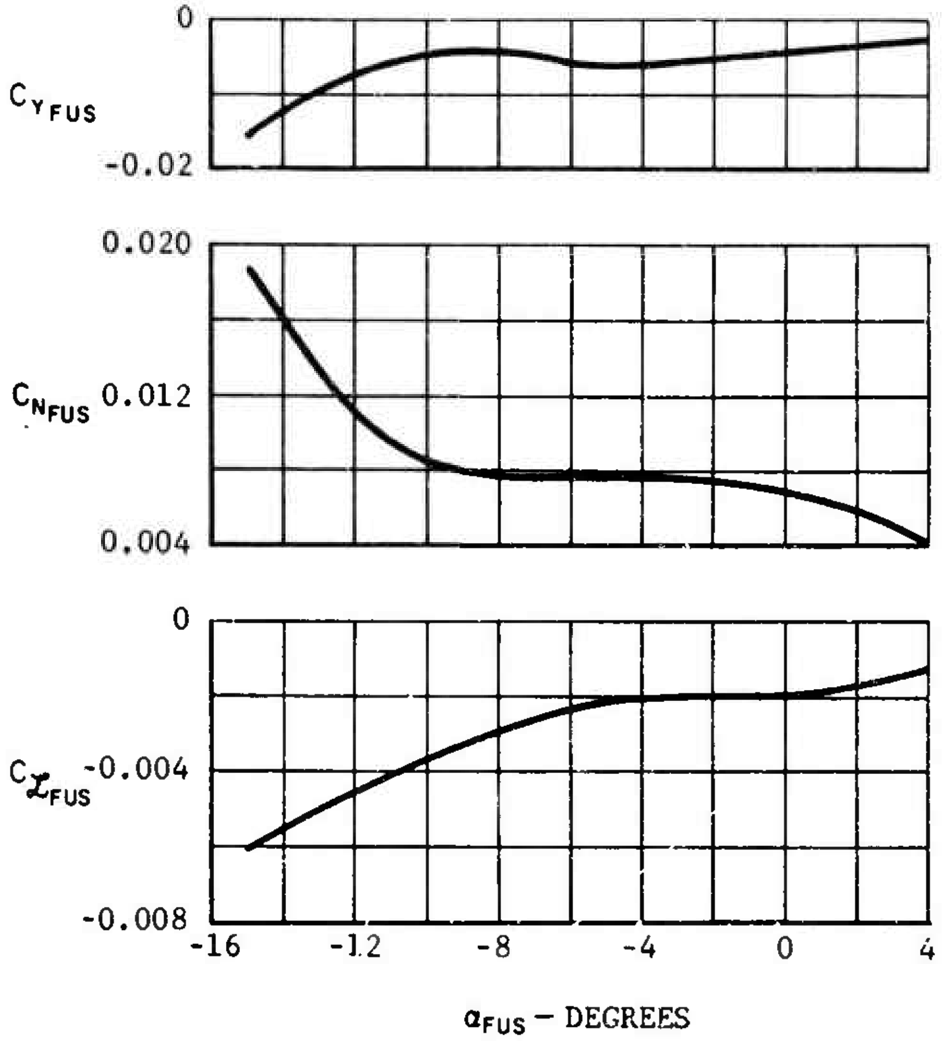


Figure 2. (Concluded).

TABLE I

DESIGN PARAMETERS FOR THE SAMPLE SINGLE ROTOR HELICOPTER

Main Rotor	Fuselage	Tail Rotor	Horizontal Tail Plane
$\theta_{1_F} = 0$	$W = 5300$ lb	$\theta_{1_{TR}} = 0$	$a_T = 3.81$
$a_{0_F} = 4°$ (preconed)	$A_{X_{FUS}} = 48$ ft^2	$a_{TR} = 5.73$	$i_T = 0$
$a = 5.73$	$A_{Y_{FUS}} = 160.7$ ft^2	$i_{TR} = 0$	$(AR)_T = 4.23$
$i_F = 0$	$A_{Z_{FUS}} = 144.5$ ft^2	$\sigma_{TR} = 0.105$	$S_T = 21.4$ ft^2
$b_F = 2$	$\ell_{FUS} = 39$ ft	$R_{TR} = 4.3$ ft	$\ell_{X_T} = -16.62$ ft
$\sigma_F = 0.051$		$\ell_{X_{TR}} = -28$ ft	$\ell_{Y_T} = 0$
$\gamma_F = 8$		$\ell_{Y_{TR}} = -1$ ft	$\ell_{Z_T} = 0$
$e_F = 0$	Fuselage aerodynamic characteristics for the model configuration D obtained from Reference 1	$\ell_{Z_{TR}} = -5$ ft	
$R_F = 22$ ft			
$\ell_{X_F} = 0$			Airfoil: NACA 0015
$\ell_{Y_F} = 0$			$C_{D_0} = 0.01$
$\ell_{Z_F} = -5.35$ ft			
$M_{S_F} = 45$ slug-ft			

(b) Determine the following operating condition for the sample aircraft:

V_o = 207 ft/sec

$(\Omega R)_F$ = 690 ft/sec

$(\Omega R)_{TR}$ = 713.8 ft/sec

Altitude = Sea level standard day (ρ = 0.002378 slug/ft^3)

Then compute

$$\mu_F = \left(\frac{V_o}{\Omega R}\right)_F = \frac{207}{690} = 0.3$$

$$(M_T)_F = \frac{V_o + (\Omega R)_F}{V_S} = \frac{207 + 690}{1118} = 0.8$$

$$\mu_{TR} = \frac{V_o}{(\Omega R)_{TR}} = \frac{207}{713.8} = 0.29$$

$$(M_T)_{TR} = \frac{V_o + (\Omega R)_{TR}}{V_S} = \frac{207 + 718.80}{1118} = 0.82$$

$$q_o = \frac{1}{2}\rho V_o^2 = \tfrac{1}{2} \times 0.002378 \times (207)^2 = 50.9 \text{ lb/ft}^2$$

$$(T.F.)_F = \left[\rho \pi R^2 (\Omega R)^2\right]_F = 0.002378 \times 3.14 \times 22^2 \times 690^2 = 1.72 \times 10^6 \text{ lb}$$

$$(T.F.)_{TR} = \left[\rho \pi R^2 (\Omega R)^2\right]_{TR} = 0.002378 \times 3.14 \times 4.3^2 \times 713.8^2 = 7.03 \times 10^6 \text{lb}$$

(c) Obtain fuselage lift and drag coefficients for α_{FUS} = 0.

Using Figure 2, and assuming α_{FUS} = 0, obtain

$C_{L_{FUS}} = -0.0060$

$C_{D_{FUS}} = 0.158$

Using the values for $A_{Z_{FUS}}$ and $A_{X_{FUS}}$ from step (a) and q_o from step (b), compute

$$L_{FUS} = C_{L_{FUS}} q_o A_{Z_{FUS}} = -0.006 \times 50.9 \times 144.5 = -44.1 \text{ lb}$$

$$D_{FUS} = C_{D_{FUS}} q_o A_{X_{FUS}} = 0.158 \times 50.9 \times 48 = 386 \text{ lb}$$

(d) Calculate the first approximation for the main rotor lift and drag forces, thus:

$$L_F = W - L_{FUS} = 5300 + 44.1 = 5344.1 \text{ lb}$$

$$D_F = -D_{FUS} = -386 \text{ lb}$$

Also compute the corresponding rotor lift and drag coefficients

$$\left(\frac{C_L'}{\sigma}\right)_F = \left[\frac{L}{(T.F.)\sigma}\right]_F = \frac{5344.1}{1.72\text{x}10^6\text{x}0.051} = 0.0609$$

$$\left(\frac{C_D'}{\sigma}\right)_F = \left[\frac{D}{(T.F.)\sigma}\right]_F = \frac{-386}{1.72\text{x}10^6\text{x}0.051} = -0.0044C$$

(e) Using Reference 2, calculate the chart values of rotor lift and drag coefficients corresponding to rotor solidity $\sigma = 0.1$, thus:

$$(\Delta\sigma)_F = \sigma_F - 0.1 = 0.051 - 0.1 = -0.049$$

$$\left[\left(\frac{C_L'}{\sigma}\right)_{0.1}\right]_F = \left(\frac{C_L'}{\sigma}\right)_F = 0.0609$$

$$\left[\left(\frac{C_D'}{\sigma}\right)_{0.1}\right]_F = \left[\frac{C_Q'}{\sigma} - \frac{\Delta\sigma}{2\mu^2}\left(\frac{C_L'}{\sigma}\right)^2\right]_F$$

$$= -0.0044 + \frac{0.049}{2(0.3)^2}(0.0609)^2 = -0.0034$$

(f) Using the values of $[(C_L'/\sigma)_{0.1}]_F$, $[(C_D'/\sigma)_{0.1}]_F$ from step (e) and θ_{1F}, μ_F, M_{T_F} from steps (a) and (b), enter Figure 44 of Reference 2 and Figure 3 of Section 5.3 and obtain the first approximations for the following main rotor trim parameters:

$$[(a_C)_{0.1}]_F = -8.2° = -0.143 \text{ rad}$$

$$a_{0_F} = 3.7° = 0.0646 \text{ rad}$$

$$a_{1_F} = 4.75° = 0.0829 \text{ rad}$$

$$b_{1_F} = 1.60° = 0.027 \text{ rad}$$

$$(\theta_{.75})_F = 7.1° = 0.124 \text{ rad}$$

$$\lambda_F = -0.052$$

$$\left(\frac{C_Q}{\sigma}\right)_F = 0.00356$$

(g) Calculate main rotor angle of attack (a_{C_F}) and rotor torque (Q_F) as follows:

$$a_{C_F} = \left[(a_C)_{0.1} + \frac{\Delta\sigma}{2\mu^2}\left(\frac{C_L'}{\sigma}\right)\right]_F$$

$$= -0.143 - \frac{0.049}{2x(0.3)^2} x 0.069 = -0.160 \text{ rad} = -9.16°$$

$$Q_F = \left[\frac{C_Q}{\sigma}(T.F.)\,\sigma R\right]_F$$

$$= 0.00356 \times 1.72 \times 10^6 \times 0.051 \times 22 = 6870 \text{ ft/lb}$$

(h) Using Section 7.6 or the pertinent test data, obtain the following downwash interference factors:

$$K_{FFUS} = K_{FT} = K_{FTR} = 1.0$$

Then compute the downwash interference factors ϵ_{FUS}, ϵ_T, and ϵ_{TR} using the values of λ_F from step (f) and α_{C_F} from step (g), respectively. Thus

$$\epsilon_{FUS} = \epsilon_T = \epsilon_{TR} = 1.0 \left(\tan \alpha_C - \frac{\lambda}{\mu}\right)_F$$

$$= 1.0\left[\tan(-9.16°) + \frac{0.052}{0.3}\right] = 0.012 \text{ rad} = 0.688°$$

(i) Using the design parameters and the initial trim values obtained in the steps above, determine the relationship between α_{FUS} and $C_{M_{FUS}}$ from the following equation:

$$\alpha_{FUS} = \frac{\left[\ell_x L - \ell_z D + \frac{eb\Omega^2 M_S}{2}(a_1 + \alpha_C - i)\right]_F + q_0\left[\ell_x S a (i-\epsilon)\right]_T}{-(\ell_x D + \ell_z L)_F - q_0(\ell_x S a)_T}$$

$$+ \frac{C_{M_{FUS}} q_0 A x_{FUS} \ell_{FUS}}{-(\ell_x D + \ell_z L)_F - q_0(\ell_x S a)_T} - \epsilon_{FUS}$$

$$\alpha_{FUS} = \frac{-5.35\text{x}386 + 50.9\text{x}16.62\text{x}21.4\text{x}3.81\text{x}0.012}{5.35\text{x}5344.1 + 50.9\text{x}16.62\text{x}21.4\text{x}3.81}$$

$$\frac{+50.9\text{x}48\text{x}39\text{x}\, C_{M_{FUS}}}{5.35\text{x}5344.1 + 50.9\text{x}16.62\text{x}21.4\text{x}3.81} - 0.012$$

$$\alpha_{FUS} = -0.0247 + 0.976\ C_{M_{FUS}}$$

$$\therefore \alpha_{FUS}^{\circ} = -1.415 + 55.92\ C_{M_{FUS}}$$

Superimpose the linear relationship between α_{FUS}° and $C_{M_{FUS}}$ from the above equation on the experimental fuselage pitching moment curve $C_{M_{FUS}}$ vs α_{FUS}° and obtain the point of intersection as shown in Figure 3. This point yields the fuselage trim angle of attack

$$\alpha_{FUS} = -3.2^{\circ} = -0.0558 \text{ rad}$$

(j) Using α_{FUS} from step (i), enter the fuselage charts of Figure 2 and obtain the following fuselage characteristics.

$$C_{L_{FUS}} = -0.0073$$

$$C_{M_{FUS}} = -0.032$$

$$C_{D_{FUS}} = 0.158$$

$$C_{N_{FUS}} = 0.0077$$

$$C_{Y_{FUS}} = -0.0057$$

$$C_{\mathcal{L}_{FUS}} = -0.002$$

Then compute the corresponding fuselage forces and moments, thus:

$$L_{FUS} = C_{L_{FUS}} q_0 A_{Z_{FUS}} = -0.0073 \times 50.9 \times 144.5 = -53.7 \text{ lb}$$

$$D_{FUS} = C_{D_{FUS}} q_0 A_{X_{FUS}} = 0.158 \times 50.9 \times 48 = 386 \text{ lb}$$

$$Y_{FUS} = C_{Y_{FUS}} q_0 A_{Y_{FUS}} = -0.0057 \times 50.9 \times 160.7 = -46.6 \text{ lb}$$

$$M_{FUS} = C_{M_{FUS}} q_0 A_{Y_{FUS}} \ell_{FUS} = -0.032 \times 50.9 \times 48 \times 39 = -3050 \text{ ft-lb}$$

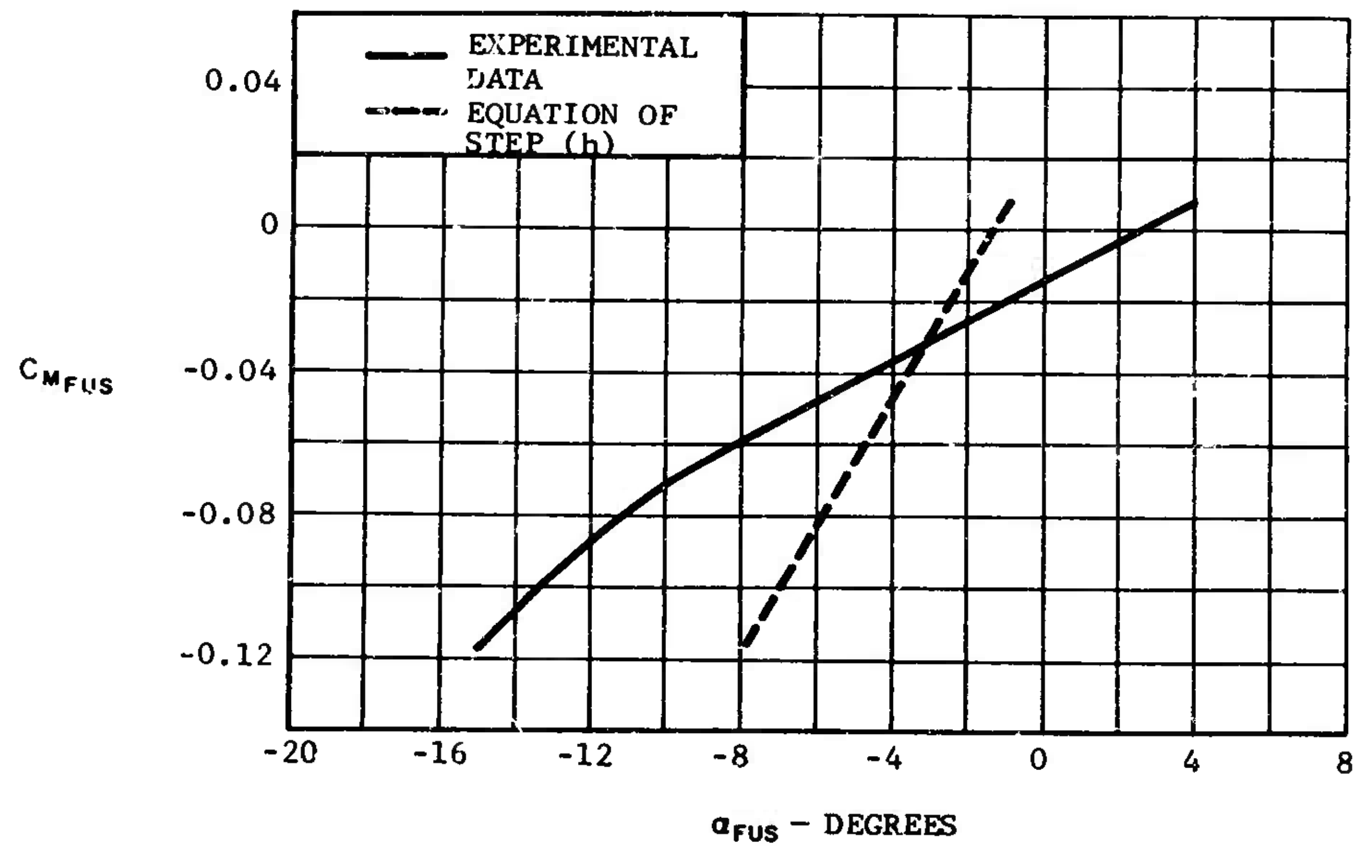


Figure 3. Superposition of the Calculated and the Experimental Fuselage Pitching Moment Data.

$$\mathcal{L}_{FUS} = C_{\mathcal{L}_{FUS}} q_0 A_{x_{FUS}} \ell_{FUS}$$
$$= -0.002 \times 50.9 \times 48 \times 39 = -191 \text{ ft-lb}$$

$$N_{FUS} = C_{N_{FUS}} q_0 A_{x_{FUS}} \ell_{FUS}$$
$$= 0.0077 \times 50.0 \times 48 \times 39 = 734 \text{ ft-lb}$$

(k) Using the values of N_{FUS} from step (j) and Q_F from step (g), determine tail rotor thrust and tail rotor lift coefficient, thus

$$T_{TR} = \frac{N_{FUS}+Q_F}{-\ell_{x_{TR}}} = \frac{734 + 6870}{28} = 272 \text{ lb}$$

$$\left(\frac{C_L'}{\sigma}\right)_{TR} = \left[\frac{T}{(T.F.)\,\sigma}\right]_{TR} = \frac{272}{7.03 \times 10^4 \times 0.105} = 0.0368$$

(l) With the values of $\theta_{I_{TR}}$, μ_{TR}, and $M_{T_{TR}}$ from steps (a) and (b), $(C_L'/\sigma)_{TR}$ from step (h), and $(\alpha_C)_{TR} = 0$, enter the appropriate performance charts of Reference 2 (interpolate between the charts if necessary), and obtain the following tail rotor parameters:

$$\left[\left(\frac{C_D'}{\sigma}\right)_{TR}\right]_{0.1} = 0.0020, \qquad \lambda_{TR} = 0.007$$

$$\left(\frac{C_Q'}{\sigma}\right)_{TR} = 0.0011, \qquad (\theta_{.75})_{TR} = 2.2°$$

Then compute

$$\left(\frac{C_D'}{\sigma}\right)_{TR} = \left[\left(\frac{C_D'}{\sigma}\right)_{0.1} + \frac{\Delta\sigma}{2\mu^2}\left(\frac{C_L'}{\sigma}\right)^2\right]_{TR}$$

$$= 0.0020 + \frac{(0.105-0.1)}{2\text{x}(0.29)^2}\,(0.0368)^2 = 0.00204$$

$$D_{TR} = \left[\frac{C_D{}'}{\sigma}(T.F.)\sigma\right]_{TR} = 0.00204 \times 7.03 \times 10^4 \times 0.105 = 15.1 \text{ lb}$$

$$Q_{TR} = \left[\frac{C_Q}{\sigma}(T.F.)\sigma R\right]_{TR} = 0.0011 \times 7.03 \times 10^4 \times 0.105 \times 4.3$$
$$= 34.9 \text{ ft-lb}$$

(m) From the trim values obtained in the steps above, determine the horizontal tail plane characteristics as follows:

$$\alpha = \alpha_{FUS} + \epsilon_{FUS} = -0.0558 + 0.012 = -0.0438 \text{ rad}$$
$$= -2.51°$$

$$\alpha_T = \alpha + i_T - \epsilon_T = -0.0438 + 0 - 0.012$$
$$= -0.0058 \text{ rad} = -3.2°$$

$$C_{L_T} = a_T \, \alpha_T = 3.81\ (-0.0558) = -0.213$$

$$C_{D_T} = \left(C_{D_0} + \frac{C_L{}^2}{\pi \mathcal{R}}\right)_T = 0.01 + \frac{(-0.213)^2}{3.14 \times 4.23} = 0.0134$$

$$L_T = C_{L_T} q_0 S_T = -0.213 \times 50.9 \times 21.4 = -232 \text{ lb}$$

$$D_T = C_{D_T} q_0 S_T = 0.0134 \times 50.9 \times 21.4 = 14.6 \text{ lb}$$

(n) Assuming $A_{1_F} = \phi = Y_{TR} = \gamma_C = 0$, solve simultaneously the X and Z equations from Section 4.0 to obtain a better approximation for the main rotor lift and drag, thus

$$K_1 = W\alpha - L_{FUS}(\alpha - \epsilon_{FUS}) - L_T(\alpha - \epsilon_T) + D_{FUS} + D_T + D_{TR}$$

$$= 5300(-0.0438) + 53.7(-0.0558) + 232(-0.0558)$$
$$+386 + 14.6 + 15.1 = 168 \text{ lb}$$

$$K_2 = D_{FUS}(\alpha - \epsilon_{FUS}) + D_T(\alpha - \epsilon_T) + D_{TR}(\alpha - \epsilon_{TR}) + L_{FUS} + L_T - W$$

$$= 386(-0.0558) + 14.6(-0.0558) + 15.1(-0.0558)$$
$$-53.7 - 232 - 5300 = -5610 \text{ lb}$$

$$L_F = \frac{K_1 \alpha - K_2}{1+\alpha^2} = \frac{168(-0.0438)+5610}{1+(-0.0438)^2} = 5590 \text{ lb}$$

$$D_F = L_F \alpha - K_1 = 5590(-0.0438) - 168 = -413 \text{ lb}$$

Then obtain

$$\left(\frac{C_L'}{\sigma}\right)_F = \left[\left(\frac{C_L'}{\sigma}\right)_{0.1}\right]_F = \left[\frac{L}{(T.F.)\sigma}\right]_F = \frac{5590}{1.72 \times 10^6 \times 0.051} = 0.0637$$

$$\left(\frac{C_D'}{\sigma}\right)_F = \left[\frac{D}{(T.F.)\,\sigma}\right]_F = -\frac{413}{1.72 \times 10^6 \times 0.051} = -0.00471$$

$$\left[\left(\frac{C_D'}{\sigma}\right)_{0.1}\right]_F = \left[\frac{C_D'}{\sigma} - \frac{\Delta\sigma}{2\mu^2}\left(\frac{C_L'}{\sigma}\right)^2\right]_F$$

$$= -0.00471 + \frac{0.049}{2 \times (0.3)^2}(0.0637)^2 = -0.00361$$

(o) Repeat steps (f) through (n) with the new values of $\left[(C_L'/\sigma)_{0.1}\right]_F$ and $\left[(C_D'/\sigma)_{0.1}\right]_F$ from step (n) until convergence is achieved, yielding the final trim values as shown in Table II below:

TABLE II

FINAL TRIM VALUES FOR THE SAMPLE SINGLE ROTOR HELICOPTER

Main Rotor	Fuselage	Tail Rotor	Tail Plane
a_{C_F} = -9.40° (-0.164 rad)	α = -2.37° (-0.0414 rad)	α_{TR} = -3.58° (-0.0625 rad)	α_T = -3.58° (-0.0625 rad)
L_F = 5620 lb	α_{FUS} = -3.58° (-0.0625 rad)	T_{TR} = 274 lb	L_T = -259 lb
D_F = -411 lb	L_{FUS} = -56.6 lb	D_{TR} = 15.8 lb	D_T = 15.6 lb
Q_F = 7040 ft-lb	D_{FUS} = 386 lb	Q_{TR} = 34.9 ft-lb	ϵ_T = 1.21° (0.0211 rad)
λ_F = -0.056	Y_{FUS} = -49.1 lb	λ_{TR} = -0.007	
$\theta_{.75_F}$ = 7.25° (0.126 rad)	M_{FUS} = -3240 ft-lb	$\theta_{.75_{TR}}$ = 2.2° (0.0384 rad)	
a_{0_F} = 3.87° (0.0675 rad)	N_{FUS} = 734 ft-lb	ϵ_{TR} = 1.21° (0.0211 rad)	
a_{1_F} = 4.85° (0.0846 rad)	$\mathcal{L}_{FUS}$ = -191 ft-lb		
b_{1_F} = 1.65° (0.0288 rad)	ϵ_{FUS} = 1.21° (0.0211 rad)		

(p) Using the final trim values from Table II, calculate main rotor side force, thus:

$$Y_F = \Big[(T.F.)\sigma \frac{a}{2}\Big(-\frac{3}{4}\mu\,\theta_{.75}a_0 + \frac{1}{3}\theta_{.75}b_1 + \frac{3}{8}\mu^2\theta_{.75}b_1$$

$$+\frac{3}{4}\lambda b_1 + \frac{1}{6}a_0a_1 - \frac{3}{2}\mu\lambda a_0 - \mu^2 a_0a_1 + \frac{1}{4}\mu a_1b_1 + \frac{1}{8}\mu^2\lambda b_1\Big)\Big]_F$$

$$= \frac{1.72\times10^6\times0.051\times5.73}{2}\Big(-\frac{3}{4}\times0.3\times0.126\times0.0675$$

$$+\frac{1}{3}\times0.126\times0.0288+\frac{3}{8}\times0.09\times0.126\times0.0288$$

$$-\frac{3}{4}\times0.056\times0.0288+\frac{1}{6}\times0.0675\times0.0846$$

$$+\frac{3}{2}\times0.3\times0.056\times0.0675-0.09\times0.0675\times0.0846$$

$$+\frac{1}{4}\times0.3\times0.0846\times0.0288-\frac{1}{8}\times0.09\times0.056\times0.0288\Big)$$

$$= 129\text{ lb}$$

(q) Also compute the main rotor lateral cyclic A_{1_F} (from the rolling moment equation) and aircraft roll altitude ϕ (from the side force equation) using the final trim values as follows:

$$A_{1_F} = \frac{\left(\ell_Z Y - \frac{eb\Omega^2 M_S}{2} b_1\right)_F + \ell_{Z_{TR}}T_{TR} + \ell_{Y_{TR}}D_{TR}(\alpha-\epsilon_{TR}) - \mathcal{L}_{FUS}}{\left[-\ell_Z L + \left(\frac{eb\Omega^2 M_S}{2}\right)_F\right]}$$

$$= \frac{-5.35\times129-5\times274+1\times15.8\times0.0625+191}{5.35\times5620}$$

$$= -0.0621\text{ rad} = -3.56^\circ$$

$$\phi = -\frac{(LA_1+Y)_F + Y_{FUS} + T_{TR}}{W}$$

$$= -\left(\frac{-5620 \times 0.0621 + 129 - 49.1 + 274}{5300}\right)$$

$$= -0.000924 \text{ rad} = -0.0529°$$

(r) Finally, compute the main rotor longitudinal cyclic pitch

$$B_{1_F} = \alpha - \alpha_{C_F} + i_F = -0.0414 + 0.164 = 0.123 \text{ rad} = 7.05°$$

10.1.2 Stability Derivatives for a Single Rotor Helicopter

The numerical procedure for computing the stability derivatives for a single rotor helicopter is very similar to that presented in Section 10.2.2 for the sample tandem rotor configuration. The main rotor and the tail rotor derivatives for the sample single rotor helicopter are obtained by following the numerical procedures for the front rotor of the tandem rotor configuration of Section 10.2.2 and by utilizing the appropriate trim conditions presented in Table II. The required fuselage derivatives are computed by graphically obtaining slopes to the fuselage data of Figure 2, and utilizing equations presented in Subsection 10.2.2.3. The horizontal tail plane derivatives are obtained from Reference 3.

The total stability derivatives for the sample single rotor helicopter for the six degrees of freedom of aircraft coupled motion are presented in Table III. This table also includes the control derivatives required for aircraft response calculations.

TABLE III

TOTAL STABILITY DERIVATIVES FOR THE SAMPLE SINGLE ROTOR HELICOPTER

Var. \ Eq.	X	Y	Z	M	N	L
θ	-5300	0	231.24	0	0	0
$\dot{\theta}$	1906.	-179.86	33730.	-7015.	166.77	-958.
$\ddot{\theta}$	0	0	0	-8400	0	0
ϕ	0	5300	0	0	0	0
$\dot{\phi}$	-198.46	-1929.	12.65	1067.	1701.	-2347.
$\ddot{\phi}$	0	0	0	0	600	-2120
ψ	0	-231.24	0	0	0	0
$\dot{\psi}$	-38.15	-33,700	-4.183	-552.	-10,000	1738.
$\ddot{\psi}$	0	0	0	0	-6950.	600
u	-6.563	1.665	18.512	13.842	-25.137	9.179
$\dot{u}$	-164.60	0	0	0	0	0
$\ddot{u}$	0	0	0	0	0	0
v	-2.077	-27.503	0.136	14.942	334.91	-85.269
$\dot{v}$	0	-164.60	0	0	0	0
$\ddot{v}$	0	0	0	0	0	0
w	-2.595	-7.211	-179.65	-150.84	-30.853	-42.48
$\dot{w}$	0	0	-164.60	0	0	0
$\ddot{w}$	0	0	0	0	0	0
A_1	-11.18	5600	-286.	59.83	0	29960
$\dot{A}_1$	-180.70	-320.	11.89	966.	0	-2039.
B_1	5854.	1643.	33267.35	-31540.	6387.	8793.
$\dot{B}_1$	-438.	177.40	71.4	2344.11	-97.8	946.

10.1.3 Stability Characteristic Equation

The stability characteristic equation for the sample single rotor helicopter can be obtained by utilizing the total aircraft stability derivatives presented in Table III and by following the analytical procedure outlined in Section 8. The numerical procedures for obtaining the coefficients of the characteristic equation extracting the stability roots and the analysis of the roots are exactly the same as those for the tandem rotor configuration presented in Section 10.2.3. Therefore, these numerical procedures will not be duplicated here.

The computations of aircraft stability characteristics involving more than three degrees of freedom of aircraft motion are most conveniently performed utilizing a digital or analog computer program. A typical analog computer program for predicting the response characteristics due to control inputs of a single rotor helicopter is described below.

10.1.4 Aircraft Response

Presented in this section are typical computations for the sample single rotor helicopter response due to control inputs. These computations were performed with the aid of the Pace 231R analog computer and include six degrees of freedom of coupled aircraft motion and two degrees of freedom of a stabilization device. The input forcing functions (step or pulse inputs) are the pilot's longitudinal and lateral cyclic controls which are programmed to be activated independently or simultaneously. The analog computer schematic diagram representing the equations of motion of the aircraft and the stabilization device is shown in Figure 4.

The sample calculation was performed utilizing the total helicopter stability derivatives presented in Table III. These derivatives were normalized by the coefficient of the highest order variable, e.g., the X equation was divided by $X_{\dot{u}}$, the M equation was divided by $M_{\ddot{\theta}}$, etc. In addition, appropriate scaling factors were utilized. The resulting settings of the potentiometers P and Q shown in Figure 4 are presented in Table IV.

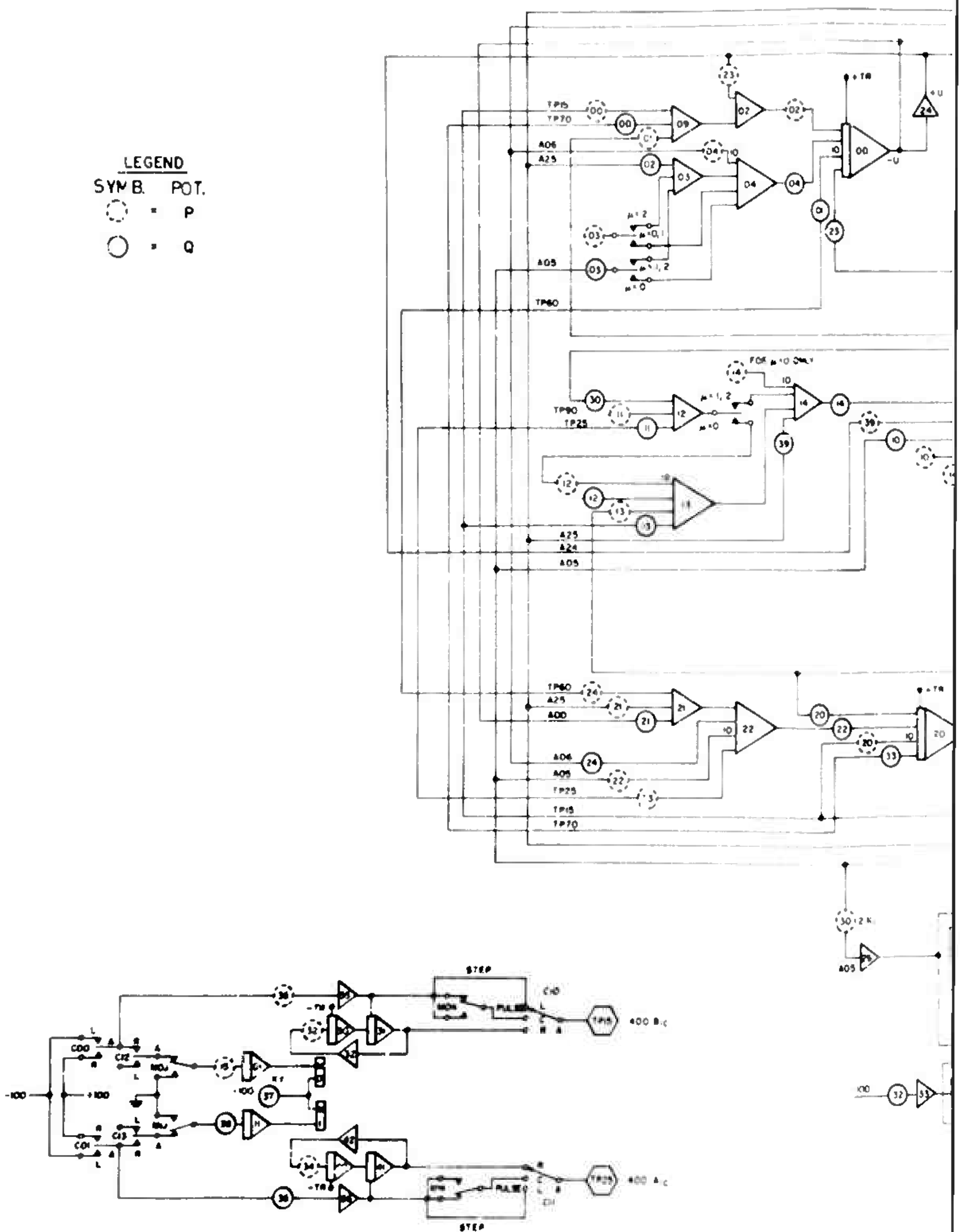


FIGURE 4. ANALOG COMPUTER SCHEMATIC

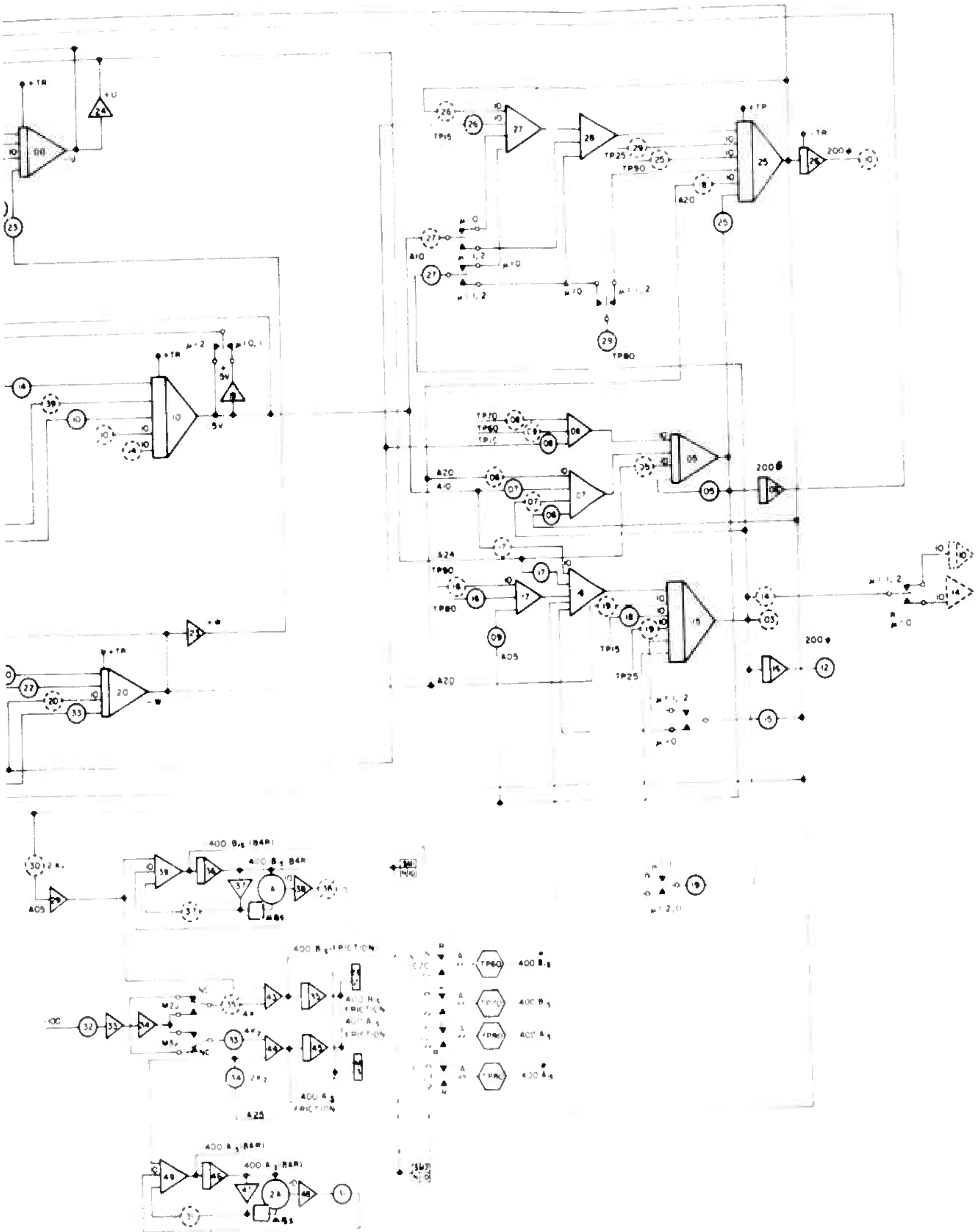


TABLE IV

ANALOG COMPUTER POTENTIOMETER SETTINGS

Pot. No.	Setting	Pot. No.	Setting	Pot. No.	Setting	Pot. No.	Setting
P00	0.7114	P10	0.0805	P20	0.0158	P30	
Q00	0.7114	Q10	0.0274	Q20	0.5455	Q30	0.2000
P01	0.0154	P11	0.0271	P21	0.0012	P31	0.5000
Q01	0.0053	Q11	0.6805	Q21	0.3374	Q31	0.1400
P02	0.1667	P12	0.1000	P22	0.3074	P32	1.0000
Q02	0.0362	Q12	0.0169	Q22	0.3333	Q32	
P03	0.0070	P13	0.2629	P23	0.2393	P33	0.0174
Q03	0.3475	Q13	0.1997	Q23	0.0158	Q33	0.6740
P04	0.0966	P14	0.5123	P24	0.0043	P34	1.0000
Q04	0.1667	Q14	0.8333	Q24	0.0211	Q34	0.1875
P05	0.0330	P15	0.1000	P25	0.4560	P35	0.3750
Q05	0.8352	Q15	0.1534	Q25	0.4560	Q35	
P06	0.3591	P16	0.0834	P26	0.1061	P36	1.0000
Q06	0.1270	Q16		Q26	0.3024	Q36	1.0000
P07	0.0657	P17	0.1836	P27	0.1000	P37	0.5000
Q07	0.1000	Q17	0.0568	Q27	0.4025	Q37	0.1000
P08	0.2503	P18	0.0834	P28	0.4382	P38	0.1406
Q08	0.2503	Q18	0.8737	Q28	0.0668	Q38	0.1000
P09	0.1860	P19	0.1266	P29	0.9665	P39	0.0502
Q09	0.0174	Q19	1.0000	Q29	0.6579	Q39	0.2433

Typical time history traces of the aircraft response due to pulse inputs of the longitudinal and lateral cyclic controls B_{1_C} and A_{1_C} are shown in Figures 5 and 6, respectively. The results presented in these figures include the effect of the stability augmentation system on the coupled modes of aircraft motion.

Figure 5 shows the coupled longitudinal and lateral aircraft response due to a longitudinal cyclic control pulse input of $B_{1_C} = 1°$ applied over a one-second time period. Examining the results of Figure 5a, it can be noted that the aircraft response in pitch and vertical perturbation velocity is represented by a stable oscillation. Specifically, after following a four-second oscillation in pitch, the aircraft attains a steady state pitch altitude of about 3 degrees nose down. Furthermore, the perturbations in vertical velocity do not exceed a maximum value of about 2.5 ft/sec. The effect of the longitudinal cyclic input on aircraft lateral response in roll, yaw, and lateral velocity can be seen from the results of Figure 5b. It can be noted that each of these lateral variables is affected by the application of the longitudinal cyclic control, indicating appreciable cross-coupling between aircraft longitudinal and lateral response modes. For example, it can be seen that the aircraft roll altitude ϕ is of the same order of magnitude as the pitch altitude θ. Also, the aircraft response in roll and in lateral perturbation velocity is represented by a slow convergence and a damped oscillation, respectively. The divergence in yaw altitude is present since no heading stabilization was applied.

The effect of lateral cyclic control pulse input of $A_{1_C} = 1°$, applied over a one-second period, on the aircraft coupled response is shown in Figure 6. Figure 6a indicates that the helicopter attains a maximum roll altitude of 12° in about 2.5 seconds after the control disturbance, and then it gradually converges toward a steady state roll. The lateral perturbation velocity is represented by a damped oscillation having a maximum amplitude of about 10 ft/sec. The aircraft pitch altitude and vertical

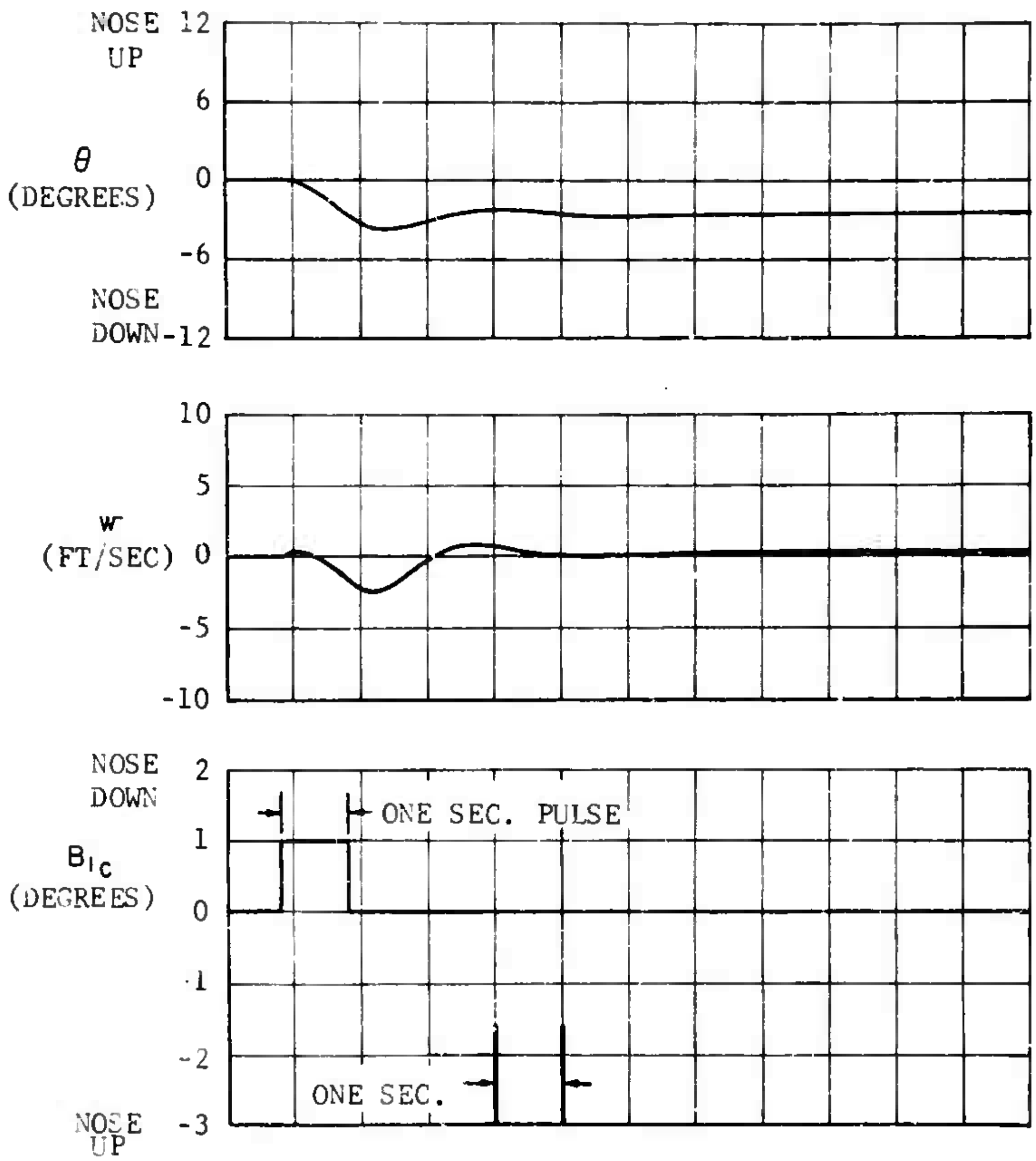


Figure 5a. Single Rotor Response Due to Longitudinal Control Input-Coupled Six Degrees of Freedom.

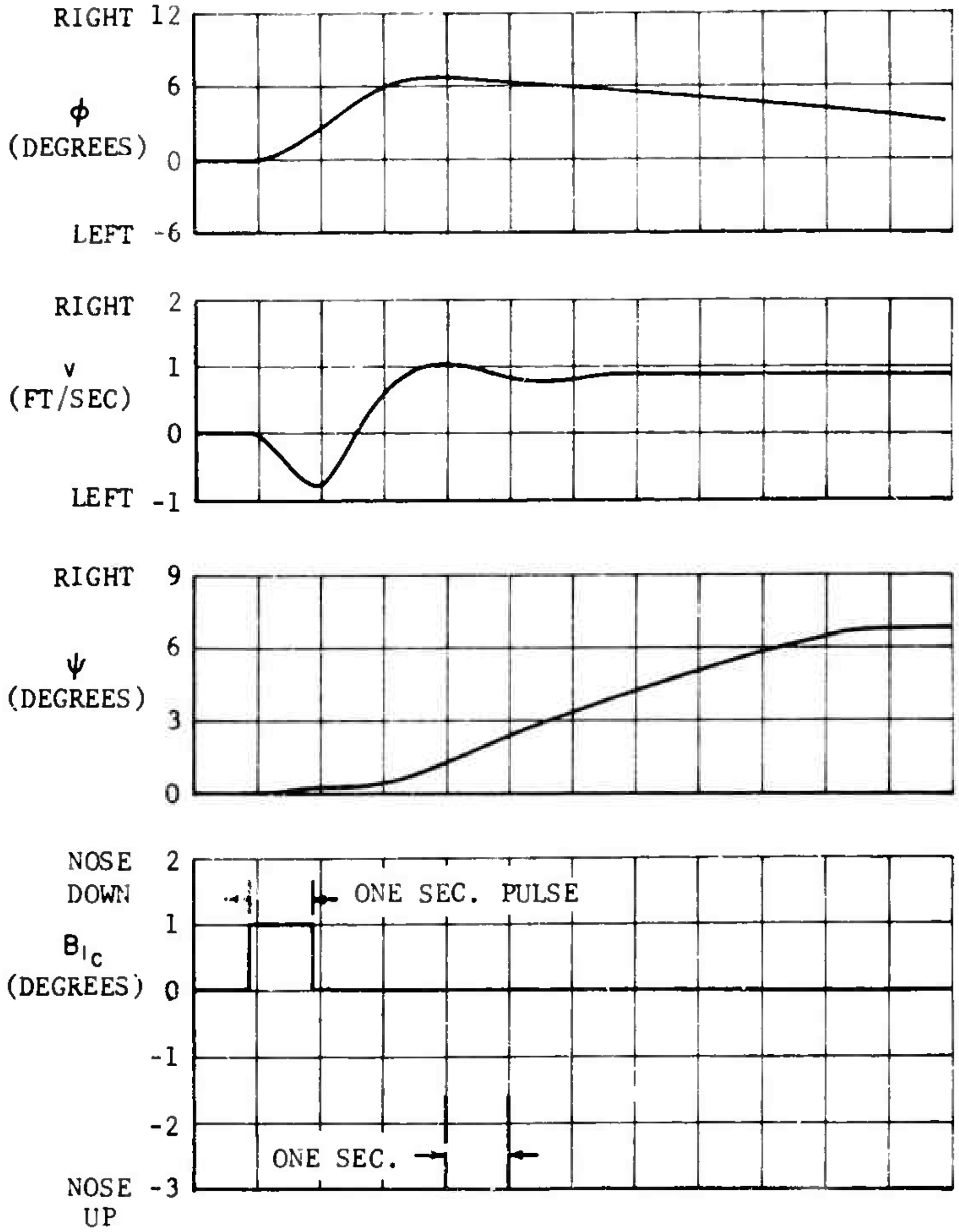


Figure 5b. Single Rotor Response Due to Longitudinal Control Input-Coupled Six Degrees of Freedom.

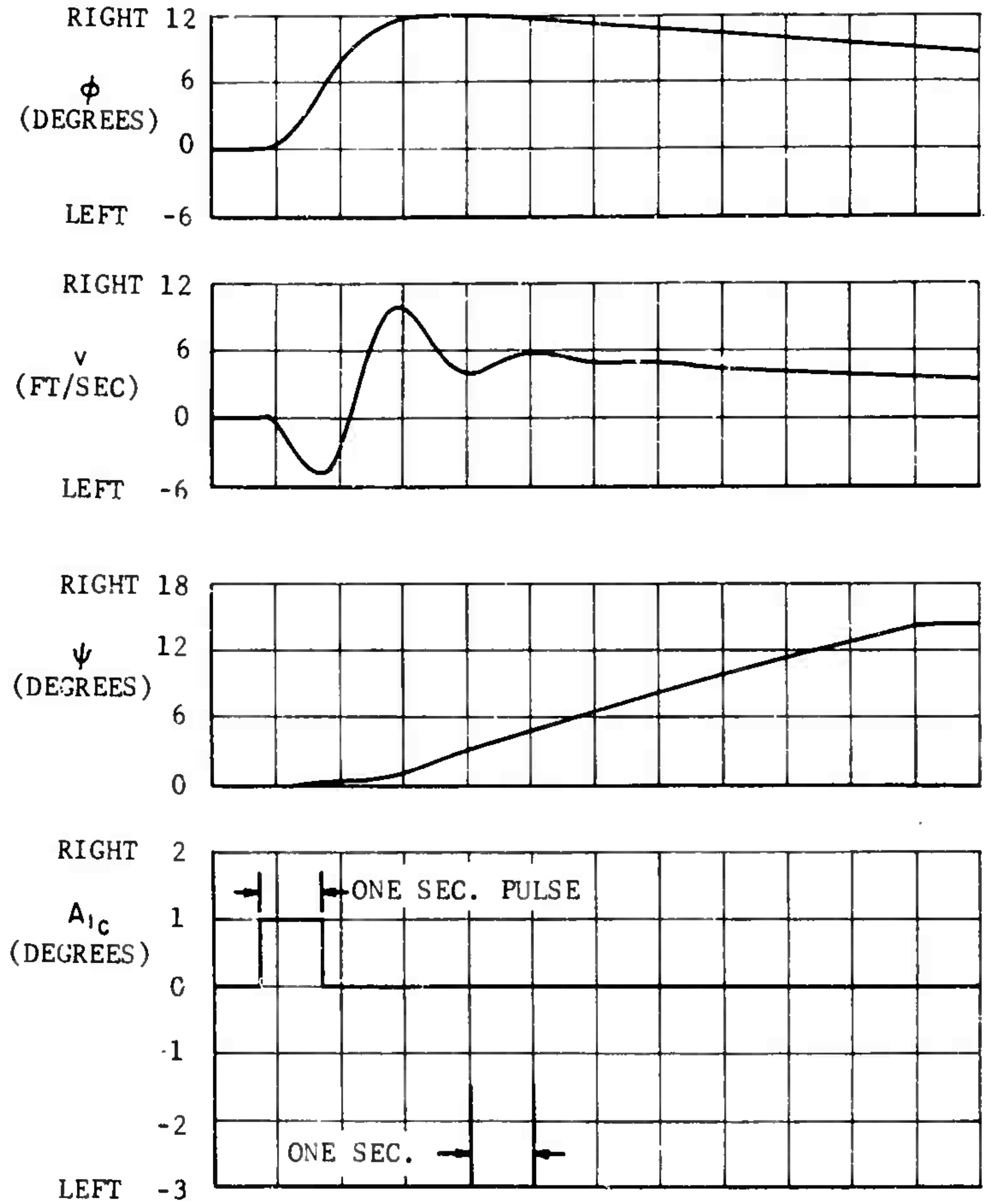


Figure 6a. Single Rotor Response Due to Lateral Control Input-Coupled Six Degrees of Freedom.

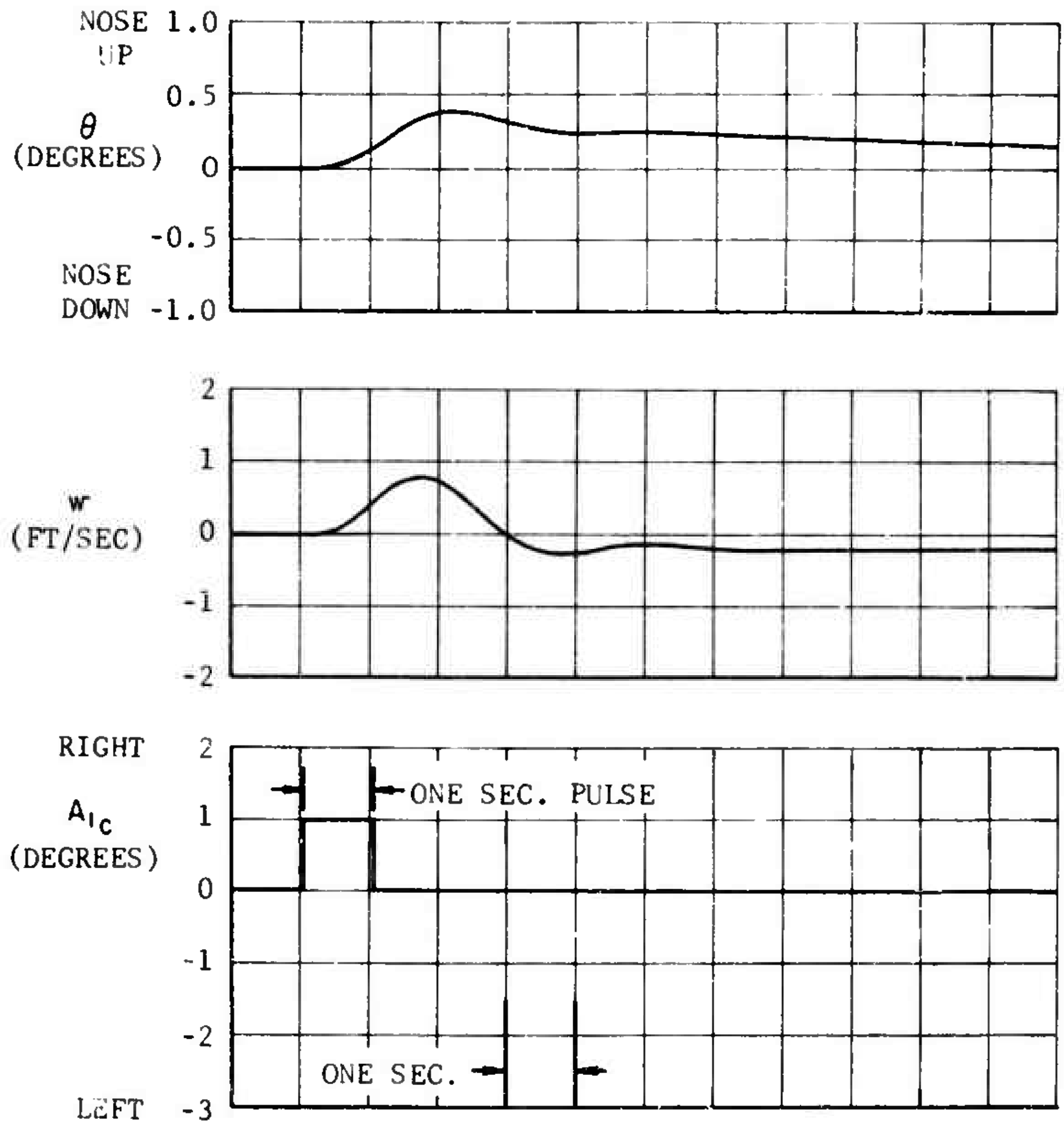


Figure 6b. Single Rotor Response Due to Lateral Control Input-Coupled Six Degrees of Freedom.

perturbation velocity, as shown in Figure 6b, are only slightly affected by the lateral control input. Thus, comparing Figures 5 and 6, it can be inferred that the longitudinal control input introduces appreciably larger cross-coupling between the longitudinal and lateral response modes of the sample helicopter than the lateral control input.

REFERENCES

1. Sweet, G. E., Jenkins, J. L., Jr., Wind Tunnel Investigation of the Drag and Static Stability Characteristics of Four Helicopter Fuselage Models, NASA Technical Note TND-1363, National Aeronautics and Space Administration, Washington, D.C., July 1962.

2. Tanner, W. H., Charts for Estimating Rotary Wing Performance in Hover and at High Forward Speeds, NASA Contractor Report CR-114, National Aeronautics and Space Administration, Washington, D.C., November 1964.

3. USAF Stability and Control Handbook (DATCOM), Flight Control Division, Air Force Flight Dynamics Laboratory, Wright-Patterson Air Force Base, Ohio, October 1960, Revised July 1963.

10.2 TANDEM ROTOR HELICOPTER

The tandem rotor helicopter considered in this sample calculation is a heavy utility type aircraft as illustrated in Figure 2 of Section 3.3. It consists of three-bladed, freely flapping, front and rear rotors of identical geometry. The front and rear rotor shafts are inclined forward relative to the fuselage datum line through angles of $i_F = -9°$ and $i_R = -4°$, respectively. The rotor shaft dihedral (Γ) is therefore

$$\Gamma = i_F - i_R = -9° - (-4°) = -5°$$

The aircraft has no horizontal or vertical tail planes. The fuselage shape resembles that shown in Figure 1. The fuselage characteristics of the sample tandem rotor helicopter are presented in coefficient form in Figure 2. These data have been nondimensionalized in exactly the same manner as the fuselage data for the sample single rotor helicopter presented in Subsection 10.1.1. It should be noted that the data presented in Figure 2 do not pertain to any existing tandem rotor helicopter but are presented herein only for the qualitative indication of trends and illustrative purposes.

The longitudinal control is applied through a differential collective pitch, i.e., reduction of collective pitch on front rotor head, and increase of collective pitch on rear rotor head for nose-down control. The longitudinal cyclic controls B_{1_F} and B_{1_R} on front and rear rotors, respectively, remain fixed for a particular flight condition. These controls are automatically preset as functions of forward speed.

The aircraft operating conditions assumed in this sample calculation correspond to a forward speed of $V = 207$ ft/sec, rotor tip speed of $(\Omega R)_F = (\Omega R)_R = 690$ ft/sec, and a pressure altitude corresponding to sea level standard day conditions.

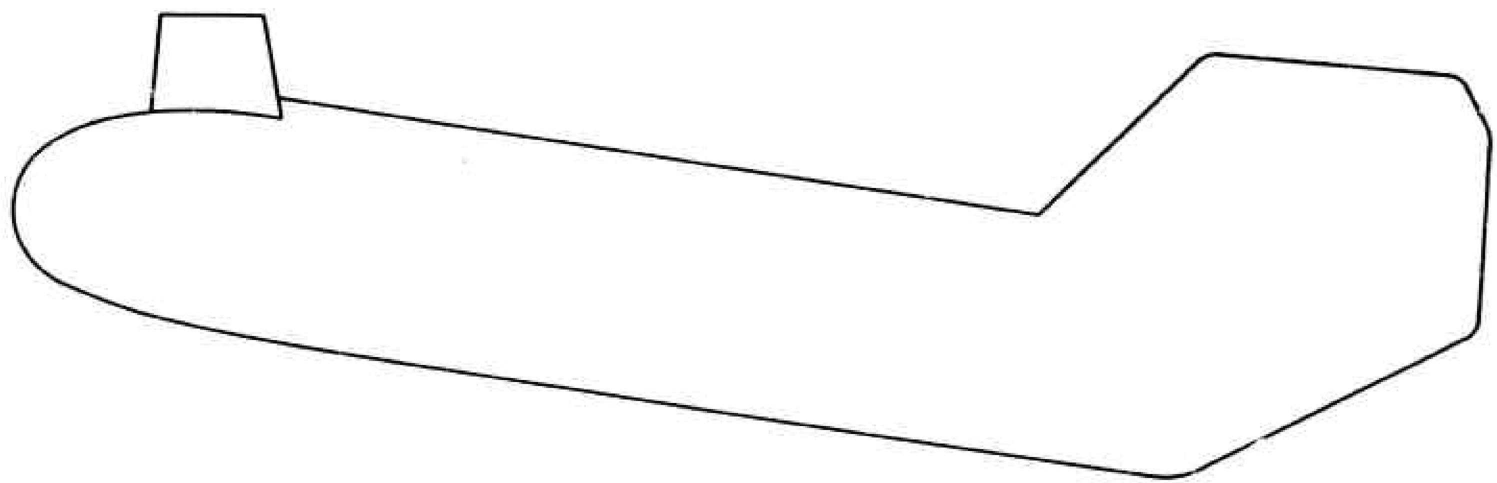


Figure 1. Fuselage Shape of the Sample Tandem Rotor Helicopter.

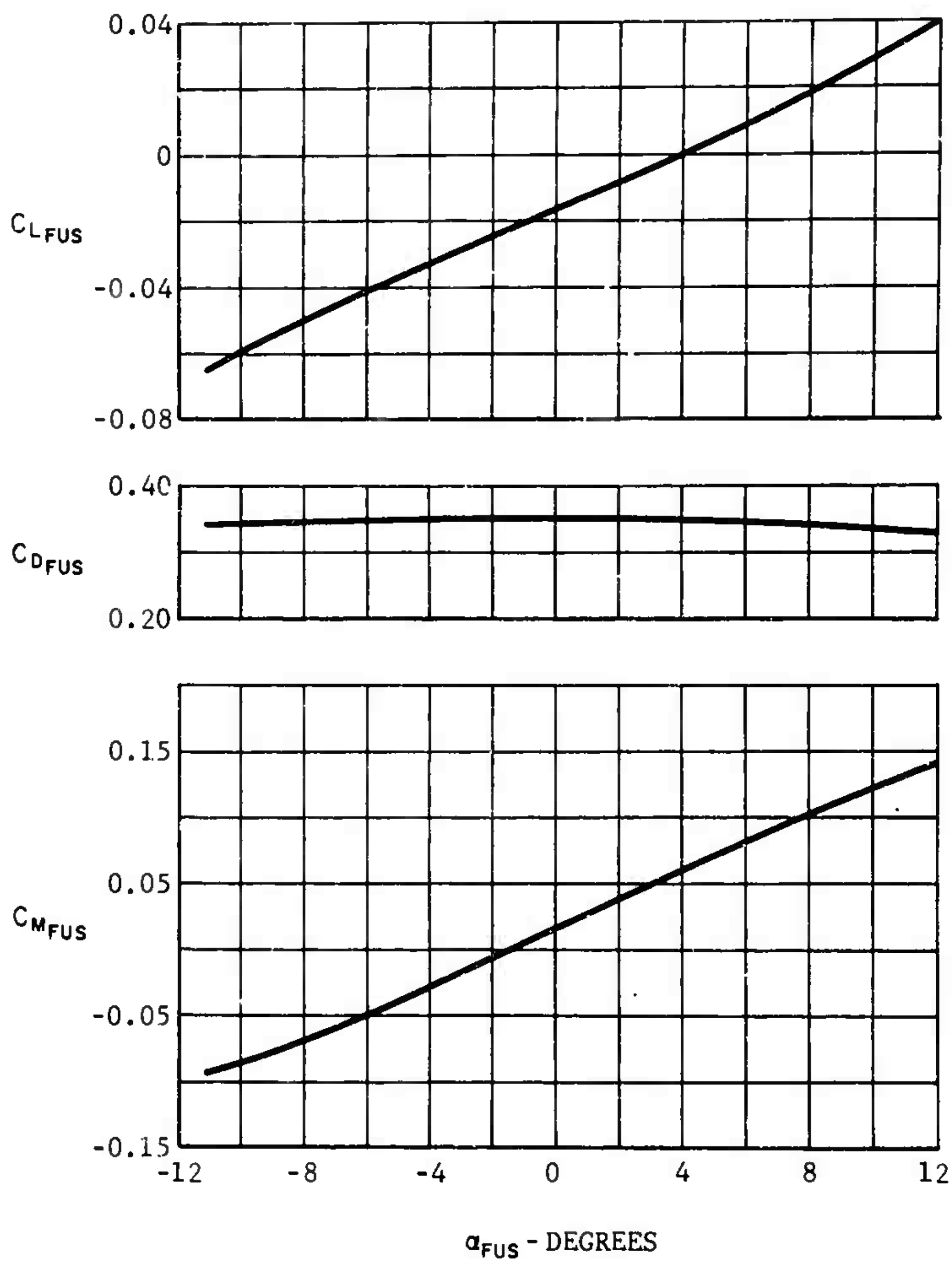


Figure 2. Fuselage Characteristics for the Sample Tandem Rotor Helicopter.

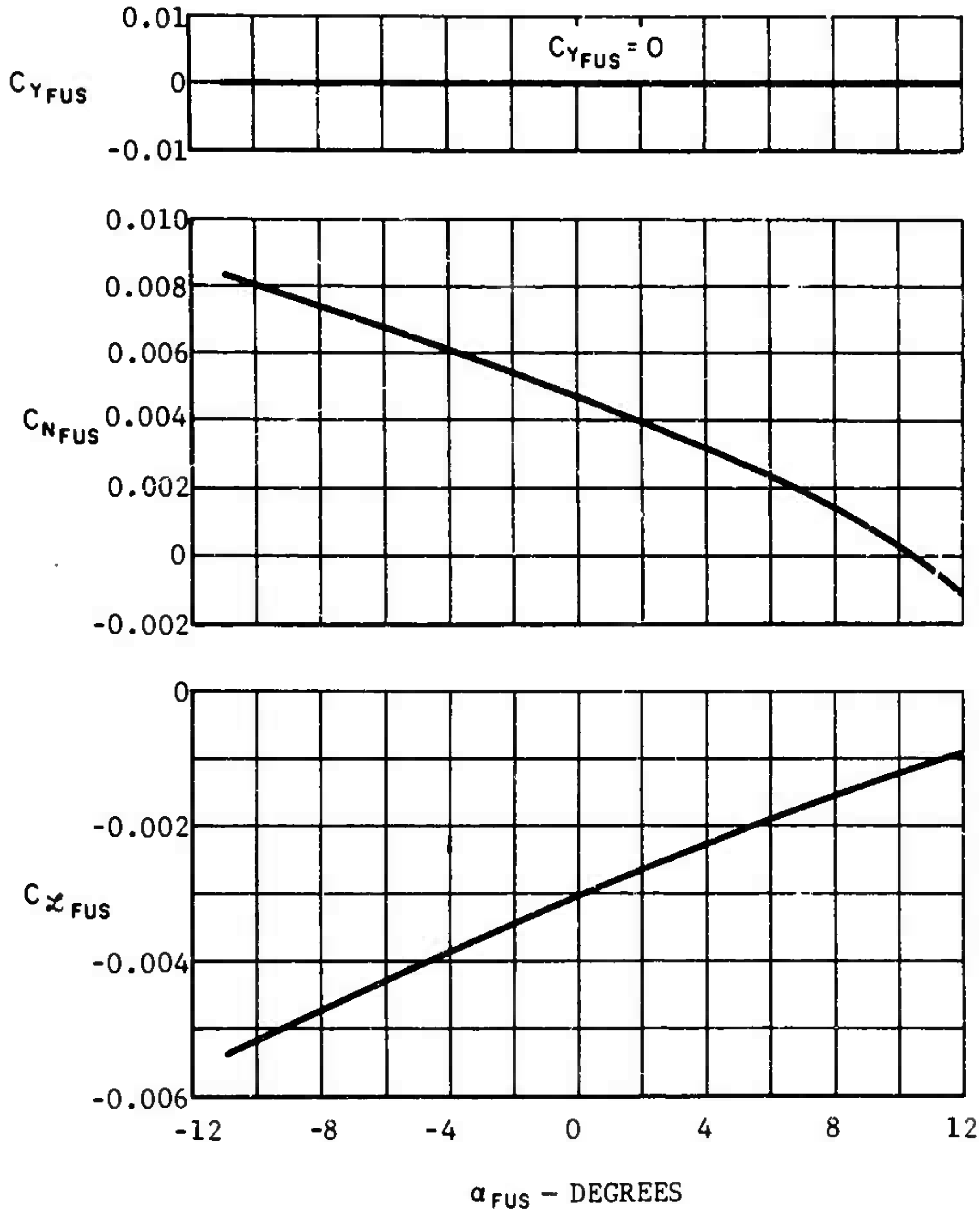


Figure 2. (Concluded).

The longitudinal cyclic controls on front and rear rotors are fixed at $B_{1_F} = 5°$ and $B_{1_R} = 3°$ respectively.

10.2.1 Trim Calculation for a Tandem Rotor Helicopter

The sample trim calculation for a tandem rotor helicopter is performed utilizing the analytical procedure outlined in Subsection 5.2.2 as follows:

(a) Determine the required design parameters for a tandem rotor configuration as shown in Table I.

TABLE I

DESIGN PARAMETERS FOR THE SAMPLE TANDEM ROTOR HELICOPTER

	Rotor		Fuselage	
	Front	Rear		
θ_1	0	0	W	28500 lb
a	5.73	5.73	$A_{X_{FUS}}$	121.6 ft^2
i	-9°	-4°	$A_{Y_{FUS}}$	511.6 ft^2
B_1	3°	5°	$A_{Z_{FUS}}$	566.8 ft^2
b	3	3	ℓ_{FUS}	51 ft
σ	0.1	0.1	Fuselage aerodynamic characteristics are presented in Figure 2.	
γ	8	8		
ϵ	0.667 ft	0.667 ft		
R	29.5 ft	29.5 ft		
ℓ_X	20 ft	-18.92 ft		
ℓ_Y	0	0		
ℓ_Z	-6.1 ft	-10.7 ft		
M_S	114.96 slug-ft	114.96 slug-ft		

(b) Establish the following helicopter operating conditions:

$V_0 = 207$ ft/sec

$\Omega R = 690$ ft/sec

Altitude = sea level standard day
($\rho = 0.002378$ slug/ft^3)

Then compute

$$\mu = \frac{V_0}{\Omega R} = \frac{207}{690} = 0.3$$

$$M_T = \frac{V_0 + \Omega R}{V_S} = \frac{207 + 690}{1118} = 0.8$$

$$T.F. = \rho \pi R^2 (\Omega R)^2 = 0.002378 \times 3.14 \times 29.5^2 \times 690^2$$

$$= 3.09 \times 10^6$$

$$q_0 = \frac{1}{2} \rho V_0^2 = \frac{1}{2} \times 0.002378 \times 207^2 = 50.9 \text{ lb/ft}^2$$

(c) Obtain $C_{L_{FUS}}$ and $C_{D_{FUS}}$ from the appropriate fuselage characteristic charts for $\alpha = \alpha_{FUS} = 0$. For the sample helicopter configuration, assume a slightly positive fuselage angle of attack to achieve the required propulsive force. Thus, assuming $\alpha = \alpha_{FUS} = 1.6°$, enter Figure 2 and obtain

$C_{L_{FUS}} = -0.01$, $C_{D_{FUS}} = 0.35$

Then calculate

$$L_{FUS} = C_{L_{FUS}} q_0 A_{Z_{FUS}} = -0.01 \times 50.9 \times 566.8 = -289 \text{ lb}$$

$$D_{FUS} = C_{D_{FUS}} q_0 A_{X_{FUS}} = 0.35 \times 50.9 \times 121.6 = 2170 \text{ lb}$$

(d) From the Z-force equation, compute $(C_L'/\sigma)_F$ assuming arbitrary values for $(C_L'/\sigma)_R$, thus:

$$\left(\frac{C_L'}{\sigma}\right)_F = \frac{W - L_{FUS} + D_{FUS}\epsilon_{FUS}}{(T.F.)\sigma} - \left(\frac{C_L'}{\sigma}\right)_R$$

$$= \frac{28500 + 289}{3.09 \times 10^6 \times 0.1} - \left(\frac{C_L'}{\sigma}\right)_R$$

$$= 0.0932 - \left(\frac{C_L'}{\sigma}\right)_R$$

The calculated results are shown in Table II below.

TABLE II

LIFT DISTRIBUTIONS ON FRONT AND REAR ROTORS

$\left(\frac{C_L'}{\sigma}\right)_R$	Calculations	$\left(\frac{C_L'}{\sigma}\right)_F$
0.03	0.0932-0.03	0.0632
0.05	0.0932-0.05	0.0432
0.06	0.0932-0.06	0.0332

Using the value of α from step (c) and assuming initially that $\epsilon_F = \epsilon_R = \epsilon_{FUS} = 0$, compute the first approximations for front and rear rotor angles of attack, thus:

$$\alpha_{C_F} = \alpha + (i - B_1 - \epsilon)_F = 1.60° - 9° - 3° = -10.4°$$

$$\alpha_{C_R} = \alpha + (i - B_1 - \epsilon)_R = 1.60° - 4° - 5° = -7.4°$$

(e) From the X-force equation, calculate $(C_D'/\sigma)^*_{TOTAL}$ using the results obtained in steps (c) and (d), thus:

$$\left(\frac{C_D'}{\sigma}\right)^*_{TOTAL} = \left(\frac{C_D'}{\sigma}\right)_F + \left(\frac{C_D'}{\sigma}\right)_R = -\left[\frac{D_{FUS} + L_{FUS}\,\epsilon_{FUS}}{(T.F.)\sigma}\right] - \epsilon_R \left(\frac{C_L'}{\sigma}\right)_R$$

$$= -\frac{2170}{3.09 \times 10^6 \times 0.1} = -0.00702$$

Then compute

$\Delta\sigma = \sigma - 0.1 = 0.1 - 0.1 = 0$

$$\left[(\alpha_C)_{0.1}\right]_F = \left[\alpha_C - \frac{\Delta\sigma}{2\mu^2}\left(\frac{C_L'}{\sigma}\right)\right]_F = -10.4°$$

$$\left[(\alpha_C)_{0.1}\right]_R = \left[\alpha_C - \frac{\Delta\sigma}{2\mu^2}\left(\frac{C_L'}{\sigma}\right)\right]_R = -7.4°$$

(f) Using values of $(C_L'/\sigma)_F$, $[(\alpha_C)_{0.1}]_F$, $(C_L'/\sigma)_R$, and $[(\alpha_C)_{0.1}]_F$ from steps (d) and (e), enter Figure 44 of Reference 1 and obtain the corresponding values of $[(C_D'/\sigma)_{0.1}]_F$ and $[(C_D'/\sigma)_{0.1}]_R$.

Then, for $\Delta\sigma = 0$ (step (e)), obtain

$$\left(\frac{C_D'}{\sigma}\right)_F = \left[\left(\frac{C_D'}{\sigma}\right)_{0.1}\right]_F$$

$$\left(\frac{C_D'}{\sigma}\right)_R = \left[\left(\frac{C_D'}{\sigma}\right)_{0.1}\right]_R$$

and

$$\left(\frac{C_D'}{\sigma}\right)_F + \left(\frac{C_D'}{\sigma}\right)_R = \left(\frac{C_D'}{\sigma}\right)_{TOTAL}$$

The drag distribution between the two rotors and the total drag corresponding to the assumed lift distributions of step (d) are presented in Table III below.

TABLE III

DRAG DISTRIBUTIONS ON FRONT AND REAR ROTORS

$[(\alpha_C)_{0.1}]_F = -10.4°$		$[(\alpha_C)_{0.1}]_R = -7.4°$		$\left(\frac{C_D'}{\sigma}\right)_{TOTAL}$
$\left(\frac{C_L'}{\sigma}\right)_F$	$\left(\frac{C_D'}{\sigma}\right)_F$	$\left(\frac{C_L'}{\sigma}\right)_R$	$\left(\frac{C_D'}{\sigma}\right)_R$	
0.0632	-0.0055	0.03	-0.0018	-0.0073
0.0432	-0.0043	0.05	-0.0025	-0.0068
0.0332	-0.0035	0.06	-0.0025	-0.0060

(g) Using the values from steps (e) and (f), obtain plots of $(C_L'/\sigma)_F$, $(C_D'/\sigma)_F$, $(C_L'/\sigma)_R$, $(C_D'/\sigma)_R$, and $(C_D'/\sigma)^*_{TOTAL}$ versus $(C_D'/\sigma)_{TOTAL}$ as shown in Figure 3. Also draw a straight line of $(C_D'/\sigma)_{TOTAL} = (C_D'/\sigma)_{TOTAL}$. At the point of intersection of this straight line with the horizontal line corresponding to $(C_D'/\sigma)^*_{TOTAL} = (C_D'/\sigma)_{TOTAL}$ read off

$$\left(\frac{C_L'}{\sigma}\right)_F = 0.048, \qquad \left(\frac{C_D'}{\sigma}\right)_F = -0.0047$$

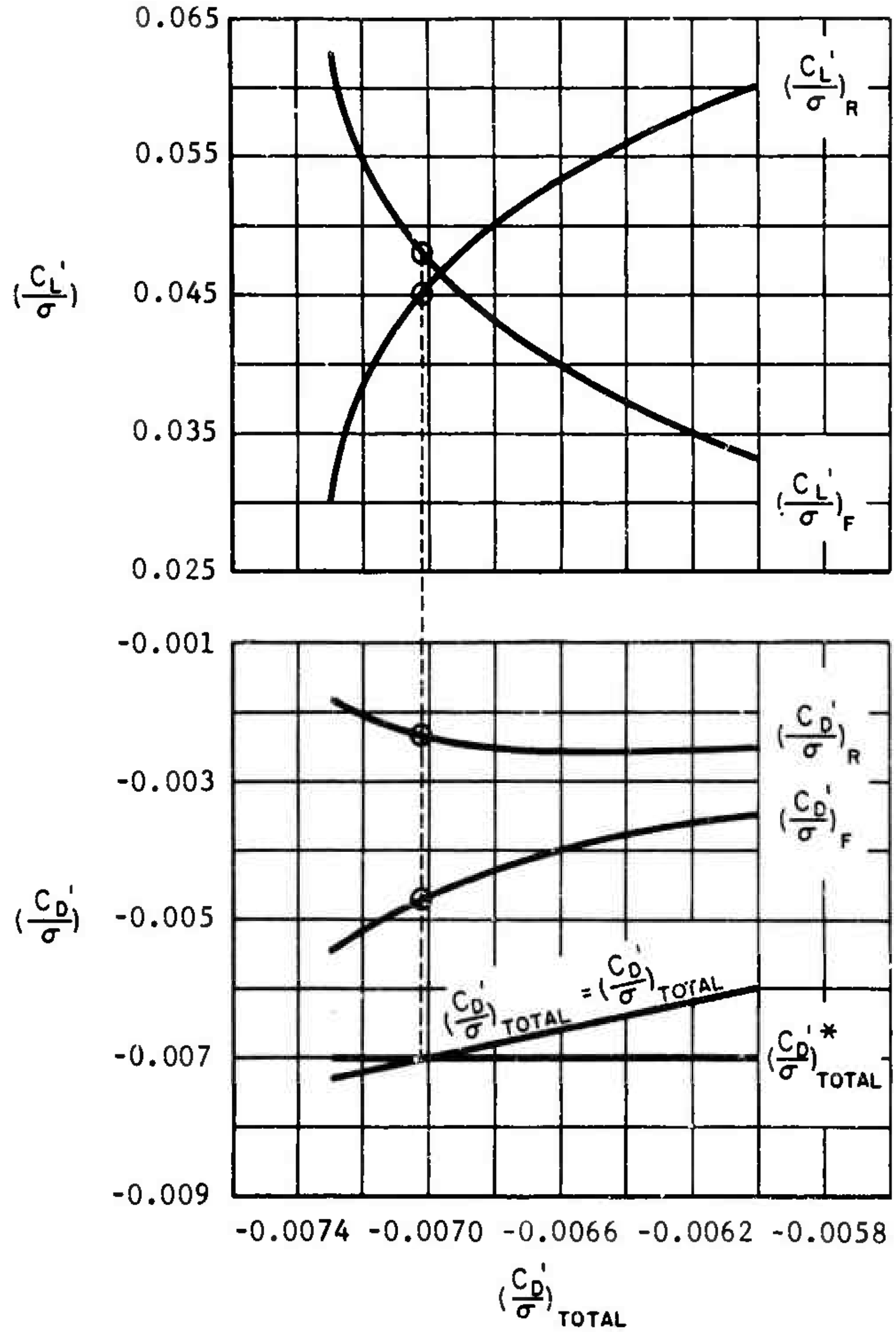


Figure 3. Auxiliary Charts for Obtaining First Approximation for Lift and Drag Distribution on Front and Rear Rotors.

and

$$\left(\frac{C_L'}{\sigma}\right)_R = 0.045, \quad \left(\frac{C_D'}{\sigma}\right)_R = -0.0023$$

Then for $\Delta\sigma = 0$, obtain the chart values for the front and rear rotor drag coefficients, thus:

$$\left[\left(\frac{C_D'}{\sigma}\right)_{0.1}\right]_F = \left(\frac{C_D'}{\sigma}\right)_F = -0.0047$$

$$\left[\left(\frac{C_D'}{\sigma}\right)_{0.1}\right]_R = \left(\frac{C_D'}{\sigma}\right)_R = -0.0023$$

(h) Using the values of $(C_L'/\sigma)_F$, $\left[(C_D'/\sigma)_{0.1}\right]_F$, $(C_L'/\sigma)_R$, $\left[(C_D'/\sigma)_{0.1}\right]_R$ from step (g) and $\left[(\alpha_C)_{0.1}\right]_F$, $\left[(\alpha_C)_{0.1}\right]_R$ from step (e), enter Figure 44a of Reference 1 and Figure 3 of Section 5.3 and obtain

$\lambda_F = -0.061$ $\quad a_{1_F} = 4.2°$

$\lambda_R = -0.045$ $\quad a_{1_R} = 3.5°$

(i) Using Section 7.6 or the pertinent test data, obtain the following downward interference factors:

$K_{RF} = 0$ $\quad K_{FR} = 1.5$

$K_{FFUS} = K_{RFUS} = 1$

Then compute the following interference angles, using α_{C_F} and α_{C_R} from step (d) and λ_F and λ_R from step (h):

$\epsilon_F = 0$

$$\epsilon_R = K_{FR}\left(\tan a_c - \frac{\lambda}{\mu}\right)_F$$

$$= 1.5\left[\tan(-10.4°)+\frac{0.061}{0.3}\right] = 0.0297 \text{ rad} = 1.70°$$

$$\epsilon_{FUS} = K_{FFUS}\left(\tan a_c - \frac{\lambda}{\mu}\right)_F + K_{RFUS}\left(\tan a_c - \frac{\lambda}{\mu}\right)_R$$

$$= \left[\tan(-10.4°)+\frac{0.061}{0.3}\right] + \left[\tan(-7.4°)+\frac{0.045}{0.3}\right]$$

$$= 0.0399 \text{ rad} = 2.29°$$

(j) Compute $C_{M_{FUS}}$ from the pitching moment equation, using the parameters determined above:

$$C_{M_{FUS}} = \frac{(T.F)\sigma}{q_0 A_{X_{FUS}} \ell_{FUS}}\left[(A\sin\alpha - B\cos\alpha) - \frac{(M_{HUB_F} + M_{HUB_R})}{(T.F)\sigma}\right]$$

where

$$A = -\left[\ell_x\left(\frac{C_D'}{\sigma}\right) + \ell_z\left(\frac{C_L'}{\sigma}\right)\right]_F - \left[\ell_x\left(\frac{C_D'}{\sigma}\right) + \ell_z\left(\frac{C_L'}{\sigma}\right)\right]_R$$

$$-\left[\ell_x\left(\frac{C_L'}{\sigma}\right) - \ell_z\left(\frac{C_D'}{\sigma}\right)\right]_F \epsilon_F - \left[\ell_x\left(\frac{C_L'}{\sigma}\right) - \ell_z\left(\frac{C_D'}{\sigma}\right)\right]_R \epsilon_R$$

$$= -\left[20(-0.0047)-6.1(0.048)\right]$$

$$-\left[-18.92(-0.0023)-10.7(0.045)\right]$$

$$-\left[-18.92(0.045)+10.7(-0.0023)\right] \times 0.0297$$

$$= 0.851$$

$$B = \left[\ell_x \left(\frac{C_L'}{\sigma}\right) - \ell_z \left(\frac{C_D'}{\sigma}\right)\right]_F + \left[\ell_x \left(\frac{C_L'}{\sigma}\right) - \ell_z \left(\frac{C_D'}{\sigma}\right)\right]_R$$

$$-\left[\ell_x \left(\frac{C_D'}{\sigma}\right) + \ell_z \left(\frac{C_L'}{\sigma}\right)\right]_F \epsilon_F - \left[\ell_x \left(\frac{C_D'}{\sigma}\right) + \ell_z \left(\frac{C_L'}{\sigma}\right)\right]_R \epsilon_R$$

$$= \left[20(0.048) + 6.1\ (-0.0047)\right]$$

$$+ \left[-18.92(0.045) + 10.7(-0.0023)\right]$$

$$- \left[-18.92(-0.0023) - 10.7(0.045)\right] \times 0.0297$$

$$= 0.0683$$

$$M_{HUB_F} + M_{HUB_R} = \frac{eb\Omega^2 M_S}{2}\ (a_{1_F} + a_{1_R} - B_{1_F} - B_{1_R})$$

$$= \frac{0.667 \times 3 \times 23.4^2 \times 114.96}{2} \left(\frac{4.2° + 3.5° - 3° - 5°}{57.3}\right)$$

$$= -330 \text{ ft-lb}$$

$$\therefore C_{M_{FUS}} = \frac{3.09\times10^6 \times 0.1}{50.9 \times 121.6 \times 51} \Big[0.851 \sin(1.6°) - 0.0683 \cos(1.6°)$$

$$+ \frac{330}{3.09\times10^6 \times 0.1}\Big]$$

$$= -0.0425$$

Also calculate

$$\alpha_{FUS} = \alpha - \epsilon_{FUS} = 1.6^\circ - 2.29^\circ = -0.69^\circ$$

(k) Repeat steps (c) through (j) for two different values of α. The results thus obtained for $\alpha = 1^\circ$, 1.4° and 1.6° are summarized in Table IV below.

TABLE IV

PRELIMINARY TRIM RESULTS FOR THE SAMPLE TANDEM ROTOR HELICOPTER

α	α_{FUS}	$(\frac{C_L'}{\sigma})_F$	$(\frac{C_D'}{\sigma})_F$	$(\frac{C_L'}{\sigma})_R$	$(\frac{C_D'}{\sigma})_R$	ϵ_R	ϵ_{FUS}	$C_{M_{FUS}}$
1.6	-0.69°	0.048	-0.0047	0.045	-0.0023	1.70°	2.29°	-0.0425
1.4	-1.05°	0.037	-0.0042	0.0562	-0.0028	1.39°	2.45°	0.377
1	-1.77°	0.0322	-0.00387	0.0613	-0.0032	1.06°	2.77°	0.564

Plot the values of $(C_L'/\sigma)_F$, $(C_D'/\sigma)_F$, $(C_L'/\sigma)_R$, $(C_D'/\sigma)_R$ ϵ_R, ϵ_{FUS} and $C_{M_{FUS}}$ versus α_{FUS} as shown in Figure 4.

(l) Superimpose the available fuselage data of $C_{M_{FUS}}$ vs. α_{FUS} from Figure 2 on the corresponding plot from step (k), as shown in Figure 5. The point of intersection (P) of the fuselage moment data with the corresponding results computed in step (k) yields the first approximation for the trim values of α_{FUS} and $C_{M_{FUS}}$, thus:

α_{FUS} = -0.72°

$C_{M_{FUS}}$ = 0.008

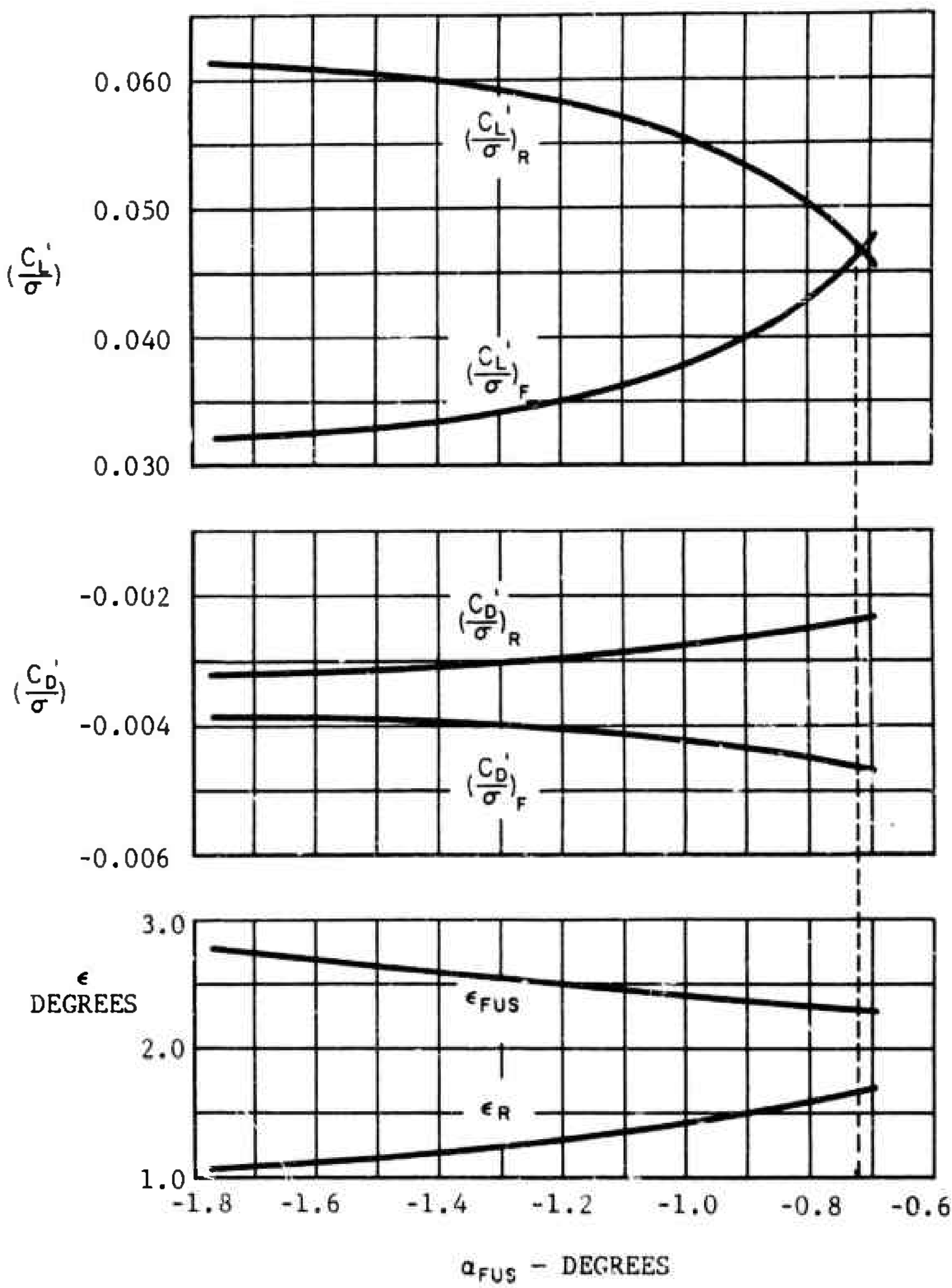


Figure 4. Auxiliary Charts for Obtaining Better Approximation of the Trim Values for Front and Rear Rotors.

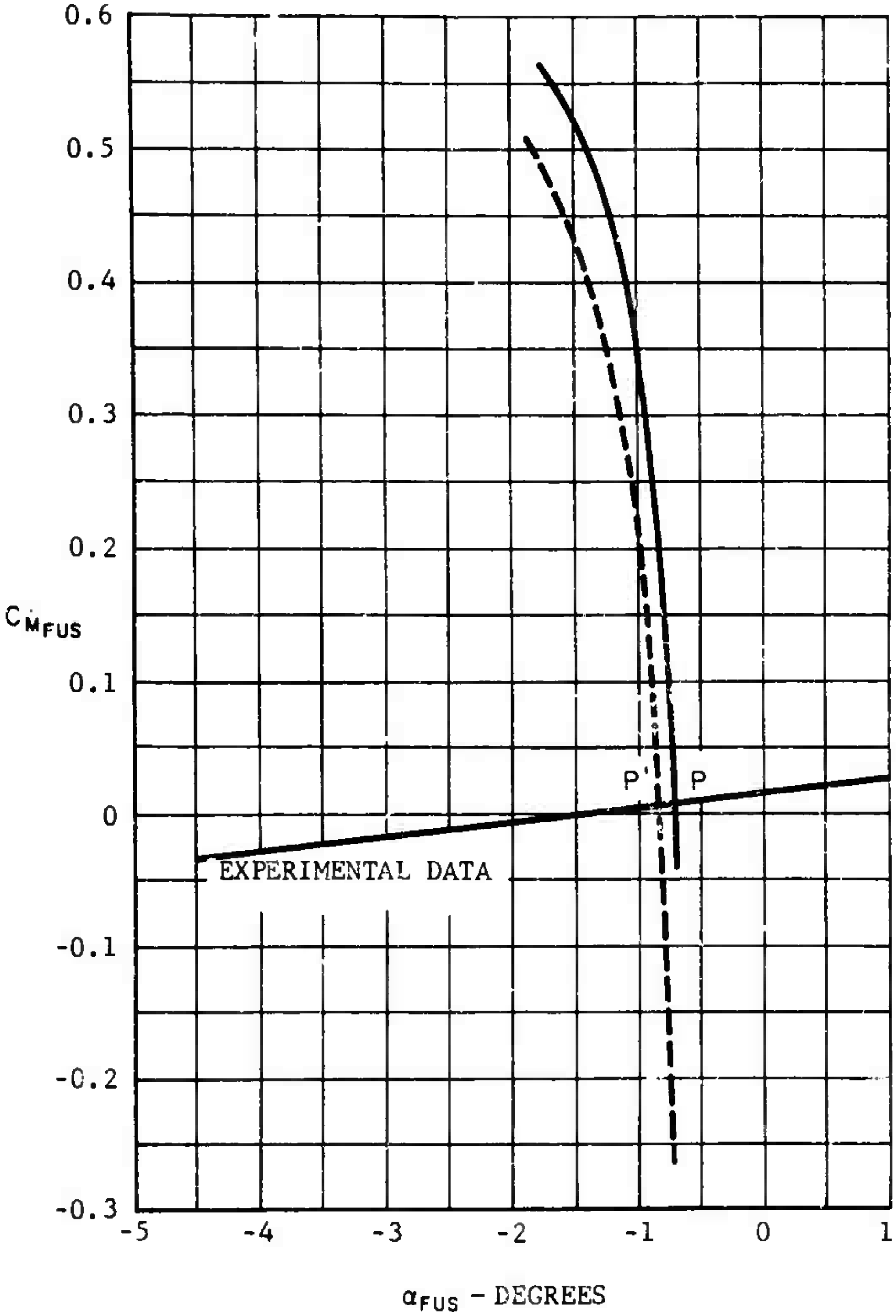


Figure 5. Auxiliary Trim Chart of $C_{M_{FUS}}$ vs. α_{FUS}.

(m) Enter α_{FUS} from step (l) in Figure 4 and read off the first approximations for the following trim values:

$$\left(\frac{C_L'}{\sigma}\right)_F = 0.0462, \left(\frac{C_D'}{\sigma}\right)_F = -0.0047, \epsilon_F = 0$$

$$\left(\frac{C_L'}{\sigma}\right)_R = 0.0468, \left(\frac{C_D'}{\sigma}\right)_R = -0.0023, \epsilon_R = 1.67°$$

$$\epsilon_{FUS} = 2.30°$$

Then calculate

$$\alpha = \alpha_{FUS} + \epsilon_{FUS} = -0.72° + 2.30° = 1.58°$$

$$\alpha_{C_F} = \alpha + (i - B_1 - \epsilon)_F = 1.58° - 9° - 3° = -10.42°$$

$$\alpha_{C_R} = \alpha + (i - B_1 - \epsilon)_R = 1.58° - 4° - 5° - 1.67° = -9.09°$$

(n) Repeat steps (c) through (m) using values of α_{FUS}, ϵ_{FUS}, ϵ_F, and ϵ_R from steps (l) and (m). The results thus obtained are presented in Table V.

TABLE V

INTERMEDIATE TRIM RESULTS

α	α_{FUS}	$(\frac{C_L'}{\sigma})_F$	$(\frac{C_D'}{\sigma})_F$	$(\frac{C_L'}{\sigma})_R$	$(\frac{C_D'}{\sigma})_R$	ϵ_R	ϵ_{FUS}	$C_{M_{FUS}}$
1.8°	-0.73°	0.0583	-0.0051	0.0359	-0.00289	2.30	2.53	-0.263
1.58°	-1.07°	0.041	-0.0044	0.0533	-0.0041	1.67	2.65	0.263
1.30°	-1.53°	0.0365	-0.0042	0.0579	-0.00443	1.52	2.83	0.434
1°	-1.88°	0.0345	-0.0041	0.0601	-0.0046	1.34	2.88	0.510

Plot the results from Table V as shown in Figure 6. Also, superimpose the computed values of $C_{M_{FUS}}$ vs. α_{FUS} from Table V on the plot of Figure 5 and obtain the point of intersection (P'). This point of intersection yields the final fuselage trim values of

$\alpha_{FUS} = -0.85°$

$C_{M_{FUS}} = 0.005$

Entering $\alpha_{FUS} = -0.85°$ into Figures 2 and 6 and utilizing the appropriate rotor performance charts of Reference 1 and Section 5.3, obtain the final trim values for the sample tandem rotor helicopter as shown in Table VI.

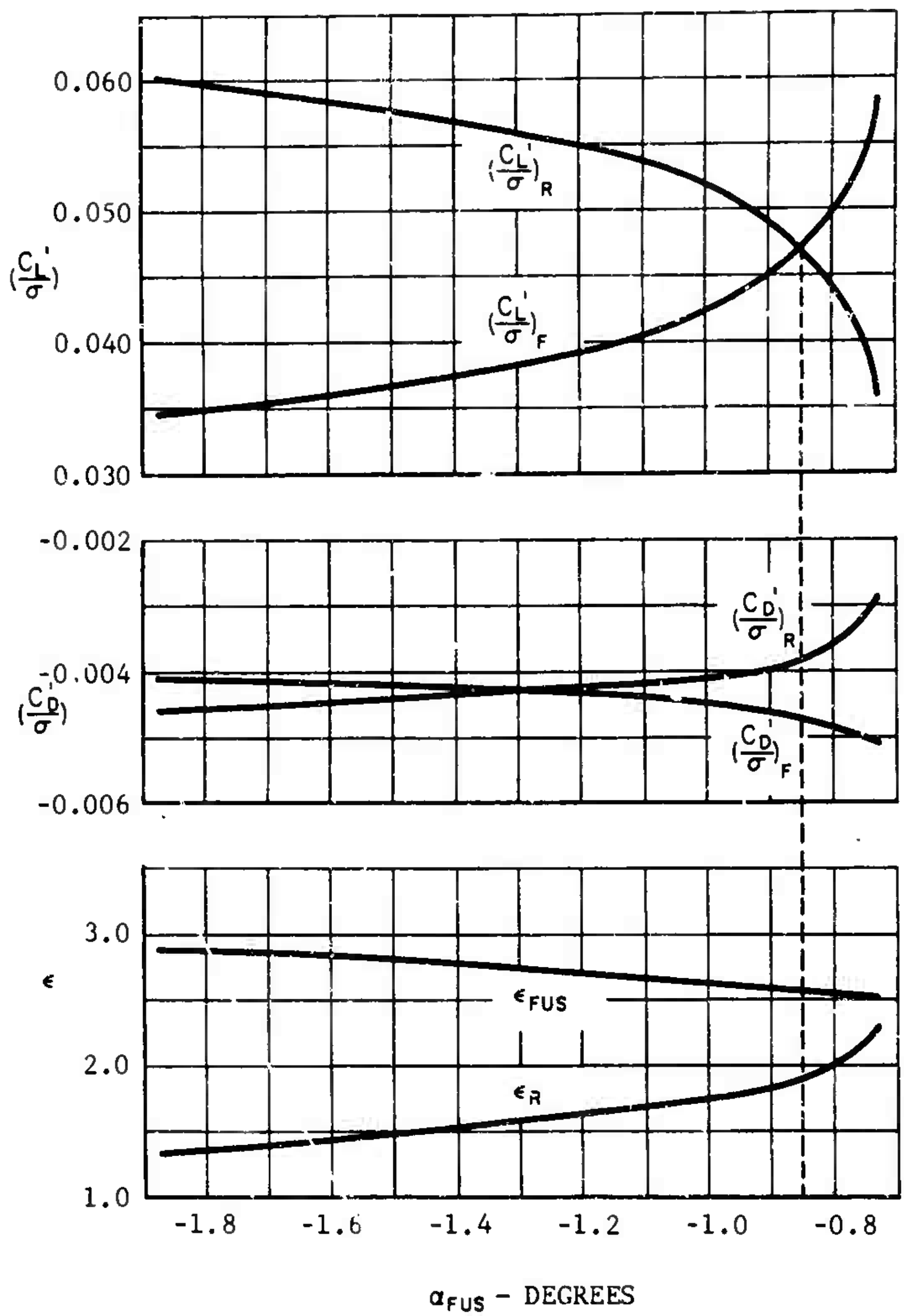


Figure 6. Auxiliary Charts for Obtaining Final Trim Values for Front and Rear Rotors.

TABLE VI

FINAL TRIM VALUES FOR THE SAMPLE
TANDEM ROTOR HELICOPTER

	Rotor		Fuselage	
	Front	Rear		
a_c	-10.29°	-9.19°	α_{FUS}	-0.85°(-0.0148 rad)
$(\frac{C_L'}{\sigma})$	0.0474	0.0469	ϵ_{FUS}	2.56°(0.0447 rad)
$(\frac{C_D'}{\sigma})$	-0.00473	-0.00380	L_{FUS}	-577 lb
$(\frac{C_Q}{\sigma})$	0.0035	0.0031	D_{FUS}	2170 lb
$\theta_{.75}$	7°(0.122 rad)	6.5° (0.113 rad)	Y_{FUS}	0
			M_{FUS}	1580 ft-lb
a_0	2.95° (0.0515 rad)	2.87° (0.0501 rad)	N_{FUS}	1580 ft-lb
a_1	4.2° (0.0733 rad)	4° (0.0698 rad)	$\mathcal{L}_{FUS}$	-1010 ft-lb
b_1	1.2° (0.0209 rad)	1.18° (0.0206 rad)		
λ	-0.061	-0.056		
ϵ	0	1.90° (0.0332 rad)		

$\alpha = \alpha_{FUS} + \epsilon_{FUS} = -0.85° + 2.56° = 1.71° \text{ (0.0298 rad)}$

(o) Calculate rotor side force (C_Y'/σ) for front and rear rotor, thus:

$$\left(\frac{C_Y'}{\sigma}\right)_F = \frac{a}{2}\left[-\frac{3}{4}\mu\theta_{75}a_0 + \frac{1}{3}\theta_{75}b_1 + \frac{3}{8}\mu^2\theta_{75}b_1 + \frac{3}{4}\lambda b_1 + \frac{1}{6}a_0a_1 - \frac{3}{2}\mu\lambda a_0 - \mu^2 a_0 a_1 + \frac{1}{4}\mu a_1 b_1 + \frac{1}{8}\mu^2\lambda b_1\right]_F$$

$$= \frac{5.73}{2}\left[-\frac{3}{4}x0.3x0.122x0.0515+\frac{1}{3}x0.122x0.0209 +\frac{3}{8}x0.09x0.122x0.0209-\frac{3}{4}x0.061x0.0209 +\frac{1}{6}x0.0515x0.0733+\frac{3}{2}x0.3x0.061x0.0515 -0.09x0.0515x0.0733+\frac{1}{4}x0.3x0.0733x0.0209 -\frac{1}{8}x0.09x0.061x0.0209\right]$$

$$= 0.00106$$

$$\left(\frac{C_Y'}{\sigma}\right)_R = \frac{5.73}{2}\left[-\frac{3}{4}x0.3x0.113x0.0501+\frac{1}{3}x0.113x0.0206 +\frac{3}{8}x0.09x0.113x0.0206-\frac{3}{4}x0.056x0.0206 +\frac{1}{6}x0.0501x0.0698+\frac{3}{2}x0.3x0.056x0.0501 -0.09x0.0501x0.0698+\frac{1}{4}x0.3x0.0698x0.0206 -\frac{1}{8}x0.09x0.056x0.0206\right]$$

$$= 0.000977$$

(p) Solve rolling and yawing moment equations of motion and obtain A_{1_F} and A_{1_R}, thus:

$$A_{1_F} = \Big\{ -\ell_{X_R}\left(\frac{C_L'}{\sigma}\right)_R \Big[-\ell_{Z_F}\left(\frac{C_Y'}{\sigma}\right)_F + \ell_{Z_R}\left(\frac{C_Y'}{\sigma}\right)_R + \frac{eb\Omega^2 M_S}{2\sigma(T.F.)}(b_{1_F}+b_{1_R})$$

$$+ \frac{\mathcal{L}_{FUS}}{\sigma(T.F.)}\Big] + \Big[-\ell_{Z_R}\left(\frac{C_L'}{\sigma}\right)_R + \frac{eb\Omega^2 M_S}{2\sigma(T.F.)}\Big]\Big[R\left(\frac{C_Q}{\sigma}\right)_F - R\left(\frac{C_Q}{\sigma}\right)_R + \ell_{X_F}\left(\frac{C_Y'}{\sigma}\right)_F$$

$$-\ell_{X_R}\left(\frac{C_Y'}{\sigma}\right)_R + \frac{N_{FUS}}{\sigma(T.F.)}\Big]\Big\} \Big/ \Big\{ \left(\frac{C_L'}{\sigma}\right)_F \left(\frac{C_L'}{\sigma}\right)_R \Big[\ell_{X_F}\ell_{Z_R} - \ell_{Z_F}\ell_{X_R} \Big]$$

$$- \frac{eb\Omega^2 M_S}{2\sigma(T.F.)}\Big[\ell_{X_F}\left(\frac{C_L'}{\sigma}\right)_F - \ell_{X_R}\left(\frac{C_L'}{\sigma}\right)_R \Big]\Big\}$$

$$= \Big\{ 18.92\text{x}0.0469\Big[6.1\text{x}0.00106 - 10.7\text{x}0.000977$$

$$+ \frac{0.667\text{x}3\text{x}23.4^2\text{x}114.96}{2\text{x}0.1\text{x}3.09\text{x}10^6}(0.0209+0.0206)$$

$$- \frac{1010}{0.1\text{x}3.09\text{x}10^6}\Big] + \Big[10.7\text{x}0.0469$$

$$+ \frac{0.667\text{x}3\text{x}23.4^2\text{x}114.96}{2\text{x}0.1\text{x}3.09\text{x}10^6}\Big]\Big[29.5\text{x}0.0035$$

$$-29.5\text{x}0.0031 + 20 + 0.00106 + 18.92\text{x}0.000977$$

$$+ \frac{1580}{0.1\text{x}3.09\text{x}10^6}\Big]\Big\} \Big/ \Big\{ 0.0474\text{x}0.0469(-20\text{x}10.7$$

$$-6.1\text{x}18.92) - \frac{0.667\text{x}3\text{x}23.4^2\text{x}114.96}{2\text{x}0.1\text{x}3.09\text{x}10^6}\Big[20\text{x}0.0474$$

$$+18.92\text{x}0.0469\Big]\Big\}$$

$$= -0.0371 \text{ rad} = -2.13°$$

$$A_{1_R} = \Big\{-\Big[-\ell_{z_R}\left(\frac{C_L'}{\sigma}\right)_F + \frac{eb\Omega^2 M_S}{2\sigma(\text{T.F.})}\Big]\Big[R\left(\frac{C_Q}{\sigma}\right)_F - R\left(\frac{C_Q}{\sigma}\right)_R + \ell_{x_F}\left(\frac{C_Y'}{\sigma}\right)_F - \ell_{x_R}\left(\frac{C_Y'}{\sigma}\right)_R$$

$$+ \frac{N_{FUS}}{\sigma(\text{T.F.})}\Big] + \ell_{x_F}\left(\frac{C_L'}{\sigma}\right)_F\Big[-\ell_{z_F}\left(\frac{C_Y'}{\sigma}\right)_F + \ell_{z_R}\left(\frac{C_Y'}{\sigma}\right)_R + \frac{eb\Omega^2 M_S}{2\sigma(\text{T.F.})}(b_{1_F} + b_{1_R})$$

$$+ \frac{\mathcal{L}_{FUS}}{\sigma(\text{T.F.})}\Big]\Big\} \Big/ \Big\{\left(\frac{C_L'}{\sigma}\right)_F\left(\frac{C_L'}{\sigma}\right)_R\Big[\ell_{x_F}\ell_{z_R} - \ell_{z_F}\ell_{x_R}\Big]$$

$$- \frac{eb\Omega^2 M_S}{2\sigma(\text{T.F.})}\Big[\ell_{x_F}\left(\frac{C_L'}{\sigma}\right)_F - \ell_{x_R}\left(\frac{C_L'}{\sigma}\right)_R\Big]\Big\}$$

$$= \Big\{-\Big[6.1\text{x}0.0474 + \frac{0.667\text{x}3\text{x}23.4^2\text{x}114.96}{2\text{x}0.1\text{x}3.09\text{x}10^6}\Big]\Big[29.5\text{x}0.0035$$

$$-29.5\text{x}0.0031 + 20\text{x}0.00106 + 18.92\text{x}0.000977$$

$$+ \frac{1580}{3.09\text{x}10^6}\Big] + 20\text{x}0.0474\Big[6.1\text{x}0.00106 - 10.7\text{x}0.000977$$

$$+ \frac{0.667\text{x}3\text{x}23.4^2\text{x}114.96}{2\text{x}0.1\text{x}3.09\text{x}10^6}(0.0209 + 0.0206)$$

$$- \frac{1010}{3.09\text{x}10^6}\Big]\Big\} \Big/ \Big\{0.0474\text{x}0.0469\Big[-20\text{x}10.7$$

$$-6.1\text{x}18.92\Big] - \frac{0.667\text{x}3\text{x}23.4^2\text{x}114.96}{2\text{x}0.1\text{x}3.09\text{x}10^6}\Big[20\text{x}0.0474$$

$$+18.92\text{x}0.0469\Big]\Big\}$$

$$= 0.0242 \text{ rad} = 1.39°$$

(q) Using the Y-Force equation, solve for the aircraft roll altitude, thus:

$$\phi = \frac{(\frac{C_Y'}{\sigma})_R - (\frac{C_Y'}{\sigma})_F - (\frac{C_L'}{\sigma})_F A_{1F} - (\frac{C_L'}{\sigma})_R A_{1R}}{\frac{W}{\sigma (T.F.)}}$$

$$= \frac{0.000977-0.00106-0.0474(-0.0371)-0.0469(0.0242)}{\frac{28500}{0.1x3.09x10^6}}$$

$$= 0.00586 \text{ rad} = 0.336°$$

10.2.2 Stability Derivatives for a Tandem Rotor Helicopter

The stability derivatives for a tandem rotor helicopter are evaluated utilizing the analytical procedure outlined in Section 7.0. These derivatives are computed at the helicopter trim conditions as obtained in Subsection 10.2.1 above. Since the rotor solidity for the sample tandem rotor helicopter considered is $\sigma = 0.1$, no solidity corrections are required for the rotor stability derivatives.

The stability derivatives for the sample tandem rotor helicopter are computed for three degrees of freedom of aircraft longitudinal motion. These derivatives are obtained as follows:

10.2.2.1 Front Rotor Isolated Derivatives

(a) Obtain the required front rotor trim parameters from Subsection 10.2.1.

$\mu = 0.3$ $M_T = 0.8$ $\theta_t = 0°$

$\sigma = 0.1$ $T.F. = 3.09x10^6$ $\Omega R = 690$ ft/sec

$(\frac{C_L'}{\sigma})_F = 0.0474$ $\qquad$ $(\frac{C_D'}{\sigma})_F = -0.00473$

$$L_F = \left[(T.F.)\,\sigma\right]_F (\frac{C_L'}{\sigma})_F = (3.09\text{x}10^6\text{x}0.1)(0.0474)$$

$$= 14600 \text{ lb}$$

$$D_F = \left[(T.F.)\,\sigma\right]_F (\frac{C_D'}{\sigma})_F = (3.09\text{x}10^6\text{x}0.1)(-0.00473)$$

$$= -1460 \text{ lb}$$

$\theta_{.75_F} = 7° = 0.122$ rad $\qquad$ $\lambda_F = -0.061$

$K_{RF} = 0$ $\qquad$ $\epsilon_F = 0$

(b) Using the trim values from step (a), enter isolated rotor derivative charts given in Section 7.5 and read off the following nondimensional isolated rotor derivatives for the front rotor:

(i) μ - Derivatives

$\left[\frac{\partial(\frac{C_L'}{\sigma})}{\partial\mu}\right]_F = -0.132$ $\qquad$ $\left[\frac{\partial(\frac{C_D'}{\sigma})}{\partial\mu}\right]_F = 0.0272$

$(\frac{\partial\lambda}{\partial\mu})_F = -0.137$ $\qquad$ $(\frac{\partial a_1}{\partial\mu})_F = 0.136$

(ii) a_c - Derivatives

$\left[\frac{\partial(\frac{C_L'}{\sigma})}{\partial a_c}\right]_F = 0.368$ $\qquad$ $\left[\frac{\partial(\frac{C_D'}{\sigma})}{\partial a_c}\right]_F = 0.020$

$(\frac{\partial\lambda}{\partial a_c})_F = 0.23$ $\qquad$ $(\frac{\partial a_1}{\partial a_c})_F = 0.20$

(c) Using the equations of Subsection 7.3.1 and the values obtained in steps (a) and (b) above, calculate the following front rotor dimensional derivatives:

(i) u_F - Derivatives

$$\frac{\partial L_F}{\partial u_F} = \left[\frac{(T.F.)\sigma}{\Omega R}\right]_F \left[\frac{\partial(\frac{C_L'}{\sigma})}{\partial\mu}\right]_F$$

$$= \frac{3.09\text{x}10^6\text{x}0.1}{690}(-0.132) = -59.1 \text{ lb-sec/ft}$$

$$\frac{\partial D_F}{\partial u_F} = \left[\frac{(T.F.)\sigma}{\Omega R}\right]_F \left[\frac{\partial(\frac{C_D'}{\sigma})}{\partial\mu}\right]_F$$

$$= \frac{3.09\text{x}10^6\text{x}0.1}{690}(0.0272) = 12.2 \text{ lb-sec/ft}$$

$$\frac{\partial a_{1_F}}{\partial u_F} = \left(\frac{1}{\Omega R}\right)_F \left(\frac{\partial a_1}{\partial\mu}\right)_F$$

$$= \frac{1}{690}(0.136) = 0.000197 \text{ rad-sec/ft}$$

$$\frac{\partial M_{HUB_F}}{\partial u_F} = \left(\frac{eb\Omega^2 M_S}{2}\right)_F \left(\frac{\partial a_{1_F}}{\partial u_F}\right)$$

$$= \frac{0.667\text{x}3\text{x}23.4^2\text{x}114.96}{2}(0.000197)$$

$$= 12.4 \text{ lb-sec}$$

(ii) a_F Derivatives

$$\frac{\partial L_F}{\partial a_F} = \left[(T.F.)\sigma\right]_F \left[\frac{\partial(\frac{C_L'}{\sigma})}{\partial a_C}\right]_F$$

$$= 3.09\text{x}10^6\text{x}0.1\text{x}0.368 = 114000 \text{ lb/rad}$$

$$\frac{\partial D_F}{\partial \alpha_F} = \left[(T.F.)\sigma\right]_F \left[\frac{\partial(\frac{C_D'}{\sigma})}{\partial \alpha_C}\right]_F$$

$$= 3.09\text{x}10^6\text{x}0.1\text{x}0.020 = 6180 \text{ lb/rad}$$

$$\frac{\partial a_{1_F}}{\partial \alpha_F} = \left(\frac{\partial a_1}{\partial \alpha_C}\right)_F = 0.20$$

$$\frac{\partial M_{HUB_F}}{\partial \alpha_F} = \left(\frac{eb\Omega^2 M_S}{2}\right)_F \left(\frac{\partial a_{1_F}}{\partial \alpha_F}\right)$$

$$= \frac{0.667\text{x}3\text{x}23.4^2\text{x}114.96}{2}(0.20)$$

$$= 12600 \text{ ft-lb/rad}$$

(iii) q - Derivatives

$$\frac{\partial a_{1_F}}{\partial q} = \left(\frac{\partial a_1}{\partial q}\right)_F = -\left[\frac{34}{\gamma\Omega(1.883-\mu^2)}\right]_F$$

$$= \frac{-34}{8\text{x}23.4(1.883-0.3^2)} = -0.101 \text{ /sec}$$

(iv) a_{1_F}- Derivatives

$$\frac{\partial L_F}{\partial a_{1_F}} = -D_F = 1460 \text{ lb/rad}$$

$$\frac{\partial D_F}{\partial a_{1_F}} = L_F = 14600 \text{ lb/rad}$$

10.2.2.2 Rear Rotor Isolated Derivatives

(a) Obtain the following rear rotor trim parameters from Subsection 10.2.1:

$\mu = 0.3$ $M_T = 0.8$ $\theta_1 = 0°$

$\sigma = 0.1$ T.F. $= 3.09\text{x}10^6$ $\Omega R = 690$ ft/sec

$(\frac{C_L'}{\sigma})_R = 0.0469$ $(\frac{C_D'}{\sigma})_R = -0.00380$

$$L_R = \left[(\text{T.F.})\sigma\right]_R (\frac{C_L'}{\sigma})_R = (3.09\text{x}10^6\text{x}0.1)(0.0469)$$

$$= 14500 \text{ lb}$$

$$D_R = \left[(\text{T.F.})\sigma\right]_R (\frac{C_D'}{\sigma})_R = (3.09\text{x}10^6\text{x}0.1)(-0.00380)$$

$$= -1170 \text{ lb}$$

$\theta_{.75_R} = 6.5° = 0.113$ rad $\lambda_R = -0.056$

$K_{FR} = 1.5$ $\epsilon_R = 1.90° = 0.0332$ rad

(b) Using the trim values from step (a), enter isolated rotor derivative charts given in Section 7.5 and read off the following nondimensional, isolated rotor derivatives for the rear rotor:

(i) μ - Derivatives

$$\left[\frac{\partial(\frac{C_L'}{\sigma})}{\partial\mu}\right]_R = -0.108 \qquad \left[\frac{\partial(\frac{C_D'}{\sigma})}{\partial\mu}\right]_R = 0.0245$$

$$(\frac{\partial\lambda}{\partial\mu})_R = -0.120 \qquad (\frac{\partial a_1}{\partial\mu})_R = 0.138$$

(ii) α_C - Derivatives

$$\left[\frac{\partial(\frac{C_L'}{\sigma})}{\partial\alpha_C}\right]_R = 0.368 \qquad \left[\frac{\partial(\frac{C_D'}{\sigma})}{\partial\alpha_C}\right]_R = 0.026$$

$$(\frac{\partial\lambda}{\partial\alpha_C})_R = 0.23 \qquad (\frac{\partial a_1}{\partial\alpha_C})_R = 0.196$$

(c) Using the equations of Subsection 7.3.1 and the values obtained in steps (a) and (b) above, compute the following rear rotor dimensional derivatives:

(i) <u>U_R - Derivatives</u>

$$\frac{\partial L_R}{\partial u_R} = -48.4 \text{ lb-sec/ft} \qquad \frac{\partial D_R}{\partial u_R} = 11.0 \text{ lb-sec/ft}$$

$$\frac{\partial a_{1_R}}{\partial u_R} = 0.00020 \text{ rad-sec/ft} \qquad \frac{\partial M_{HUB_R}}{\partial u_R} = 12.6 \text{ lb-sec}$$

(ii) <u>α_R - Derivatives</u>

$$\frac{\partial L_R}{\partial \alpha_R} = 114000 \text{ lb/rad} \qquad \frac{\partial D_R}{\partial \alpha_R} = 8030 \text{ lb/rad}$$

$$\frac{\partial a_{1_R}}{\partial \alpha_R} = 0.196 \qquad \frac{\partial M_{HUB_R}}{\partial \alpha_R} = 12300 \text{ ft-lb/rad}$$

(iii) <u>q - Derivatives</u>

$$\frac{\partial a_{1_R}}{\partial q} = -0.101 \text{ /sec}$$

(iv) <u>a_{1_R} - Derivatives</u>

$$\frac{\partial L_R}{\partial a_{1_R}} = 1170 \text{ lb/rad} \qquad \frac{\partial D_R}{\partial a_{1_R}} = 14500 \text{ lb/rad}$$

<u>10.2.2.3 Fuselage Isolated Derivatives</u>

(a) Obtain the following fuselage trim parameter from Subsection 10.2.1:

$V_o = 207$ ft/sec $\qquad q_o = 50.9$ lb/ft^2

$\alpha = 1.71° = 0.0298$ rad

$\alpha_{FUS} = -0.85° = -0.0148$ rad

$K_{FFUS} = K_{RFUS} = 1.0$

$\epsilon_{FUS} = 2.56° = 0.0447$ rad

$L_{FUS} = -577$ lb $\qquad D_{FUS} = 2170$ lb

$M_{FUS} = 1580$ ft-lb

Also determine fuselage pitching moment of inertia

$I_{YY} = 158041$ slug-ft^2

(b) Using the fuselage trim values from step (a) above, enter fuselage charts given in Figure 2, Subsection 10.2.1 and determine

$$\frac{\partial C_{L_{FUS}}}{\partial \alpha_{FUS}} = 0.239/\text{rad}$$

$$\frac{\partial C_{D_{FUS}}}{\partial \alpha_{FUS}} = 0$$

$$\frac{\partial C_{M_{FUS}}}{\partial \alpha_{FUS}} = 0.614/\text{rad}$$

(c) Using the equations of Subsection 7.3.1 and the values obtained in steps (a) and (b) above, compute the following rear rotor dimensional derivatives:

(i) <u>u_{FUS} - Derivatives</u>

$$\frac{\partial L_{FUS}}{\partial u_{FUS}} = \frac{2}{V_0} L_{FUS} = \frac{2}{207}(-577) = -5.57 \text{ lb-sec/ft}$$

$$\frac{\partial D_{FUS}}{\partial u_{FUS}} = \frac{2}{V_0} D_{FUS} = \frac{2}{207}(2170) = 21.0 \text{ lb-sec/ft}$$

$$\frac{\partial M_{FUS}}{\partial u_{FUS}} = \frac{2}{V_O} M_{FUS} = \frac{2}{207}(1580) = 15.3 \text{ lb-sec}$$

(ii) α_{FUS} - Derivatives

$$\frac{\partial L_{FUS}}{\partial \alpha_{FUS}} = q_O A_{Z_{FUS}} \left(\frac{\partial C_{D_{FUS}}}{\partial \alpha_{FUS}}\right)$$

$$= 50.9 \times 566.8 \times 0.239 = 6900 \text{ lb/rad}$$

$$\frac{\partial D_{FUS}}{\partial \alpha_{FUS}} = q_O A_{X_{FUS}} \left(\frac{\partial C_{D_{FUS}}}{\partial \alpha_{FUS}}\right) = 0$$

$$\frac{\partial M_{FUS}}{\partial \alpha_{FUS}} = q_O A_{X_{FUS}} \ell_{FUS} \left(\frac{\partial C_{M_{FUS}}}{\partial \alpha_{FUS}}\right)$$

$$= 50.9 \times 121.6 \times 51 \times 0.614 = 194000 \text{ ft-lb/rad}$$

10.2.2.4 Total Stability Derivatives

The total stability derivatives for the sample tandem rotor helicopter are evaluated utilizing the analytical procedures outlined in Section 7.1 and the isolated derivatives computed in Subsections 10.2.2.1 through 10.2.2.3. These derivatives for three degrees of freedom of aircraft longitudinal motion are obtained as follows:

(a) X_u

$$X_{u_F} = \frac{\partial L_F}{\partial u_F}(\alpha - \epsilon_F) - \frac{\partial D_F}{\partial u_F}$$

$$= -59.1 \times 0.0298 - 12.2 = -14.0 \text{ lb-sec/ft}$$

$$X_{\alpha_F} = \frac{\partial L_F}{\partial \alpha_F}(\alpha-\epsilon_F) - \frac{\partial D_F}{\partial \alpha_F} + L_F + D_F(\alpha-\epsilon_F)$$

$$= 114000 \times 0.0298 - 6180 + 14600 - 1460 \times 0.0298$$

$$= 11800 \text{ lb/rad}$$

$$\frac{\partial \alpha_F}{\partial u} = -\frac{K_{RF}}{V_0}\left(\frac{\lambda}{\mu} - \frac{\partial \lambda}{\partial \mu}\right)_R = 0$$

$$(X_u)_F = X_{u_F} + X_{\alpha_F}\left(\frac{\partial \alpha_F}{\partial u}\right) = -14.0 \text{ lb-sec/ft}$$

$$X_{u_R} = \frac{\partial L_R}{\partial u_R}(\alpha-\epsilon_R) - \frac{\partial D_R}{\partial u_R}$$

$$= -48.4(0.0298-0.0332)-11.0 = -10.8$$

$$X_{\alpha_R} = \frac{\partial L_R}{\partial \alpha_R}(\alpha-\epsilon_R) - \frac{\partial D_R}{\partial \alpha_R} + L_R + D_R(\alpha-\epsilon_R)$$

$$= 114000(0.0298-0.0332)-8030+14500$$

$$-1170(0.0298-0.0332) = 6090 \text{ lb/rad}$$

$$\frac{\partial \alpha_R}{\partial u} = -\frac{K_{FR}}{V_0}\left(\frac{\lambda}{\mu} - \frac{\partial \lambda}{\partial \mu}\right)_F$$

$$= -\frac{1.5}{207}\left(\frac{-0.061}{0.3}+0.137\right) = 0.000481 \text{ rad-sec/ft}$$

$$(X_u)_R = X_{u_R} + X_{\alpha_R}\left(\frac{\alpha\alpha_R}{\partial u}\right)$$

$$= -10.8 + 6090 \times 0.000481 = -7.87 \text{ lb-sec/ft}$$

$$X_{u_{FUS}} = \frac{\partial L_{FUS}}{\partial u_{FUS}}(\alpha - \epsilon_{FUS}) - \frac{\partial D_{FUS}}{\partial u_{FUS}}$$

$$= -5.57(0.0298-0.0447)-21.0$$

$$= -20.9 \text{ lb-sec/ft}$$

$$X_{\alpha_{FUS}} = \frac{\partial L_{FUS}}{\partial \alpha_{FUS}}(\alpha - \epsilon_{FUS}) - \frac{\partial D_{FUS}}{\partial \alpha_{FUS}} + L_{FUS} + D_{FUS}(\alpha - \epsilon_{FUS})$$

$$= 6900(0.0298-0.0447)-577$$

$$+2170(0.0298-0.0447) = -712 \text{ lb/rad}$$

$$\frac{\partial \alpha_{FUS}}{\partial u} = -\frac{K_{FFUS}}{V_0}\left(\frac{\lambda}{\mu} - \frac{\partial \lambda}{\partial \mu}\right)_F - \frac{K_{RFUS}}{V_0}\left(\frac{\lambda}{\mu} - \frac{\partial \lambda}{\partial \mu}\right)_R$$

$$= -\frac{1}{207}\left(\frac{-0.061}{0.3}+0.137\right) - \frac{1}{207}\left(\frac{-0.056}{0.3}+0.12\right)$$

$$= 0.000643 \text{ rad-sec/ft}$$

$$(X_u)_{FUS} = X_{u_{FUS}} + X_{\alpha_{FUS}}\left(\frac{\partial \alpha_{FUS}}{\partial u}\right)$$

$$= -20.9 - 712 \times 0.000643 = -21.4 \text{ lb-sec/ft}$$

$$\therefore X_u = (X_u)_F + (X_u)_R + (X_u)_{FUS}$$

$$= -14-7.87-21.4 = -43.3 \text{ lb-sec/ft}$$

(b) $X_{\dot{u}}$

$$X_{\dot{u}} = -\frac{W}{g} = -\frac{28500}{32.2} = -885 \text{ slug}$$

(c) X_w

$$X_{w_F} = \frac{1}{V_0} X_{\alpha_F} = \frac{1}{207} \times 11800 = 57.0 \text{ lb-sec/ft}$$

$$\frac{\partial \alpha_F}{\partial \alpha} = 1 - K_{RF}\left[1 - \frac{1}{\mu}\left(\frac{\partial \lambda}{\partial \alpha_C}\right)\right]_R = 1$$

$$(X_w)_F = X_{w_F}\left(\frac{\partial \alpha_F}{\partial \alpha}\right) = 57.0 \text{ lb-sec/ft}$$

$$X_{w_R} = \frac{1}{V_0} X_{\alpha_R} = \frac{1}{207} \times 6090 = 29.4 \text{ lb-sec/ft}$$

$$\frac{\partial \alpha_R}{\partial \alpha} = 1 - K_{FR}\left[1 - \frac{1}{\mu}\left(\frac{\partial \lambda}{\partial \alpha_C}\right)\right]_F$$

$$= 1-1.5\left(1-\frac{1}{0.3} \times 0.23\right) = 0.650$$

$$(X_w)_R = X_{w_R}\left(\frac{\partial \alpha_R}{\partial \alpha}\right) = 29.4 \times 0.65 = 19.1 \text{ lb-sec/ft}$$

$$X_{w_{FUS}} = \frac{1}{V_0} X_{\alpha_{FUS}} = \frac{1}{207} \times (-712) = -3.44$$

$$\frac{\partial \alpha_{FUS}}{\partial \alpha} = 1 - K_{FFUS}\left(1 - \frac{1}{\mu}\frac{\partial \lambda}{\partial \alpha_{C_F}}\right) - K_{RFUS}\left(1 - \frac{1}{\mu}\frac{\partial \lambda}{\partial \alpha_{C_R}}\right)$$

$$= 1-\left(1-\frac{1}{0.3} \times 0.23\right)-\left(1-\frac{1}{0.3} \times 0.23\right) = 0.533$$

$$(X_{\dot{w}})_{FUS} = X_{\dot{w}_{FUS}}\left(\frac{\partial \alpha_{FUS}}{\partial \alpha}\right) = -3.44 \times 0.533 = -1.83 \text{ lb-sec/ft}$$

$$\therefore X_{\dot{w}} = (X_{\dot{w}})_F + (X_{\dot{w}})_R + (X_{\dot{w}})_{FUS}$$

$$= 57.0 + 19.1 - 1.83 = 74.3 \text{ lb-sec/ft}$$

(d) $X_{\dot{w}}$

$$\frac{\partial \alpha_R}{\partial \dot{\alpha}} = -K_{FR}\left(1 - \frac{1}{\mu}\frac{\partial \lambda}{\partial \alpha_{CF}}\right)\left(\frac{\ell_{x_F} - \ell_{x_R}}{V_0}\right)$$

$$= -1.5\left(1 - \frac{1}{0.3} \times 0.23\right)\left(\frac{20+18.92}{207}\right) = -0.0658 \text{ sec}$$

$$(X_{\dot{w}})_R = \frac{1}{V_0} X_{\alpha_R}\left(\frac{\partial \alpha_R}{\partial \dot{\alpha}}\right) = \frac{1}{207} \times 6090(-0.0658)$$

$$= -1.94 \text{ lb-sec}^2\text{/ft}$$

$$\therefore X_{\dot{w}} = (X_{\dot{w}})_R = -1.94 \text{ lb-sec}^2\text{/ft}$$

(e) X_θ

$$X_\theta = -W = -28500 \text{ lb/rad}$$

(f) $X_{\dot{\theta}}$

$$\left(\frac{\partial X}{\partial a_1}\right)_F = \frac{\partial L_F}{\partial a_{1_F}}(\alpha - \epsilon_F) - \frac{\partial D_F}{\partial a_{1_F}}$$

$$= 1460 \times 0.0298 - 14600 = -14600 \text{ lb/rad}$$

$$(X\dot{\theta})_F = X_{u_F}\ell_{z_F} - X_{w_F}\ell_{x_F} + \left(\frac{\partial X}{\partial a_1}\right)_F \frac{\partial a_{1_F}}{\partial q}$$

$$= -14(-6.1)-57.0(20)-14600(-0.101)$$

$$= 420 \text{ lb-sec/rad}$$

$$\left(\frac{\partial X}{\partial a_1}\right)_R = \frac{\partial L_R}{\partial a_{1_R}}(\alpha-\epsilon_R) - \frac{\partial D_R}{\partial a_{1_R}}$$

$$= 1170\text{x}(0.0298-0.0332)-14500=-14500 \text{ lb/rad}$$

$$(X\dot{\theta})_R = X_{u_R}\ell_{z_R} - X_{w_R}\ell_{x_R} + \left(\frac{\partial X}{\partial a_1}\right)_R \frac{\partial a_{1_R}}{\partial q}$$

$$= -10.8(-10.7)-29.4(-18.92)-14500(-0.101)$$

$$= 2140 \text{ lb/rad}$$

$$\therefore X\dot{\theta} = (X\dot{\theta})_F + (X\dot{\theta})_R - \frac{W}{g} V_0 \alpha$$

$$= 420+2140-\frac{28500}{32.2}\text{x}207\text{x}0.0298 = -2900 \text{ lb-sec/rad}$$

(g) Z_u

$$Z_{u_F} = -\left[\frac{\partial D_F}{\partial u_F}(\alpha-\epsilon_F) + \frac{\partial L_F}{\partial u_F}\right]$$

$$= -(12.2\text{x}0.0298-59.1) = 58.7 \text{ lb-sec/ft}$$

$$Z_{\alpha_F} = -\left[\frac{\partial D_F}{\partial \alpha_F}(\alpha-\epsilon_F) + \frac{\partial L_F}{\partial \alpha_F} + D_F - L_F(\alpha-\epsilon_F)\right]$$

$$= -(6180 \text{ x } 0.0298+114000-1460-14600\text{x}0.0298)$$

$$= -112000 \text{ lb/rad}$$

$$(Z_u)_F = Z_{u_F} + Z_{\alpha_F}\left(\frac{\partial \alpha_F}{\partial u}\right) = 58.7 \text{ lb-sec/ft}$$

$$Z_{u_R} = -\left[\frac{\partial D_R}{\partial u_R}(\alpha - \epsilon_R) + \frac{\partial L_R}{\partial u_R}\right]$$

$$= -\left[11.0(0.0298-0.0332)-48.4\right]$$

$$= 48.4 \text{ lb-sec/ft}$$

$$Z_{\alpha_R} = -\left[\frac{\partial D_R}{\partial \alpha_R}(\alpha - \epsilon_R) + \frac{\partial L_R}{\partial \alpha_R} + D_R - L_R(\alpha - \epsilon_R)\right]$$

$$= -\left[8030(0.0298-0.0332)+114000-1170 -14500(0.0298-0.0332)\right] = -113000 \text{ lb/rad}$$

$$(Z_u)_R = Z_{u_R} + Z_{\alpha_R}\left(\frac{\partial \alpha_R}{\partial u}\right)$$

$$= 48.4-113000 \times 0.000481 = -5.95 \text{ lb-sec/ft}$$

$$Z_{u_{FUS}} = -\left[\frac{\partial D_{FUS}}{\partial u_{FUS}}(\alpha - \epsilon_{FUS}) + \frac{\partial L_{FUS}}{\partial u_{FUS}}\right]$$

$$= -\left[21.0(0.0298-0.0447)-5.57\right]$$

$$= 5.88 \text{ lb-sec/ft}$$

$$Z_{\alpha_{FUS}} = -\left[\frac{\partial D_{FUS}}{\partial \alpha_{FUS}}(\alpha - \epsilon_{FUS}) + \frac{\partial L_{FUS}}{\partial \alpha_{FUS}} + D_{FUS} - L_{FUS}(\alpha - \epsilon_{FUS})\right]$$

$$= -\left[6900+2170+577(0.0298-0.0447)\right]$$

$$= -9060 \text{ lb/rad}$$

$$(Z_u)_{FUS} = Z_{u_{FUS}} + Z_{\alpha_{FUS}}\left(\frac{\partial \alpha_{FUS}}{\partial u}\right)$$

$$= 5.88-9060\text{x}0.000643 = 0.0544 \text{ lb-sec/ft}$$

$$\therefore Z_u = (Z_u)_F + (Z_u)_R + (Z_u)_{FUS}$$

$$= 58.7-5.95+0.0544 = 52.8 \text{ lb-sec/ft}$$

(h) Z_w

$$Z_{w_F} = \frac{1}{V_0} Z_{\alpha_F} = \frac{1}{207}\text{x}(-112000) = -541 \text{ lb-sec/ft}$$

$$(Z_w)_F = Z_{w_F}\left(\frac{\partial \alpha_F}{\partial \alpha}\right) = -541 \text{ x } 1 = -541 \text{ lb-sec/ft}$$

$$Z_{w_R} = \frac{1}{V_0} Z_{\alpha_R} = \frac{1}{207}\text{x}(-113000) = -546 \text{ lb-sec/ft}$$

$$(Z_w)_R = Z_{w_R}\left(\frac{\partial \alpha_R}{\partial \alpha}\right) = -546 \text{ x } 0.650 = -355 \text{ lb-sec/ft}$$

$$Z_{w_{FUS}} = \frac{1}{V_0} Z_{\alpha_{FUS}} = \frac{1}{207}\text{x}(-9060) = -43.8 \text{ lb-sec/ft}$$

$$(Z_w)_{FUS} = Z_{w_{FUS}}\left(\frac{\partial \alpha_{FUS}}{\partial \alpha}\right) = -43.8\text{x}0.533 = -23.3 \text{ lb-sec/ft}$$

$$\therefore Z_w = (Z_w)_F + (Z_w)_R + (Z_w)_{FUS}$$

$$= -541-355-23.3 = -919 \text{ lb-sec/ft}$$

(i) $Z_{\dot{w}}$

$$(Z_{\dot{w}})_R = \frac{1}{V_0} Z_{\alpha_R}\left(\frac{\partial \alpha_R}{\partial \dot{\alpha}}\right) = \frac{1}{207}(-113000)(-0.0658)$$

$$= 35.9 \text{ lb-sec}^2\text{/ft}$$

$$\therefore Z_{\dot{w}} = (Z_{\dot{w}})_R - \frac{W}{g} = 35.9-\frac{28500}{32.2} = -849 \text{ lb-sec}^2\text{/ft}$$

(j) Z_θ

$$Z_\theta = -W\theta = -28500 \times 0.0298 = -849 \text{ lb/rad}$$

(k) $Z_{\dot{\theta}}$

$$\left(\frac{\partial Z}{\partial a_1}\right)_F = -\left[\frac{\partial D_F}{\partial a_{1_F}}(\alpha-\epsilon_F)+\frac{\partial L_F}{\partial a_{1_F}}\right] = -(14600\text{x}0.0298+1460)$$

$$= -1900 \text{ lb/rad}$$

$$(Z_{\dot{\theta}})_F = Z_{u_F}\ell_{z_F} - Z_{w_F}\ell_{x_F} + \left(\frac{\partial Z}{\partial a_1}\right)_F \frac{\partial a_{1_F}}{\partial q}$$

$$= 58.7(-6.1)+541\text{x}20+1900\text{x}0.101=10700 \text{ lb-sec/rad}$$

$$\left(\frac{\partial Z}{\partial a_1}\right)_R = -\left[\frac{\partial D_R}{\partial a_{1_R}}(\alpha-\epsilon_R)+\frac{\partial L_R}{\partial a_{1_R}}\right]$$

$$= -\left[14500(0.0298-0.0332)+1170\right]$$

$$= -1120 \text{ lb/rad}$$

$$(Z_{\dot{\theta}})_R = Z_{u_R}\ell_{z_R} - Z_{w_R}\ell_{x_R} + \left(\frac{\partial Z}{\partial a_1}\right)_R \frac{\partial a_{1_R}}{\partial q}$$

$$= 48.4(-10.7)+546(-18.92)+1120\text{x}0.101$$

$$= -10700 \text{ lb-sec/rad}$$

$$\therefore Z_{\dot{\theta}} = (Z_{\dot{\theta}})_F + (Z_{\dot{\theta}})_R + \frac{W}{g} V_a$$

$$= 10700-10700+\frac{28500}{32.2}\text{x}207 = 183000 \text{ lb-sec/rad}$$

(l) M_u

$$\frac{\partial M_{FUS}}{\partial u} = \frac{\partial M_{FUS}}{\partial u_{FUS}} + \frac{\partial M_{FUS}}{\partial \alpha_{FUS}} \frac{\partial \alpha_{FUS}}{\partial u}$$

$$= 15.3+194000\text{x}0.000643 = 140 \text{ lb-sec}$$

$$\frac{\partial M_{HUB_F}}{\partial u} = \frac{\partial M_{HUB_F}}{\partial u_F} + \frac{\partial M_{HUB_F}}{\partial \alpha_F} \frac{\partial \alpha_F}{\partial u}$$

$$= 12.4 \text{ lb-sec}$$

$$\frac{\partial M_{HUB_R}}{\partial u} = \frac{\partial M_{HUB_R}}{\partial u_R} + \frac{\partial M_{HUB_R}}{\partial \alpha_R} \frac{\partial \alpha_R}{\partial u}$$

$$= 12.6+12300\text{x}0.000481 = 18.5 \text{ lb-sec}$$

$$\therefore M_u = (X_u)_F \ell_{Z_F} - (Z_u)_F \ell_{X_F} + (X_u)_R \ell_{Z_R} - (Z_u)_R \ell_{X_R}$$

$$+ \frac{\partial M_{FUS}}{\partial u} + \frac{\partial M_{HUB_F}}{\partial u} + \frac{\partial M_{HUB_R}}{\partial u}$$

$$= -14(-6.1)-58.7\text{x}20-7.87(-10.7)+5.95(-18.92)$$

$$+140+12.4+18.5 = -946 \text{ lb-sec}$$

(m) M_w

$$\frac{\partial M_{FUS}}{\partial \alpha} = \frac{\partial M_{FUS}}{\partial \alpha_{FUS}} \frac{\partial \alpha_{FUS}}{\partial \alpha}$$

$$= 194000\text{x}0.533 = 103000 \text{ ft-lb/rad}$$

$$\frac{\partial M_{HUB_F}}{\partial \alpha} = \frac{\partial M_{HUB_F}}{\partial \alpha_F} \frac{\partial \alpha_F}{\partial \alpha}$$

$$= 12600\text{x}1 = 12600 \text{ ft-lb/rad}$$

$$\frac{\partial M_{HUB_R}}{\partial \alpha} = \frac{\partial M_{HUB_R}}{\partial \alpha_R}\frac{\partial \alpha_R}{\partial \alpha}$$

$$= 12300 \times 0.650 = 8000 \text{ ft-lb/rad}$$

$$\therefore M_w = (X_w)_F \ell_{z_F} - (Z_w)_F \ell_{x_F} + (X_w)_R \ell_{z_R} - (Z_w)_R \ell_{x_R}$$

$$+ \frac{1}{V_0}\left(\frac{\partial M_{FUS}}{\partial \alpha} + \frac{\partial M_{HUB_F}}{\partial \alpha} + \frac{\partial M_{HUB_R}}{\partial \alpha}\right)$$

$$= 57(-6.1)+541\times20+19.1(-10.7)+355(-18.92)$$

$$+\frac{1}{207}(103000+12600+8000) = 4150 \text{ lb-sec}$$

(n) $M_{\dot{w}}$

$$M_{\dot{w}} = (X_{\dot{w}})_R \ell_{z_R} - (Z_{\dot{w}})_R \ell_{x_R}$$

$$= -1.94(-10.7)+35.9\times18.92 = 700 \text{ lb-sec}^2$$

(o) $M_{\dot{\theta}}$

$$\frac{\partial M_{HUB_F}}{\partial q} = \frac{\partial M_{HUB_F}}{\partial u}\ell_{z_F} - \frac{1}{V_0}\frac{\partial M_{HUB_F}}{\partial \alpha}\ell_{x_F} + \frac{eb\Omega^2 M_S}{2}\left(\frac{\partial a_{1_F}}{\partial q}\right)$$

$$= 12.4(-6.1)-\frac{1}{207}\times12600\times20$$

$$+\frac{0.667\times3\times23.4^2\times114.96}{2}(-0.101)$$

$$= -7650 \text{ ft-lb-sec/rad}$$

$$\frac{\partial M_{HUB_R}}{\partial q} = \frac{\partial M_{HUB_R}}{\partial u}\ell_{z_R} - \frac{1}{V_0}\frac{\partial M_{HUB_R}}{\partial \alpha}\ell_{x_R} + \frac{eb\Omega\ M_S}{2}\left(\frac{\partial a_{1_R}}{\partial q}\right)$$

$$= 18.5(-10.7) - \frac{1}{207} \times 8000 \times (-18.92)$$

$$+ \frac{0.667 \times 3 \times 23.4^2 \times 114.96}{2}(-0.101)$$

$$= -5830 \text{ ft-lb-sec/rad}$$

$$\therefore M_{\dot{\theta}} = (X_{\dot{\theta}})_F \ell_{Z_F} - (Z_{\dot{\theta}})_F \ell_{X_F} + (X_{\dot{\theta}})_R \ell_{Z_R} - (Z_{\dot{\theta}})_R \ell_{X_R}$$

$$+ \frac{\partial M_{HUB_F}}{\partial q} + \frac{\partial M_{HUB_R}}{\partial q}$$

$$= 420(-6.1) - 10700 \times 20 + 2140(-10.7) + 10700(-18.92)$$

$$-7650 - 5830 = -455000 \text{ ft-lb-sec/rad}$$

(p) $M_{\ddot{\theta}}$

$$M_{\ddot{\theta}} = -I_{YY} = -158041 \text{ slug-ft}^2$$

The total stability derivatives for the sample tandem rotor helicopter are summarized .n Table VII below.

TABLE VII

SUMMARY OF THE TOTAL STABILITY DERIVATIVES FOR THE SAMPLE TANDEM ROTOR HELICOPTER

Eq. / Var.	X	Z	M
u	-43.3 lb-sec/ft	52.8 lb-sec/ft	-946 lb-sec
$\dot{u}$	-885 slug	0	0
$\ddot{u}$	0	0	0
w	74.3 lb-sec/ft	-919 lb-sec/ft	4150 lb-sec
$\dot{w}$	-1.94 lb-sec^2/ft	-849 lb-sec^2/ft	700 lb-sec^2
$\ddot{w}$	0	0	0
θ	-28500 lb/rad	-849 lb/rad	0
$\dot{\theta}$	-2900 lb-sec/rad	183000 lb-sec/rad	-455000 ft-lb-sec/rad
$\ddot{\theta}$	0	0	-158041 slug-ft^2

10.2.3 Stability Characteristic Equation

The dynamic stability of an aircraft is assessed by examining the coefficients and the roots of the stability characteristic equation. This characteristic equation for the sample tandem rotor helicopter can be obtained by following the analytical procedure outlined in Section 8.0 and by utilizing the aircraft total stability derivatives computed in Subsection 10.2.2.

10.2.3.1 Coefficients of the Characteristic Equation

(a) Utilizing the total aircraft stability derivatives presented in Table VI of Subsection 10.2.2, compute the following terms:

$$G_1 = Z_{\dot{w}} M_{\ddot{\theta}} - M_{\dot{w}} Z_{\ddot{\theta}} = (-849)(-158041) = 134 \times 10^6$$

$$G_2 = Z_w M_{\ddot{\theta}} + Z_{\dot{w}} M_{\dot{\theta}} - M_w Z_{\ddot{\theta}} - M_{\dot{w}} Z_{\dot{\theta}}$$

$$= (-919)(-158041) - 849(-455000) - 700(183000)$$

$$= 403 \times 10^6$$

$$G_3 = Z_{\ddot{\theta}} M_u - Z_u M_{\ddot{\theta}} = -52.8(-158041) = 8.34 \times 10^6$$

$$G_4 = Z_u M_{\dot{w}} - M_u Z_{\dot{w}}$$

$$= 52.8(700) + 946(-849) = -0.766 \times 10^6$$

$$G_5 = Z_w M_{\dot{\theta}} - M_w Z_{\dot{\theta}} - Z_{\theta} M_{\dot{w}}$$

$$= -919(-455000) - 4150(183000) + 849(700) \quad -341 \times 10^6$$

$$G_6 = M_u Z_{\dot{\theta}} - Z_u M_{\dot{\theta}}$$

$$= -946(183000) - 52.8(-455000) = -149 \times 10^6$$

$$G_7 = Z_u M_w - M_u Z_w$$

$$= 52.8(4150) + 946(-919) = -0.65 \times 10^6$$

$$G_8 = M_u X_{\dot{w}} - M_w X_{\dot{u}}$$

$$= -946(-1.94) - 4150(-885) = 3.67 \times 10^6$$

$$G_9 = X_w M_u - X_u M_w$$

$$74.3(-946) + 43.3(4150) = 0.109 \times 10^6$$

(b) Calculate the coefficients of the characteristic equation as follows:

$$A = G_1 X_{\dot{u}} = 134(-885) \times 10^6 = -119 \times 10^9$$

$$B = G_1 X_u + G_2 X_{\dot{u}} + G_3 X_{\dot{w}} + G_4 X_{\ddot{\theta}}$$

$$\left[134(-43.3) + 403(-885) + 8.34(-1.94)\right] \times 10^6$$

$$-362 \times 10^9$$

$$C = G_2 X_u + G_3 X_w + G_4 X_{\dot{\theta}} + G_5 X_{\dot{u}} + G_6 X_{\dot{w}} + G_7 X_{\ddot{\theta}}$$

$$\left[403(-43.3) + 8.34(74.3) - 0.766(-2900) - 341(-885) - 149(-1.94)\right] \times 10^6 = 287 \times 10^9$$

$$D = G_4 X_\theta + G_5 X_u + G_6 X_w + G_7 X_{\dot{\theta}} + G_8 Z_\theta$$

$$\left[-0.766(-28500) - 341(-43.3) - 149(74.3) - 0.65(-2900) + 3.67(-849)\right] \times 10^6 = 24.3 \times 10^9$$

$$E = G_7 X_\theta + G_9 Z_\theta$$

$$= \left[-0.65(-28500) + 0.109(-849)\right] \times 10^6 = 18.4 \times 10^9$$

(c) Divide all the coefficients by the coefficient A; obtain the following stability characteristic equation for the sample tandem rotor helicopter:

$$\Lambda^4 + 3.04\Lambda^3 - 2.41\Lambda^2 - 0.204\Lambda - 0.155 = 0$$

10.2.3.2 Criteria for Stability

As discussed in Section 8.4, the necessary and sufficient conditions for stability are that all the coefficients of the characteristic equation (B, C, D and E) be greater than zero when $A > 0$, and also the Routh discriminant $R^* > 0$.

It can be noted that in the sample case, the normalized coefficients C, D, and E are smaller than zero when A and B are greater than zero. Since there is only one sign change in the normalized coefficients A, B.....E, there will exist at least one positive (unstable) real root.

Therefore, the sample aircraft will possess at least one unstable aperiodic longitudinal mode, regardless of whether the Routh discriminant is positive or negative.

10.2.3.3 Solution of the Characteristic Equation

The solution of the stability characteristic equation (quartic) for the sample tandem rotor helicopter can be obtained utilizing the analytical method of Section 8.5. The calculation procedure of this method is as follows:

(a) Determine the normalized coefficients (A, B.....E) of the characteristic equation

$$\Lambda^4 + B\Lambda^3 + C\Lambda^2 + D\Lambda + E = 0$$

where

$A = 1$, $B = 3.04$, $C = -2.41$, $D = -0.204$, $E = -0.155$

(b) Calculate

$$S^* = BD + C^2 - 4E$$

$$= 3.04(-0.204) + (-2.41)^2 - 4(-0.155) = 5.80794$$

$$R^* = BCD - EB^2 - D^2$$

$$= 3.04(-2.41)(-0.204) + 0.155(3.04)^2 - (-0.204)^2$$

$$= 2.88541$$

(c) Compute

$$h_1 = \frac{1}{3}(3S^* - 4C^2)$$

$$= \frac{1}{3}\left[3(5.80794) - 4(-2.41)^2\right] = -1.93619$$

$$h_2 = \frac{1}{27}(18CS^* - 16C^3 - 27R^*) = \frac{1}{27}\Big[18(-2.41)(5.80794)$$

$$-16(-2.41)^3 - 27(2.88541)\Big] = -3.922$$

(d) Evaluate the discriminant (Δ)

$$\Delta = \frac{h_2^2}{4} + \frac{h_1^3}{27}$$

$$= \frac{(-3.922)^2}{4} + \frac{(-1.93619)^3}{27} = 3.57669$$

(e) Determine the value of (Πn).

Since Δ in step (d) is greater than zero, then

$$\Pi_n = \sqrt[3]{-\frac{h_2}{2} + \sqrt{\Delta}} + \sqrt[3]{-\frac{h_2}{2} - \sqrt{\Delta}}$$

$$= \sqrt[3]{\frac{3.922}{2} + \sqrt{3.57669}} + \sqrt[3]{\frac{3.922}{2} - \sqrt{3.57669}} = 1.97933$$

(f) Compute

$$\zeta\eta = \Pi_n + \frac{2C}{3} \leq \frac{B^2}{4}$$

$$= 1.97933 + \frac{2(-2.41)}{3} = 0.37267$$

(g) Calculate

$$\varsigma = \frac{C-\zeta\eta}{2} + \sqrt{\left(\frac{C-\zeta\eta}{2}\right) - E}$$

$$= \frac{-2.41-0.37267}{2} + \sqrt{\left(\frac{-2.41-0.37267}{2}\right)^2 + 0.155}$$

$$= 0.05462$$

$$\nu = \frac{E}{\varsigma} = \frac{-0.155}{0.05462} = -2.83778$$

$$\eta = \frac{D-B\varsigma}{\nu-\varsigma} = \frac{-0.204-3.04(0.05462)}{-2.83778-0.05462} = 0.12793$$

$$\zeta = B-\eta = 3.04 - 0.12793 = 2.91207$$

(h) Finally, determine the four roots of the stability quartic, thus:

$$\Lambda_{1,2} = -\frac{\zeta}{2} \pm \sqrt{\left(\frac{\zeta}{2}\right)^2 - \nu}$$

$$= -\frac{2.91207}{2} \pm \sqrt{\left(\frac{2.91207}{2}\right)^2 + 2.83778}$$

$$\therefore \Lambda_1 = 0.771 \qquad \Lambda_2 = -3.683$$

and,

$$\Lambda_{3,4} = -\frac{\eta}{2} \pm \sqrt{\left(\frac{\eta}{2}\right)^2 - \varsigma}$$

$$= \frac{-0.12793}{2} \pm \sqrt{\left(\frac{0.12793}{2}\right)^2 - 0.05462}$$

$$= -0.06396 \pm 0.22478\,i$$

$$\therefore \Lambda_3 = -0.0640 + 0.225\,i$$

$$\Lambda_4 = -0.0640 - 0.225\,i$$

10.2.3.4 Roots of the Characteristic Equation

The roots of the characteristic equation computed above consist of one positive real root ($\Lambda_1 = 0.771$), one negative real root ($\Lambda_2 = -3.683$), and a pair of complex roots ($\Lambda_{3,4} = -0.0640 \pm 0.225i$).

The positive real root (Λ_1) corresponds to aperiodic divergence having time to double the amplitude of about 0.90 sec. The negative root (Λ_2) corresponds to aperiodic convergence having time to half the amplitude of about 0.188 sec., which corresponds to a time constant of 0.272 sec. The complex pair ($\Lambda_{3,4}$) corresponds to a slowly convergent oscillation having a period of about 27.9 sec. and a time to half the amplitude of 10.8 sec. This corresponds to a frequency of the oscillation of 0.0358 c.p.s. and a time constant of 18.5 sec.

It can be concluded, therefore, that the sample tandem rotor helicopter is longitudinally unstable.

10.2.4 Aircraft Response

The most convenient way of computing aircraft response due to control inputs or external disturbances is through the use of an analog computer program. Such a program,which is described in detail in Subsection 10.1.4,for a single rotor helicopter can be easily applied to a variety of V/STOL aircraft by utilizing the appropriate total stability derivatives corresponding to the type of aircraft considered.

REFERENCE

1. Tanner, W. H., Charts for Estimating Rotary Wing Performance in Hover and at High Forward Speeds, NASA Contractor Report CR-114, National Aeronautics and Space Administration, Washington, D.C., November 1964.

SECTION 11. THEORETICAL DERIVATION OF EQUATIONS OF MOTION

The object of this section is to provide the reader with a better explanation of theoretical aspects of stability and control concepts and to supply a formal proof of the equations of motions used in Section 4.

11.1 GENERALIZED EQUATIONS OF MOTIONS

An aircraft in flight is subjected to:

(a) Inertia forces and moments
(b) Applied external forces and moments
(c) Generalized control forces

The inertia forces and moments arise from the aircraft's resistance to change its state of a uniform motion when subjected to changes of linear and angular velocities and accelerations in space.

The applied external forces acting upon an aircraft are of two kinds: gravitational and aerodynamic. These forces are generally dependent on the aircraft configuration and the particular flight condition. Since gravity vectors do not contribute to the moments about C.G., then the applied external moments are entirely aerodynamic.

The generalized control forces, on the other hand, are internal forces and are arbitrary functions of the control commands of a human or automatic pilot. For the purpose of the derivation of the generalized equation of motion, it will be assumed in this analysis that all controls are fixed.

11.1.1 The Inertia Forces and Moments

The complete derivation of inertia forces and moments of a rigid body in space can be found in any standard textbook on aircraft dynamics. A typical example of such an analysis is presented on page 94 of Reference 1 and will not be duplicated in this section. However, the summary of the final results of Reference 1 is presented below.

11.1.1.1 The Inertia Forces

$$X_I = \frac{W}{g}(\dot{u} + qw - rv) \tag{1}$$

$$Y_I = \frac{W}{g}(\dot{v} + ru - pw) \tag{2}$$

$$Z_I = \frac{W}{g}(\dot{w} + pv - qu) \tag{3}$$

11.1.1.2 The Inertia Moments

$$\mathcal{L}_I = \dot{p} I_{xx} - I_{xz}(\dot{r} + pq) - rq(I_{yy} - I_{zz})$$

$$- I_{xy}(\dot{q} - rp) - I_{yz}(q^2 - r^2) \tag{4}$$

$$M_I = \dot{q} I_{yy} - I_{xz}(r^2 - p^2) + rp(I_{xx} - I_{zz})$$

$$- I_{xy}(\dot{p} + rq) - I_{yz}(\dot{r} - pq) \tag{5}$$

$$N_I = \dot{r} I_{zz} - I_{xz}(\dot{p} - qr) + pq(I_{yy} - I_{xx})$$

$$- I_{xy}(p^2 - q^2) - I_{yz}(\dot{q} + pr) \tag{6}$$

11.1.2 The Applied External Forces and Moments

The resultant motion of an aircraft (free body in space) can be described by superposition of perturbation motions about each axis. In doing so, it is essential to maintain a right-handed (Maxwell corkscrew rule) system

of axes. The initial disturbance can be applied about any axis so long as the proper cyclic order of perturbation in positive directions is maintained.

For example, the initial disturbance and the cyclic order of perturbation can be considered as follows:

Perturbation about X-axis, Y-axis, Z-axis
or Y-axis, Z-axis, X-axis
or Z-axis, X-axis, Y-axis

Once an initial disturbance and a certain cyclic order of perturbation are selected, they must be maintained throughout the analysis.

The following cyclic order of perturbation is adopted herein:

(a) Rotate Y-Z axes about X-axis through an angle X_x ,

(b) Rotate X'-Z' axes about the perturbed Y-axis, Y', through an angle X_y,

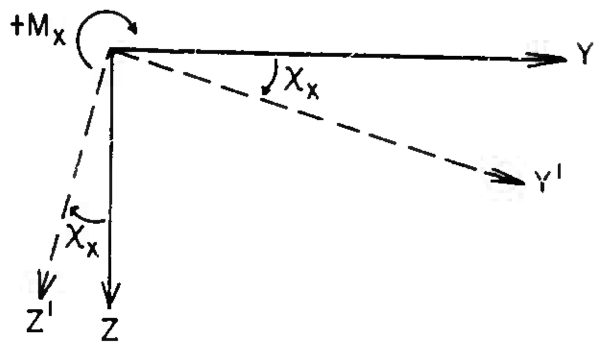
(c) Finally, rotate X"-Y" axes about the perturbed Z-axis, Z", through an angle X_z ,

where ()', ()", etc., represent forces along appropriate axes after first, second, or higher order cyclic rotations; and X_x , X_y , X_z pertain to perturbation angles of the force vectors about X, Y, and Z-axes.

11.1.2.1 The Aerodynamic and Gravitational Forces

All forces are positive if they act in the positive direction of body axes system as described in Section 3.

(a) Rotation about X-Axis

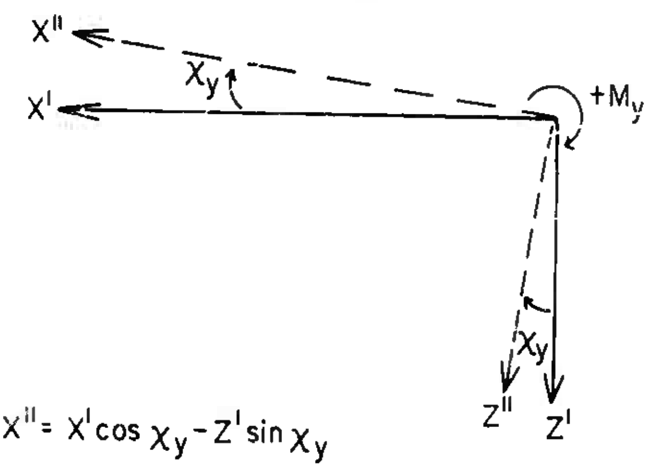


$$X' = X \tag{7}$$

$$Y' = Y\cos\chi_x + Z\sin\chi_x \tag{8}$$

$$Z' = Z\cos\chi_x - Y\sin\chi_x \tag{9}$$

(b) Rotation about Y'-Axis

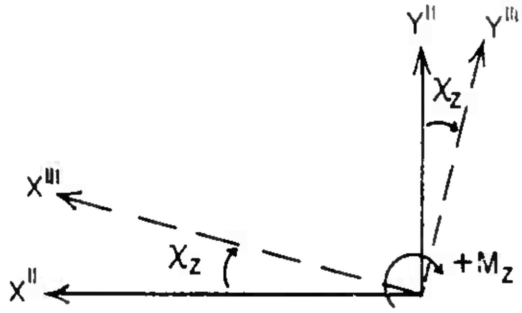


$$X'' = X'\cos\chi_y - Z'\sin\chi_y \tag{10}$$

$$Y'' = Y' \tag{11}$$

$$Z'' = Z'\cos\chi_y + X'\sin\chi_y \tag{12}$$

(c) Rotation about Z''-Axis



$$X''' = X'' \cos\chi_z + Y''\sin\chi_z \tag{13}$$

$$Y''' = Y''\cos\chi_z - X''\sin\chi_z \tag{14}$$

$$Z''' = Z'' \tag{15}$$

Substituting equations (10) through (12) into equations (13) through (15) yields

$$X''' = (X'\cos\chi_y - Z'\sin\chi_y)\cos\chi_z + Y'\sin\chi_z \tag{16}$$

$$Y''' = Y'\cos\chi_z - (X'\cos\chi_y - Z'\sin\chi_y)\sin\chi_z \tag{17}$$

$$Z''' = Z'\cos\chi_y + X'\sin\chi_y \tag{18}$$

Expanding equations (16) through (18), there follows

$$X''' = X'\cos\chi_y\cos\chi_z + Y'\sin\chi_z - Z'\sin\chi_y\cos\chi_z \tag{19}$$

$$Y''' = -X'\cos\chi_y \sin\chi_z + Y'\cos\chi_z + Z'\sin\chi_y \sin\chi_z \qquad (20)$$

$$Z''' = X'\sin\chi_y + Z'\cos\chi_y \qquad (21)$$

Substituting equations (7) through (9) into equations (19) through (21) yields

$$X''' = X\cos\chi_y \cos\chi_z + (Y\cos\chi_x + Z\sin\chi_x)\sin\chi_z$$

$$-(Z\cos\chi_x - Y\sin\chi_x)\sin\chi_y \cos\chi_z \qquad (22)$$

$$Y''' = -X\cos\chi_y \sin\chi_z + (Y\cos\chi_x + Z\sin\chi_x)\cos\chi_z$$

$$+(Z\cos\chi_y - Y\sin\chi_x)\sin\chi_y \sin\chi_z \qquad (23)$$

$$Z''' = X\sin\chi_y + (Z\cos\chi_x - Y\sin\chi_x)\cos\chi_y \qquad (24)$$

Expanding equations (22) through (24) and combining similar terms, the generalized force equations of disturbed motions are

$$X''' = X\cos\chi_y \cos\chi_z + Y(\cos\chi_x \sin\chi_z + \sin\chi_x \sin\chi_y \cos\chi_z)$$

$$+Z(\sin\chi_x \sin\chi_z - \cos\chi_x \sin\chi_y \cos\chi_z) \qquad (25)$$

$$Y''' = -X\cos\chi_y \sin\chi_z + Y(\cos\chi_x \cos\chi_z - \sin\chi_x \sin\chi_y \sin\chi_z)$$

$$+Z(\sin\chi_x \cos\chi_z + \cos\chi_x \sin\chi_y \sin\chi_z) \qquad (26)$$

$$Z''' = X\sin\chi_y - Y\sin\chi_x\cos\chi_y + Z\cos\chi_x\cos\chi_y \quad (27)$$

The final force equations, equations (25), (26), and (27), are now expressed as functions of undisturbed forces X, Y, and Z and the perturbation angles χ_x, χ_y, and χ_z related to the appropriate body system of axes.

Assuming a generalized aircraft under consideration to consist of single rotor (or front rotor of tandem rotor configuration), rear rotor (of tandem rotor configuration), fuselage, horizontal tail plane, vertical tail, tail rotor, wings, propellers or jet engines, the undisturbed force equations along the body axes are

$$X = -D_F - D_R - D_{FUS} - D_W - D_T - D_{VT} - D_{TR}$$

$$+\sum_{i=1}^{n} T_{P_i}\cos i_{P_i} \quad (28)$$

$$Y = Y_F - Y_R + Y_{FUS} - L_{VT} + \sum_{i=1}^{n} Y_{P_i} + T_{TR} \quad (29)$$

$$Z = W - L_F - L_R - L_{FUS} - L_W - L_T - Y_{TR}$$

$$-\sum_{i=1}^{n} (N_{P_i}\cos i_{P_i} + T_{P_i}\sin i_{P_i}) \quad (30)$$

The perturbation angles about the body X, Y and Z axes, relative to gravitational or aerodynamic forces, are easily identified as follows:

$\chi_x \equiv \phi$ - for gravitational force vectors,

$\chi_y \equiv -A_{1_F}, -A_{1_R}$ - for aerodynamic force vectors,

$\chi_y \equiv \theta$ - for gravitational force vectors,

$\chi_y \equiv (\alpha - \epsilon_i)$ - for aerodynamic force vectors,

$\chi_z \equiv \psi$ - for gravitational force vectors,

and

$\chi_z \equiv -\beta_s$ - for aerodynamic force vectors.

Substituting equations (28), (29), and (30), together with the appropriate perturbation angles χ_x, χ_y and χ_z, into equation (25) yields

$$
\begin{aligned}
X''' = & - D_F \cos(\alpha - \epsilon_F)\cos\beta_S - D_R \cos(\alpha - \epsilon_R)\cos\beta_S \\
& - D_{FUS}\cos(\alpha - \epsilon_{FUS})\cos\beta_S - D_W \cos(\alpha - \epsilon_W)\cos\beta_S \\
& - D_T \cos(\alpha - \epsilon_T)\cos\beta_S - D_{TR}\cos(\alpha - \epsilon_{TR})\cos\beta_S \\
& - D_{VT}\cos(\alpha - \epsilon_{VT})\cos\beta_S + \sum_{i=1}^{n} (T_{P_i}\cos i_{P_i} - N_{P_i}\sin i_{P_i}) \\
& - Y_F \cos A_{1_F}\sin\beta_S + Y_R \cos A_{1_R}\sin\beta_S \\
& - Y_{FUS}\sin\beta_S - T_{TR}\sin\beta_S + L_{VT}\sin\beta_S
\end{aligned}
$$

$$-Y_F \sin A_{I_F} \sin(\alpha - \epsilon_F)\cos\beta_S + Y_R \sin A_{I_R} \sin(\alpha - \epsilon_R)\cos\beta_S$$

$$+W \sin\phi \sin\psi - L_F \sin A_{I_F} \sin\beta_S - L_R \sin A_{I_R} \sin\beta_S$$

$$-W\cos\phi \sin\theta \cos\psi + L_F \cos A_{I_F} \sin(\alpha - \epsilon_F)\cos\beta_S$$

$$+L_R \cos A_{I_R} \sin(\alpha - \epsilon_R)\cos\beta_S + L_{FUS} \sin(\alpha - \epsilon_{FUS})\cos\beta_S$$

$$+L_W \sin(\alpha - \epsilon_W)\cos\beta_S + L_T \sin(\alpha - \epsilon_T)\cos\beta_S$$

$$+Y_{TR} \sin(\alpha - \epsilon_{TR})\cos\beta_S \qquad (31)$$

Simplifying equation (31) and dropping $(\ )'''$ sign, the final equation of motion of aerodynamic and gravitational forces acting along body X-axis is

$$X_{A+G} = \left[-D_F \cos(\alpha - \epsilon_F) + (L_F \cos A_{I_F} - Y_F \sin A_{I_F})\sin(\alpha - \epsilon_F)\right]\cos\beta_S$$

$$-(L_F \sin A_{I_F} + Y_F \cos A_{I_F})\sin\beta_S$$

$$+\left[-D_R \cos(\alpha - \epsilon_R) + (L_R \cos A_{I_R} + Y_R \sin A_{I_R})\sin(\alpha - \epsilon_R)\right]\cos\beta_S$$

$$-(L_R \sin A_{I_R} - Y_R \cos A_{I_R})\sin\beta_S$$

$$+\left[L_{FUS} \sin(\alpha - \epsilon_{FUS}) - D_{FUS}\cos(\alpha - \epsilon_{FUS})\right]\cos\beta_S - Y_{FUS}\sin\beta_S$$

$$+\left[L_W \sin(\alpha - \epsilon_W) - D_W \cos(\alpha - \epsilon_W)\right]\cos\beta_S$$

$$+\left[L_T \sin(\alpha-\epsilon_T)-D_T\cos(\alpha-\epsilon_T)\right]\cos\beta_S$$

$$+\left[Y_{TR}\sin(\alpha-\epsilon_{TR})-D_{TR}\cos(\alpha-\epsilon_{TR})\right]\cos\beta_S - T_{TR}\sin\beta_S$$

$$-D_{VT}\cos(\alpha-\epsilon_{VT})\cos\beta_S + L_{VT}\sin\beta_S$$

$$+\sum_{i=1}^{n}(T_{P_i}\sin i_{P_i} - N_{P_i}\sin i_{P_i})$$

$$+W\sin\phi\sin\psi - W\cos\phi\sin\theta\cos\psi \qquad (32)$$

Similarly, substituting equations (28), (29), and (30), together with the appropriate perturbation angles, into equation (26) yields

$$Y''' = -D_F\cos(\alpha-\epsilon_F)\sin\beta_S - D_R\cos(\alpha-\epsilon_R)\sin\beta_S$$

$$-D_{FUS}\cos(\alpha-\epsilon_{FUS})\sin\beta_S - D_W\cos(\alpha-\epsilon_W)\sin\beta_S$$

$$-D_T\cos(\alpha-\epsilon_T)\sin\beta_S - D_{TR}\cos(\alpha-\epsilon_{TR})\sin\beta_S$$

$$-D_{VT}\cos(\alpha-\epsilon_{VT})\sin\beta_S + Y_F\cos A_{I_F}\cos\beta_S$$

$$-Y_R\cos A_{I_R}\cos\beta_S + Y_{FUS}\cos\beta_S + T_{TR}\cos\beta_S$$

$$+\sum_{i=1}^{n} Y_{P_i} - L_{VT}\cos\beta_S - Y_F\sin A_{I_F}\sin(\alpha-\epsilon_F)\sin\beta_S$$

$$+Y_R\sin A_{I_R}\sin(\alpha-\epsilon_R)\sin\beta_S + W\sin\phi\cos\psi$$

$$+L_F \sin A_{I_F} \cos\beta_S + L_R \sin A_{I_R} \cos\beta_S$$

$$+W\cos\phi \sin\theta \sin\psi + L_F \cos A_{I_F} \sin(\alpha - \epsilon_F) \sin\beta_S$$

$$+L_R \cos A_{I_R} \sin(\alpha - \epsilon_R) \sin\beta_S + L_{FUS} \sin(\alpha - \epsilon_{FUS}) \sin\beta_S$$

$$+L_W \sin(\alpha - \epsilon_W) \sin\beta_S + L_T \sin(\alpha - \epsilon_T) \sin\beta_S$$

$$+Y_{TR} \sin(\alpha - \epsilon_{TR}) \sin\beta_S \qquad (33)$$

Simplifying equation (33) and dropping $(\ )'''$ sign, the final equation of motion of aerodynamic and gravitational forces acting along body Y-axis is

$$Y_{A+G} = \left[-D_F \cos(\alpha - \epsilon_F) + (L_F \cos A_{I_F} - Y_F \sin A_{I_F}) \sin(\alpha - \epsilon_F)\right] \sin\beta_S$$

$$+(L_F \sin A_{I_F} + Y_F \cos A_{I_F}) \cos\beta_S$$

$$+\left[-D_R \cos(\alpha - \epsilon_R) + (L_R \cos A_{I_R} + Y_R \sin A_{I_R}) \sin(\alpha - \epsilon_R)\right] \sin\beta_S$$

$$+(L_R \sin A_{I_R} - Y_R \cos A_{I_R}) \cos\beta_S$$

$$+\left[L_{FUS} \sin(\alpha - \epsilon_{FUS}) - D_{FUS} \cos(\alpha - \epsilon_{FUS})\right] \sin\beta_S + Y_{FUS} \cos\beta_S$$

$$+\left[L_W \sin(\alpha - \epsilon_W) - D_W \cos(\alpha - \epsilon_W)\right] \sin\beta_S$$

$$+\left[L_T \sin(\alpha - \epsilon_T) - D_T \cos(\alpha - \epsilon_T)\right] \sin\beta_S$$

$$+\left[Y_{TR}\sin(\alpha-\epsilon_{TR})-D_{TR}\cos(\alpha-\epsilon_{TR})\right]\sin\beta_S$$

$$+T_{TR}\cos\beta_S-D_{VT}\cos(\alpha-\epsilon_{VT})\sin\beta_S-L_{VT}\cos\beta_S+\sum_{i=1}^{n}Y_{P_i}$$

$$+W\sin\phi\cos\psi+W\cos\phi\sin\theta\sin\psi \qquad (34)$$

Finally, substituting equations (28), (29), and (30), together with the appropriate perturbation angles, into equation (27) yields

$$Z''' = -D_F\sin(\alpha-\epsilon_F)-D_R\sin(\alpha-\epsilon_R)-D_{FUS}\sin(\alpha-\epsilon_{FUS})$$

$$-D_W\sin(\alpha-\epsilon_W)-D_T\sin(\alpha-\epsilon_T)-D_{TR}\sin(\alpha-\epsilon_{TR})$$

$$-D_{VT}\sin(\alpha-\epsilon_{VT})+Y_F\sin A_{1_F}\cos(\alpha-\epsilon_F)$$

$$-Y_R\sin A_{1_R}\cos(\alpha-\epsilon_R)+W\cos\phi\cos\theta$$

$$-L_F\cos A_{1_F}\cos(\alpha-\epsilon_F)-L_R\cos A_{1_R}\cos(\alpha-\epsilon_R)$$

$$-L_{FUS}\cos(\alpha-\epsilon_{FUS})-L_W\cos(\alpha-\epsilon_W)$$

$$-L_T\cos(\alpha-\epsilon_T)-Y_{TR}\cos(\alpha-\epsilon_{TR})$$

$$-\sum_{i=1}^{n}(N_{P_i}\cos i_{P_i}+T_{P_i}\sin i_{P_i}) \qquad (35)$$

Simplifying equation (35) and eliminating $(\ )'''$ sign, the final equation of motion of aerodynamic and gravitational forces acting along body Z-axis is

$$
\begin{aligned}
Z_{A+G} = & -\left[D_F \sin(\alpha-\epsilon_F) + (L_F \cos A_{I_F} - Y_F \sin A_{I_F}) \cos(\alpha-\epsilon_F)\right] \\
& -\left[D_R \sin(\alpha-\epsilon_R) + (L_R \cos A_{I_R} + Y_R \sin A_{I_R}) \cos(\alpha-\epsilon_R)\right] \\
& -\left[L_{FUS} \cos(\alpha-\epsilon_{FUS}) + D_{FUS} \sin(\alpha-\epsilon_{FUS})\right] \\
& -\left[L_W \cos(\alpha-\epsilon_W) + D_W \sin(\alpha-\epsilon_W)\right] \\
& -\left[L_T \cos(\alpha-\epsilon_T) + D_T \sin(\alpha-\epsilon_T)\right] \\
& -\left[Y_{TR} \cos(\alpha-\epsilon_{TR}) + D_{TR} \sin(\alpha-\epsilon_{TR})\right] - D_{VT} \sin(\alpha-\epsilon_{VT}) \\
& -\sum_{i=1}^{n} (N_{P_i} \cos i_{P_i} + T_{P_i} \sin i_{P_i}) + W \cos\phi \cos\theta
\end{aligned}
\qquad (36)
$$

11.1.2.2 The Aerodynamic Moments

Aerodynamic moments are generated by aerodynamic torques as well as by the aerodynamic forces acting on the various aircraft components.

The aerodynamic moments due to aerodynamic forces are expressed in terms of the displacement vector of the force from the aircraft C.G. and the force itself as follows:

$$\vec{M}_i = \vec{r}_i \times \vec{F}_i \qquad (37)$$

where

$$\vec{r}_i = \ell_{X_i} \vec{i} + \ell_{Y_i} \vec{j} + \ell_{Z_i} \vec{k}$$

$$\vec{F}_i = X_i \vec{i} + Y_i \vec{j} + Z_i \vec{k}$$

and $\vec{i}, \vec{j}, \vec{k}$ are unit vectors along the X, Y, Z axes located at aircraft C.G.

The suffix i pertains to the i^{th} aerodynamic component.

Performing vector multiplication, equation (37) can be expressed as follows:

$$\vec{M}_i = \mathcal{L}_i \vec{i} + M_i \vec{j} + N_i \vec{k} \tag{38}$$

where

$$\mathcal{L}_i = Z_i \ell_{Y_i} - Y_i \ell_{Z_i}$$

$$M_i = X_i \ell_{Z_i} - Z_i \ell_{X_i}$$

$$N_i = Y_i \ell_{X_i} - X_i \ell_{Y_i}$$

The aerodynamic moments due to aerodynamic torques for a generalized aircraft configuration are

$$\mathcal{L}_{O_i} = \mathcal{L}_{FUS} + \mathcal{L}_{HUB_F} - \mathcal{L}_{HUB_R} + \sum_{i=1}^{n} Q_{P_i} \tag{39}$$

$$M_{O_i} = M_{FUS} + M_{HUB_F} + M_{HUB_R} + Q_{TR} \tag{40}$$

$$N_{O_i} = N_{FUS} + Q_F - Q_R \tag{41}$$

Adding all the rolling moments due to aerodynamic forces and torques, there results

$$
\begin{aligned}
\mathcal{L}_A = -\ell_{Z_F}\Big\{&\Big[-D_F\cos(\alpha-\epsilon_F)+(L_F\cos A_{1_F}-Y_F\sin A_{1_F})\sin(\alpha-\epsilon_F)\Big]\sin\beta_S \\
&+(L_F\sin A_{1_F}+Y_F\cos A_{1_F})\cos\beta_S\Big\} \\
-\ell_{Z_R}\Big\{&\Big[-D_R\cos(\alpha-\epsilon_R)+(L_R\cos A_{1_R}+Y_R\sin A_{1_R})\sin(\alpha-\epsilon_R)\Big]\sin\beta_S \\
&+(L_R\sin A_{1_R}-Y_R\cos A_{1_R})\cos\beta_S\Big\} \\
-\ell_{Z_W}&\Big[L_W\sin(\alpha-\epsilon_W)-D_W\cos(\alpha-\epsilon_W)\Big]\sin\beta_S \\
-\ell_{Z_T}&\Big[L_T\sin(\alpha-\epsilon_T)-D_T\cos(\alpha-\epsilon_T)\Big]\sin\beta_S \\
-\ell_{Z_{VT}}&\Big[-D_{VT}\cos(\alpha-\epsilon_{VT})\sin\beta_S-L_{VT}\cos\beta_S\Big] \\
-\ell_{Z_{TR}}&\Big\{\Big[Y_{TR}\sin(\alpha-\epsilon_{TR})-D_{TR}\cos(\alpha-\epsilon_{TR})\Big]\sin\beta_S+T_{TR}\cos\beta_S\Big\}-\sum_{i=1}^{n}\ell_{Z_{P_i}}Y_{P_i} \\
-\ell_{Y_F}&\Big[D_F\sin(\alpha-\epsilon_F)+(L_F\cos A_{1_F}-Y_F\sin A_{1_F})\cos(\alpha-\epsilon_F)\Big] \\
-\ell_{Y_R}&\Big[D_R\sin(\alpha-\epsilon_R)+(L_R\cos A_{1_R}+Y_R\sin A_{1_R})\cos(\alpha-\epsilon_R)\Big] \\
-\ell_{Y_W}&\Big[L_W\cos(\alpha-\epsilon_W)+D_W\sin(\alpha-\epsilon_W)\Big] \\
-\ell_{Y_T}&\Big[L_T\cos(\alpha-\epsilon_T)+D_T\sin(\alpha-\epsilon_T)\Big] \\
-\ell_{Y_{VT}}&\Big[D_{VT}\sin(\alpha-\epsilon_{VT})\Big] \\
-\ell_{Y_{TR}}&\Big[Y_{TR}\cos(\alpha-\epsilon_{TR})+D_{TR}\sin(\alpha-\epsilon_{TR})\Big] \\
-\sum_{i=1}^{n}&\Big[\ell_{Y_{P_i}}(N_{P_i}\cos i_{P_i}+T_{P_i}\sin i_{P_i})\Big] \\
+\mathcal{L}_{FUS}&+\mathcal{L}_{HUB_F}-\mathcal{L}_{HUB_R}+\sum_{i=1}^{n}Q_{P_i}
\end{aligned}
$$

Regrouping terms of equation (42), the final aerodynamic rolling moment equation, $\mathcal{L}_A$, about body X-axis becomes

$$
\begin{aligned}
\mathcal{L}_A = & -\ell_{Z_F}\Big[-D_F\cos(\alpha-\epsilon_F)\sin\beta_S + (L_F\sin A_{I_F} + Y_F\cos A_{I_F})\cos\beta_S \\
& \quad + (L_F\cos A_{I_F} - Y_F\sin A_{I_F})\sin(\alpha-\epsilon_F)\sin\beta_S\Big] \\
& -\ell_{Y_F}\Big[D_F\sin(\alpha-\epsilon_F) + (L_F\cos A_{I_F} - Y_F\sin A_{I_F})\cos(\alpha-\epsilon_F)\Big] \\
& -\ell_{Z_R}\Big[-D_R\cos(\alpha-\epsilon_R)\sin\beta_S + (L_R\sin A_{I_R} - Y_R\cos A_{I_R})\cos\beta_S \\
& \quad + (L_R\cos A_{I_R} + Y_R\sin A_{I_R})\sin(\alpha-\epsilon_R)\sin\beta_S\Big] \\
& -\ell_{Y_R}\Big[D_R\sin(\alpha-\epsilon_R) + (L_R\cos A_{I_R} + Y_R\sin A_{I_R})\cos(\alpha-\epsilon_R)\Big] \\
& -\ell_{Z_W}\Big[L_W\sin(\alpha-\epsilon_W) - D_W\cos(\alpha-\epsilon_W)\Big]\sin\beta_S \\
& -\ell_{Y_W}\Big[L_W\cos(\alpha-\epsilon_W) + D_W\sin(\alpha-\epsilon_W)\Big] \\
& -\ell_{Z_T}\Big[L_T\sin(\alpha-\epsilon_T) - D_T\cos(\alpha-\epsilon_T)\Big]\sin\beta_S \\
& -\ell_{Y_T}\Big[L_T\cos(\alpha-\epsilon_T) + D_T\sin(\alpha-\epsilon_T)\Big] \\
& -\ell_{Z_{VT}}\Big[-D_{VT}\cos(\alpha-\epsilon_{VT})\sin\beta_S - L_{VT}\cos\beta_S\Big] \\
& -\ell_{Y_{VT}}\Big[D_{VT}\sin(\alpha-\epsilon_{VT})\Big] \\
& -\ell_{Z_{TR}}\Big\{\Big[Y_{TR}\sin(\alpha-\epsilon_{TR}) - D_{TR}\cos(\alpha-\epsilon_{TR})\Big]\sin\beta_S + T_{TR}\cos\beta_S\Big\} \\
& -\ell_{Y_{TR}}\Big[Y_{TR}\cos(\alpha-\epsilon_{TR}) + D_{TR}\sin(\alpha-\epsilon_{TR})\Big] \\
& -\sum_{i=1}^{n}\Big[\ell_{Z_{P_i}} Y_{P_i} + \ell_{Y_{P_i}}(N_{P_i}\cos i_{P_i} + T_{P_i}\sin i_{P_i})\Big] \\
& +\mathcal{L}_{FUS} + \mathcal{L}_{HUB_F} - \mathcal{L}_{HUB_R} + \sum_{i=1}^{n} Q_{P_i}
\end{aligned}
$$

Similarly, summing all the pitching moments due to aerodynamic forces and torques, the final aerodynamic pitching moment equation, M_A , about body Y-axis becomes

$$
\begin{aligned}
M_A = &+ \ell_{X_F}\left[D_F \sin(\alpha-\epsilon_F) + (L_F \cos A_{1_F} - Y_F \sin A_{1_F}) \cos(\alpha-\epsilon_F)\right] \\
&- \ell_{Z_F}\Big[D_F \cos(\alpha-\epsilon_F)\cos\beta_S + (L_F \sin A_{1_F} + Y_F \cos A_{1_F}) \sin\beta_S \\
&\qquad - (L_F \cos A_{1_F} - Y_F \sin A_{1_F}) \sin(\alpha-\epsilon_F)\cos\beta_S\Big] \\
&+ \ell_{X_R}\left[D_R \sin(\alpha-\epsilon_R) + (L_R \cos A_{1_R} + Y_R \sin A_{1_R}) \cos(\alpha-\epsilon_R)\right] \\
&- \ell_{Z_R}\Big[D_R \cos(\alpha-\epsilon_R)\cos\beta_S + (L_R \sin A_{1_R} - Y_R \cos A_{1_R}) \sin\beta_S \\
&\qquad - (L_R \cos A_{1_R} + Y_R \sin A_{1_R}) \sin(\alpha-\epsilon_R)\cos\beta_S\Big] \\
&+ \ell_{X_W}\left[L_W \cos(\alpha-\epsilon_W) + D_W \sin(\alpha-\epsilon_W)\right] \\
&- \ell_{Z_W}\left[D_W \cos(\alpha-\epsilon_W) - L_W \sin(\alpha-\epsilon_W)\right]\cos\beta_S \\
&+ \ell_{X_T}\left[L_T \cos(\alpha-\epsilon_T) + D_T \sin(\alpha-\epsilon_T)\right] \\
&- \ell_{Z_T}\left[D_T \cos(\alpha-\epsilon_T) - L_T \sin(\alpha-\epsilon_T)\right]\cos\beta_S \\
&+ \ell_{X_{VT}}\left[D_{VT} \sin(\alpha-\epsilon_{VT})\right] \\
&- \ell_{Z_{VT}}\left[D_{VT} \cos(\alpha-\epsilon_{VT})\cos\beta_S - L_{VT}\sin\beta_S\right] \\
&+ \ell_{X_{TR}}\left[Y_{TR} \cos(\alpha-\epsilon_{TR}) + D_{TR} \sin(\alpha-\epsilon_{TR})\right] \\
&- \ell_{Z_{TR}}\left\{\left[D_{TR} \cos(\alpha-\epsilon_{TR}) - Y_{TR} \sin(\alpha-\epsilon_{TR})\right]\cos\beta_S + T_{TR}\sin\beta_S\right\} \\
&+ \sum_{i=1}^{n}\left[\ell_{X_{P_i}}(N_{P_i}\cos i_{P_i} + T_{P_i}\sin i_{P_i}) - \ell_{Z_{P_i}}(N_{P_i}\sin i_{P_i} - T_{P_i}\cos i_{P_i})\right] \\
&+ M_{FUS} + M_{HUB_F} + M_{HUB_R} + Q_{TR}
\end{aligned}
$$

Finally, the aerodynamic yawing moment equation, N_A, about body Z-axis can be expressed as follows:

$$
\begin{aligned}
N_A = -\ell_{X_F}\Big\{ & \Big[D_F\cos(\alpha-\epsilon_F)-(L_F\cos A_{I_F}-Y_F\sin A_{I_F})\sin(\alpha-\epsilon_F)\Big]\sin\beta_S \\
& -(L_F\sin A_{I_F}+Y_F\cos A_{I_F})\cos\beta_S\Big\} \\
-\ell_{Y_F}\Big\{ & \Big[-D_F\cos(\alpha-\epsilon_F)+(L_F\cos A_{I_F}-Y_F\sin A_{I_F})\sin(\alpha-\epsilon_F)\Big]\cos\beta_S \\
& -(L_F\sin A_{I_F}+Y_F\cos A_{I_F})\sin\beta_S\Big\} \\
-\ell_{X_R}\Big\{ & \Big[D_R\cos(\alpha-\epsilon_R)-(L_R\cos A_{I_R}+Y_R\sin A_{I_R})\sin(\alpha-\epsilon_R)\Big]\sin\beta_S \\
& -(L_R\sin A_{I_R}-Y_R\cos A_{I_R})\cos\beta_S\Big\} \\
-\ell_{Y_R}\Big\{ & \Big[-D_R\cos(\alpha-\epsilon_R)+(L_R\cos A_{I_R}+Y_R\sin A_{I_R})\sin(\alpha-\epsilon_R)\Big]\cos\beta_S \\
& -(L_R\sin A_{I_R}-Y_R\cos A_{I_R})\sin\beta_S\Big\} \\
& -\ell_{X_W}\Big[D_W\cos(\alpha-\epsilon_W)-L_W\sin(\alpha-\epsilon_W)\Big]\sin\beta_S \\
& -\ell_{Y_W}\Big[L_W\sin(\alpha-\epsilon_W)-D_W\cos(\alpha-\epsilon_W)\Big]\cos\beta_S \\
& -\ell_{X_T}\Big[D_T\cos(\alpha-\epsilon_T)-L_T\sin(\alpha-\epsilon_T)\Big]\sin\beta_S \\
& -\ell_{Y_T}\Big[L_T\sin(\alpha-\epsilon_T)-D_T\cos(\alpha-\epsilon_T)\Big]\cos\beta_S \\
& -\ell_{X_{VT}}\Big[D_{VT}\cos(\alpha-\epsilon_{VT})\sin\beta_S+L_{VT}\cos\beta_S\Big] \\
& -\ell_{Y_{VT}}\Big[-D_{VT}\cos(\alpha-\epsilon_{VT})\cos\beta_S+L_{VT}\sin\beta_S\Big] \\
& -\ell_{X_{TR}}\Big\{\Big[D_{TR}\cos(\alpha-\epsilon_{TR})-Y_{TR}\sin(\alpha-\epsilon_{TR})\Big]\sin\beta_S-T_{TR}\cos\beta_S\Big\} \\
& +\ell_{Y_{TR}}\Big\{\Big[Y_{TR}\sin(\alpha-\epsilon_{TR})-D_{TR}\cos(\alpha-\epsilon_{TR})\Big]\cos\beta_S-T_{TR}\sin\beta_S\Big\} \\
& +\sum_{i=1}^{n}\Big[\ell_{X_{P_i}}Y_{P_i}-\ell_{Y_{P_i}}(T_{P_i}\cos i_{P_i}-N_{P_i}\sin i_{P_i})\Big]+N_{FUS}+Q_F-Q_R
\end{aligned}
$$

11.1.3 The Final Equations of Motion

The final equations of motion can now be written as summations of all aerodynamic, gravitational and inertia forces and moments along and about the body axes as follows:

$$X = X_{A+G} - X_I = 0 \tag{46}$$

$$Y = Y_{A+G} - Y_I = 0 \tag{47}$$

$$Z = Z_{A+G} - Z_I = 0 \tag{48}$$

$$\mathcal{L} = \mathcal{L}_A - \mathcal{L}_I = 0 \tag{49}$$

$$M = M_A - M_I = 0 \tag{50}$$

$$N = N_A - N_I = 0 \tag{51}$$

Substituting the expressions for aerodynamic, gravitational, and inertia forces and moments presented above into equations (46) through (51) yields the final generalized equations of motion of completely arbitrary aircraft geometry. These equations derived for the aircraft body system of axes are presented in Section 4.0.

REFERENCE

1. Etkin, B., Dynamics of Flight, New York, John Wiley & Sons, Inc., London, Chapman & Hall, Ltd., 1959.

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Analytical methods are presented for determining the dynamic stability and control characteristics of generalized procedures which are considerably simplified through the extensive use of information presented in graphs and charts. These charts are applicable to flight conditions from hover to high forward speeds.

The charts for low forward speeds (advance ratios, $\mu \leq 0.2$) were obtained from the rotor performance data based on classical rotor theory. However, the high-speed charts ($\mu \geq 0.3$) exclude the major assumptions of classical theory and include blade compressibility, stall, reverse flow, large inflow ratios, etc.

This handbook contains information suitable for extensive digital and analog computer studies and also provides rapid procedures for predicting helicopter stability and control characteristics for preliminary design applications.

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